ELECTRICAL ENGINEERING TEXTS

HARRY E. CLIFFORD, Consulting Editor (deceased)

A COURSE IN
ELECTRICAL ENGINEERING

VOLUME II

ALTERNATING CURRENTS

ELECTRICAL ENGINEERING TEXTS

HARRY E. CLIFFORD, Consulting Editor (deceased)

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ELECTRICAL ENGINEERING TEXTS

A COURSE IN ELECTRICAL ENGINEERING

VOLUME II

ALTERNATING CURRENTS

BY

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FOURTH EDITION

NEW YORK AND LONDON

McGRAW-HILL BOOK COMPANY, INC.

1947

A COURSE IN ELECTRICAL ENGINEERING

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE TO THE FOURTH EDITION

Since the last revision of this volume, in 1934, there have been many important developments in the field of electrical engineering, a great number being stimulated by war. In addition, the great importance of science in our present-day life and economy has increased greatly the number of those studying in the field of electrical engineering. These factors not only have made essential a broader foundation in the education of the electrical engineer, but also have made it necessary to raise the level of the electrical-engineering curriculum. In the present revision the author has attempted to meet these requirements by expanding the fundamental material and by showing in more detail its application to the study of electrical machinery and apparatus.

For example, the scope of the properties of alternating-current circuits has been expanded to include further developments in series and parallel resonance, harmonics, power as related to circuit parameters, and conjugate method of computing watts and vars; and the application of subscript notation and complex operators to polyphase circuits has been extended further than in prior editions.

In the field of instruments, there have been added the thermal and rectifier types of voltmeter and ammeter and the cathode-ray oscilloscope, all of which are now in common use. Also, late improvements in instrument design have been described.

The illustrations showing the design and construction of apparatus such as alternators, transformers, induction motors, and rectifiers are representative of modern practice. For example, in the chapter on Transformers the development of cold-rolled high-reduction transformer steel and its effect in making a radical change in the construction of transformer cores from the method of using flat iron punchings to that of using rolled and bent iron are discussed in some detail.

In the analysis of alternator testing and operation there are now included single-phase pulsating armature reaction, the Potier method, and its development into the American Standards Association method for determining alternator regulation.

The operation of the synchronous motor is studied in greater detail, and in view of the wide use of selsyns, and the application of

synchronous motors to electric ship propulsion, these also have been added. The large number of long, very high-voltage transmission lines, such as those at Boulder Dam, has led to the inclusion of line calculations that take into consideration their distributed capacitance. Likewise, the rapid expansion of the low-voltage three-phase networks has prompted the addition of these in the text.

In the chapter on Electron Tubes, the discussion of the theory of emission and vacuum-tube operation has been expanded, together with the measurements of amplification and the dynamic characteristics of tubes. The methods of measuring transconductance, amplification, and plate resistance have been given in greater detail, particularly in the matter of dynamic measurements. Frequency modulation and frequency-modulation detection have been added. New developments in rectifier practice, such as the selenium rectifier, the ignitron, and the electronic control of motors have also been added to the chapter on Rectifiers.

All the problems are entirely new and, as in former editions, they follow closely the analyses developed in the text.

The author is indebted to so many who have made helpful suggestions, or who have assisted him in other ways in this revision, that he can hope to include the names of only a limited number.

Chapter XIV on Electron Tubes was written by R. F. Field and A. G. Bousquet, both of the General Radio Company of Cambridge, Mass.

The author has been helped by R. T. Gibbs, Dr. E. C. Easton, and John P. Newton of the Graduate School of Engineering at Harvard University, and by A. L. Russell of the Franklin Technical Institute of Boston, who contributed to the preparation and solution of the problems.

The author is indebted also to the Department of Chemistry and Electricity of the United States Military Academy at West Point, formerly in charge of Col. C. L. Fenton and now in charge of Col. B. W. Bartlett. The members of the instructing staff have contributed much useful material and have reviewed parts of the manuscript during its preparation. The names of Lt. Col. R. I. Heinlein, Jr., Associate Professor of Electricity, Lt. Col. C. R. Nichols, Assistant Professor of Electricity, and Lt. Col. L. E. Johnson, Assistant Professor of Electricity, should be mentioned particularly.

Helpful suggestions were received from Professor C. V. O. Terwilliger of the United States Naval Academy and from Comdr. W. E. Creeden and Lt. E. P. Rivard of the United States Coast Guard Academy.

The author is indebted to Professor R. W. Ahlquist of Iowa State College, Professor Harry Baum of the College of the City of New York, Professor Paul C. Cromwell of New York University, Professor Segismundo Gerszonowicz of the University of Montevideo, Professor J. Hugo Johnson of the University of Idaho, Dean J. H. Lampe of North Carolina State College, Professor T. C. Seidell of the Georgia School of Technology, Professor F. N. Tompkins of Brown University, and Professor Gordon F. Tracy of the University of Wisconsin.

Also, the assistance of the several manufacturers who contributed data and illustrations is acknowledged.

The author must express his deep appreciation and gratitude to Professor H. E. Clifford, Consulting Editor, formerly Dean of the Graduate School of Engineering, Harvard University, for his valuable collaboration and assistance throughout the preparation of this revision.

CHESTER L. DAWES.

CAMBRIDGE, MASS.,
December, 1946.

PREFACE TO THE FIRST EDITION

This volume is intended for those who have such a knowledge of direct currents as is given by Volume I. It presupposes no knowledge of alternating currents. The first two chapters are devoted to the development of the fundamental laws of alternating currents and alternating-current circuits. Subsequent chapters consider the application of these fundamental laws to alternating-current measurements, to polyphase circuits, to alternating-current machinery, and to power transmission. A chapter on illumination and photometry has been included, as a brief discussion of the underlying principles of light and of light measurements is important in a general course in electrical engineering.

The development of the various alternating-current formulas and of the operation of various types of machinery, transmission lines, etc., are based on the fundamental laws of electricity and magnetism as set forth in Volume I. Mathematical developments are occasionally introduced, as supplementary to the descriptive matter. As in Volume I, numerous illustrative problems and methods of making laboratory tests are given throughout the text.

This volume is intended to be elementary in character and to act as a stepping stone to the more advanced texts of this series. In many cases rigorous and detailed analysis is not given, particularly in the chapter on alternating-current measurements and in the discussion of certain types of alternating-current apparatus. A thorough analysis of these subjects is found in "Electrical Measurements" by F. A. Laws, and "Principles of Alternating-current Machinery" by R. R. Lawrence, both of which volumes are included in this series of Electrical Engineering Texts.

The author is indebted to various manufacturing companies for their cooperation in supplying material and illustrations for the text; to Professor R. R. Lawrence of the Massachusetts Institute of Technology, for his careful review of the manuscript and his many helpful suggestions given during its preparation; and particularly to Professor H. E. Clifford of The Harvard Engineering School, for his helpful advice during the preparation of the manuscript and for the thorough manner in which he has edited the material contained in this volume.

Chester L. Dawes.

Cambridge, Mass.,
January, 1922.

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A COURSE IN
ELECTRICAL ENGINEERING

VOLUME II
ALTERNATING CURRENTS

CHAPTER I

ALTERNATING CURRENT AND VOLTAGE

1. General Field of Use of Alternating Current.—At the present time over 90 per cent of the electrical energy used for commercial purposes is generated as alternating current. This is not due primarily to any superiority of alternating over direct current so far as applicability to industrial and domestic uses is concerned. In fact, there are many instances where direct current is absolutely necessary for industrial purposes, such as municipal traction, electrolytic processes, and certain types of arc lamps; also, direct-current motors are superior for elevators, printing presses, and many variable-speed drives. However, for these various purposes the energy is generated and transmitted almost always as alternating current and then converted to direct current.

Some of the reasons for generating electrical energy as alternating current are the following:

Alternating current can be generated at comparatively high voltages, and these voltages can be raised and lowered readily by means of static transformers. This permits the economical transmission of alternating-current energy over considerable distances by using high transmission voltages, since the weight of transmission conductor varies inversely as the *square* of the transmission voltage, when the power, distance, and loss are fixed (Vol. I, Chap. XV), and high transmission voltages can be reduced efficiently at the receiving end of the transmission line. So far (1946) no practical method has been devised for raising and lowering direct-current voltage involving large amounts of power. Rotating commutators can be used to raise and lower the voltage, but both voltage and power are limited.[1] There are experi-

[1] ALEXANDERSON, E. F. W. and E. L. PHILLIPI, "Electronic Power Converters, Their History and Development," *Gen. Elec. Rev.*, September, 1944, p. 41.

mental lines in which high-voltage alternating current is rectified by vapor-type electronic rectifiers for transmission as direct current. At the receiving end of the transmission line the direct-current power is inverted by electronic inverters to alternating-current power for commercial uses (see Chap. XV). However, this system thus far has not been applied on a large scale.

It is possible to build alternating-current generators in large units to run at high speeds so that the construction and operating costs per kilowatt are low, and such generators are admirably adapted to high-speed turbine drive. The largest alternators operating today (1946) have a rating of 200,000 kva.[1] Owing to commutation difficulties, direct-current generators cannot be designed in large units, particularly for high speeds. At 1,000 rpm, it is difficult to design a direct-current generator having a rating of even 1,000 kw. On the other hand, alternators with ratings as high as 81,250 kva now (1946) operate at 3,600 rpm, and a 100,000-kva 0.85-power-factor 3,600-rpm unit is under construction.

For constant-speed work, the alternating-current induction motor is more efficient than the direct-current motor and is less in first cost and in maintenance, owing in part to the fact that the induction motor has no commutator. It is occasionally desirable, therefore, to generate power as alternating current in order to be able to use induction motors.

The high transmission efficiencies obtainable with alternating current make it economical to generate electrical energy in large quantities in a single station and to distribute it over a large territory. The large boilers, automatic stokers, superheaters, recording instruments, etc., that are possible in large stations result in high boiler-room efficiency. Large turbines have an economy which may be three or four times as good as that of the steam units in a small plant. The alternating-current generator in the larger sizes has an efficiency of 96 to 98.5 per cent (see pp. 231, 232). Then, again, as the boilers and large turbine units require few attendants per kilowatt, the labor and supervision charges per kilowatt-hour are small.

For these reasons, it is often more economical to generate electrical energy with large units, to transmit it long distances, and even to convert it into direct current rather than to generate direct current at the place where it is to be utilized.

[1] There are two 200,000-kva 0.8-power-factor 4-pole 60-cycle 1,800-rpm 16,500-volt single-shaft turbine alternators in operation at the Hudson Avenue Station of the Consolidated Edison Company, New York. They were manufactured by the General Electric Company.

It must be remembered, however, that the reduced generating costs are balanced in part at least by distribution costs resulting from investment charges in lines, cables, substations, machinery, etc., in addition to the labor and maintenance costs of the distribution system.

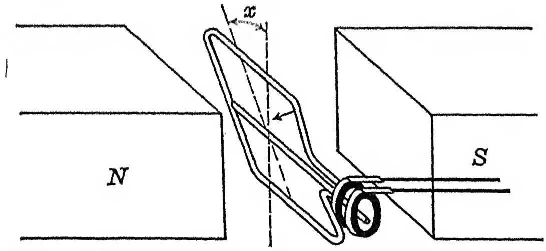


FIG. 1.—Coil rotating in uniform field.

Alternating current owes its importance to the fact that it can be generated economically with large units; its voltage can be readily raised and lowered, so that energy can be transmitted economically for considerable distances. Alternating-current motors for constant-speed work are usually preferable to direct-current motors.

2. Sine Waves.—It is shown in Vol. I, Chap. XI, that when a single coil rotates at constant speed in a uniform field, Fig. 1, an alternating

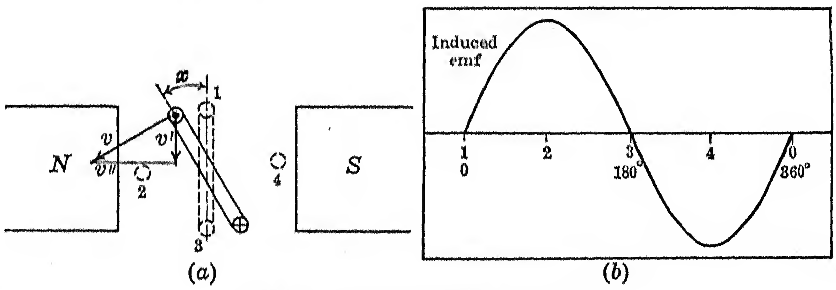


FIG. 2.—Coil inducing sine-wave emf.

emf is generated. The successive values of the emf may be represented by a smooth curve called a sine wave, Fig. 2(*b*), since the values of the emf are proportional to the sine of the angle x that the coil makes with a plane through its axis and perpendicular to the direction of the magnetic field, Fig. 1. This may be shown as follows:

The emf induced in a single conductor that cuts a magnetic field (Vol. I, Chap. XI) is given by

$$e = Blv \cdot 10^{-8} \qquad \text{volts,} \tag{1}$$

where B, l, v are mutually perpendicular. However, when the conductor is in the position with respect to the flux shown in Fig.

2(a), the velocity v is not perpendicular to the direction of the flux. It may be resolved, however, into two components, v'' parallel to the direction of the flux and v' perpendicular to this direction. Since the component v'' is parallel to the direction of the flux, it cannot cause an induced emf. The component $v' = v \sin x$, being perpendicular to the flux, does produce an emf. Hence, from (1), the induced emf is given by

$$e = Blv \sin x \cdot 10^{-8} \qquad \text{volts,} \tag{2}$$

where x is the angle through which the conductor has moved from position 1. Thus the emf induced in such a conductor may be represented by a sine wave. When the top of the coil, Fig. 2(a), is at

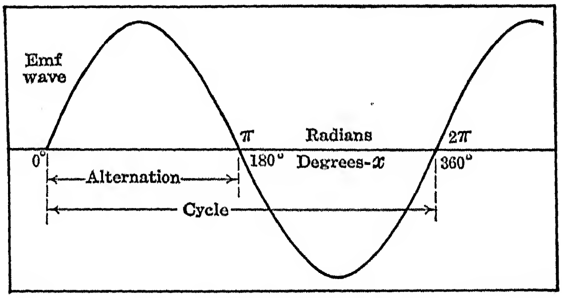


FIG. 3.—Sine-wave induced emf.

position 1, the emf is zero; when at position 2, the emf is a positive maximum; when at position 3, the emf is zero; when at position 4, the emf is a negative maximum, Fig. 2(b). When a periodic wave, such as a sine wave, has gone through one complete set of positive or of negative values, Figs. 2(b) and 3, it is said to have completed an alternation, Fig. 3. If it has gone through one complete set of positive and one complete set of negative values, it is said to have completed a cycle.

The emf waves of some commercial alternators, particularly the older ones, may differ materially from a sine wave, but with most commercial alternators the emf wave is sufficiently near to a sine wave to warrant its being treated as such. (For wave shapes in alternators, see Sec. 115, p. 179.)

Alternating-current theory and analysis are based on sine (or cosine) waves of voltage, current, and power. This is due to the fact that the sine and cosine functions are simple and accordingly are readily expressed mathematically. Also, sine and cosine waves of voltage and current are the only types of waves that can pass through all types of linear circuits (that is, circuits whose parameters such as resistance and inductance do not change) without distortion.

If a periodic wave is not a sine wave, it may be resolved into a series of sine waves of fundamental and higher frequencies. Each one of these sinusoidal components, or *harmonics*, then may be treated as a sine wave at its particular frequency (see Sec. 40, p. 67). Unless otherwise specified, the methods of analysis and the equations that follow apply to sine waves of voltage and current.

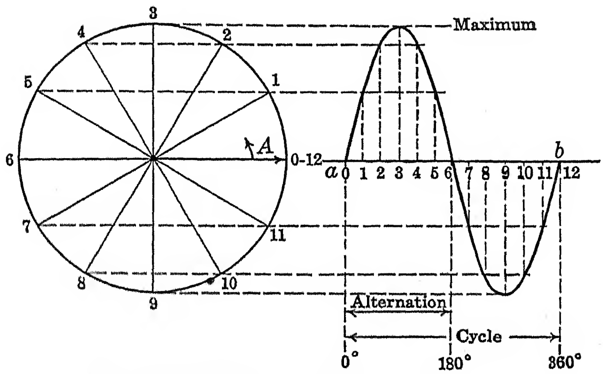


FIG. 4.—Graphical construction of sine wave.

The sine wave may be produced graphically as follows: Draw a circle, Fig. 4, whose radius A is equal to the maximum value of the sine wave. Divide the circumference of this circle into any number of equal parts, in this case 12, and number them 1, 2, . . . 12. Draw a horizontal line ab that, if extended, would pass through the center of the circle. Divide ab into the same number of equal parts as there are

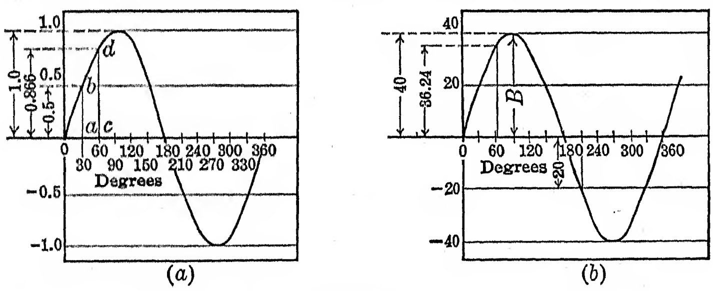


FIG. 5.—Numerical values of ordinates of sine waves for definite angles.

on the circumference of the circle, and give the points corresponding numbers. Erect a perpendicular ordinate at each point. Project the points on the circle horizontally to intersect perpendiculars having corresponding numbers. A smooth curve drawn through the intersections will be a sine wave.

The sine wave may also be plotted from a table of sines (Appendix E, p. 606). Mark a horizontal axis, Fig. 5(a), in degrees. At

each point erect an ordinate equal to the sine of the corresponding angle. Thus, at 30° the ordinate ab is 0.5; at 60° the ordinate cd is 0.866; at 90° it is 1.0; etc. The wave passes through zero at 180°, because the sine of 180° is zero. When the angle becomes greater than 180°, the sine becomes negative and the wave falls below the line, as the sine is negative between 180° and 360° (see p. 604). The above is equivalent to plotting the sine of the angle x, Fig. 2(a).

If the wave has a maximum value B, Fig. 5(b), the value of the ordinate at any point may be found by multiplying B into the sine of the corresponding angle. That is,

$$y = B \sin x, \tag{3}$$

where x is expressed in degrees.

Example.—Find the ordinates of a sine wave at points corresponding to 65° and 210°, the maximum ordinate being 40 units, Fig. 5(b).

From p. 607, $\sin 65° = 0.906$.

$$40 \cdot 0.906 = 36.24. \quad \textit{Ans.}$$
$$\sin 210° = -(\sin 210° - 180°) = -\sin 30° = -0.5 \ [(31), \text{p. } 604].$$
$$40 \cdot (-0.5) = -20. \quad \textit{Ans.}$$

These values are shown in Fig. 5(b).

3. Cycle; Frequency.—When the conductor has completed 1 revolution, Fig. 2(a), it has gone through an angle of 360° or 2π radians. The emf wave then has gone through an angle of 360°, Fig. 2(b), or 2π radians, Fig. 3. If the speed in revolutions per second (rps) is s, the frequency of the emf wave in cycles per second f is equal to s, since for each revolution the emf induced in the conductor goes through one complete set of positive and one complete set of negative values. If the conductor has been rotating for a time t sec from position 1, it will have gone through st revolutions, or ft cycles. Hence,

$$x = 2\pi st = 2\pi ft \text{ radians, or } 360ft \text{ deg.} \tag{I}$$

Since at constant speed or frequency, $2\pi f$ or $360f$ is constant, alternating-current waves may be plotted with time as abscissas as well as with radians or degrees.

If the angular velocity is ω (in radians per second), then from (I),

$$\omega = 2\pi f \quad \text{radians per sec,} \tag{II}$$

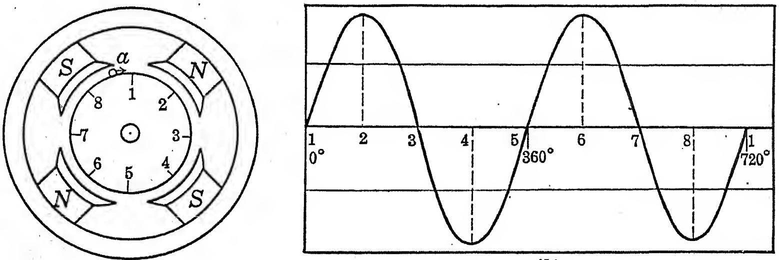
or

$$360f \quad \text{deg per sec} \tag{III}$$

since $2\pi f$ is the radians per second through which the wave goes and $360f$ is the degrees per second through which the wave goes.

If the alternator is a multipolar machine, for example, 4 poles,

Fig. 6(*a*), as soon as the conductor *a* has passed a north and a south pole, that is, has gone from 1 to 5, the emf wave has completed 1 cycle, or 360 electrical time degrees. Thus a cycle is completed every time the conductor passes one *pair* of poles. Therefore the frequency in



(*a*) (*b*)

FIG. 6.—Two cycles per revolution in 4-pole alternator.

cycles per second is equal to the number of *pairs* of poles passed per second. That is,

$$f = \frac{P}{2} \cdot s, \qquad \text{or} \qquad f = \frac{PS}{120} \quad \text{cycles per sec} \qquad (4)$$

where s = rps, S = revolutions per minute (rpm), and P = number of poles. Thus if the speed of a 2-pole alternator is 60 rps, or 3,600 rpm, the frequency is 60 cycles per sec.

The following table shows the relation of speed, frequency, and number of poles for a few typical cases.

Poles	Speed, rpm	
	60 cycles	25 cycles
2	3,600	1,500
4	1,800	750
6	1,200	500
8	900	375
40	180	75

Example.—A 60-cycle engine-driven alternator has a speed of 120 rpm. How many poles has it?

Using (4) and solving for P,

$$P = \frac{120f}{S} = \frac{120 \cdot 60}{120} = 60 \text{ poles.} \quad \textit{Ans.}$$

This example may be solved also without using (4) directly. A 2-pole 60-cycle alternator rotates at 3,600 rpm. Therefore the alternator must have

$$\frac{3{,}600}{120} 2 = 60 \text{ poles.} \quad \textit{Ans.}$$

In practice, nearly all alternators have stationary armatures and rotating fields, and the above relations apply.

When the conductor a has gone from 1 to 5, Fig. 6(a), that is, through 180 space degrees, the emf has gone through 360 electrical degrees, Fig. 6(b). When the coil has completed 1 revolution, it has gone through 360 space degrees and the emf has gone through 720 electrical degrees, Fig. 6(b). With a 4-pole machine 1 space degree equals 2 electrical degrees. With 6 poles, 1 space degree equals 3 electrical degrees, etc.

4. Commercial Frequencies.—In the United States, frequencies are standardized at 60 cycles and at 25 cycles per sec, although other frequencies are used. In California, for example, and also in Mexico, 50 cycles is used on some of the large transmission systems. In the early days of alternating-current development 133 cycles was common, but few if any plants now generate at this frequency. The principal advantage of higher frequencies is that transformers require less iron and copper and so are lighter and cheaper. The flicker of lamps is not perceptible at 60 cycles, but at 25 cycles it is evident. On the other hand, the voltage drop in transmission lines and in apparatus varies almost directly as the frequency, so that better voltage regulation throughout the system is obtained with low frequency. Power apparatus, such as induction motors, synchronous converters, and alternating-current commutator motors, operates better at low frequencies. With one or two exceptions, however, the operation is satisfactory at 60 cycles per sec. A power and lighting company would operate ordinarily at 60 cycles per sec, because the flicker of lamps at 25 cycles per sec is objectionable and the transformers at this lower frequency are heavier and more costly than they are at the higher frequency. On the other hand, an electric utility generating strictly for power purposes may use 25 cycles. This frequency is used by the New York, New Haven & Hartford Railroad for its electric locomotives; by the Norfolk and Western Railway for operating electric locomotives; and by the Boston Elevated Railway Company for transmitting high-voltage power to its direct-current substations. In Europe, frequencies as low as $16\frac{2}{3}$, 15, and even $12\frac{1}{2}$ cycles per sec are common.

5. Equation of Sine Wave of Current.—If $2\pi\, ft$ [Eq. (I), p. 6] is substituted in Eq. (3) or if $\omega = 2\pi f$ [Eq. (II)] is used, the equation of a sine wave of alternating current may be written

$$i = I_m \sin 2\pi ft = I_m \sin \omega t, \tag{5}$$

where i is the value of the current at any time t, I_m is the maximum value of the current, and $\omega = 2\pi f$. The quantity ω is equal to 2π times

the frequency f and is the *angular velocity* in radians per second of the rotating vector that may be used to construct the sine wave (Appendix, p. 601).

For example, if the vector A, Fig. 4, be considered as rotating in counterclockwise direction and taking successive positions 1, 2, 3. it will produce 1 cycle for each revolution. In each revolution, it goes an angular distance of 2π radians. If it rotates 60 times a second, its angular velocity is $2\pi 60$, or 377, radians per sec. The sine wave produced from this rotating vector has a frequency of 60 cycles per sec. Hence, for a 60-cycle wave, $\omega = 377$. For a 25-cycle wave, $\omega = 2\pi 25$, or 157, radians per sec.

Similarly the equation of a sine wave of emf will be given by

$$e = E_m \sin \omega t \text{ [see (6a), p. 16].} \tag{6}$$

Example.—What is the equation of a 25-cycle-current sine wave, having an rms value of 30 amp, and what is the value of the current when the time is 0.005 sec? Assume that the wave crosses the time axis in a positive direction when the time is equal to zero.

$$\begin{aligned} I_m &= 30\sqrt{2} = 42.4 \text{ amp.} \\ 2\pi 25 &= 157 = \omega. \\ i &= 42.4 \sin 157t. \quad \textit{Ans.} \\ i &= 42.4 \sin 157 \cdot 0.005 \\ &= 42.4 \sin 0.785 \text{ radian} \\ 2\pi &= 6.28 \text{ radians} = 360° \text{ (p. 6).} \end{aligned}$$

$(0.785/6.28) \cdot 360° = 45°$. Also, as the wave completes 360° in $\frac{1}{25}$, or 0.04 sec, in 0.005 sec, it will have completed $0.005/0.040 = \frac{1}{8}$ cycle.

$$\frac{360°}{8} = 45° \text{ (\textit{check}).}$$

$$i' = 42.4 \sin 45° = 42.4 \cdot 0.707 = 30 \text{ amp.} \quad \textit{Ans.}$$

6. Alternating-current Ampere.—Figure 7(a) shows an alternating-current sine wave, having a maximum value of 1.414 amp. At first thought it might seem that the value in amperes of such a wave should be based on the *average* value. If the wave is considered over one complete cycle, the average value is zero, as there is just as much negative as positive current. A direct-current ammeter, if connected to measure this current, would indicate zero, as such an instrument measures *average* values.

The value of an alternating current is based not on its average value but on its *heating* effect and may be defined as follows:

An alternating-current ampere is that current which, flowing through a given ohmic resistance, will produce heat at the same rate as a direct-current ampere.

Assume that a resistance unit is immersed in a calorimeter and that when a direct-current ampere is sent through this resistance the temperature of the water is raised 20° in 10 min. An alternating-current ampere, if sent through this same resistance unit, will raise the temperature of the water by the same amount in the same time, other conditions such as radiation, for example, being the same; that is, both currents produce heat at the same rate.

The heating effect varies as the *square* of the current, that is, at any instant it is proportional to i^2R.

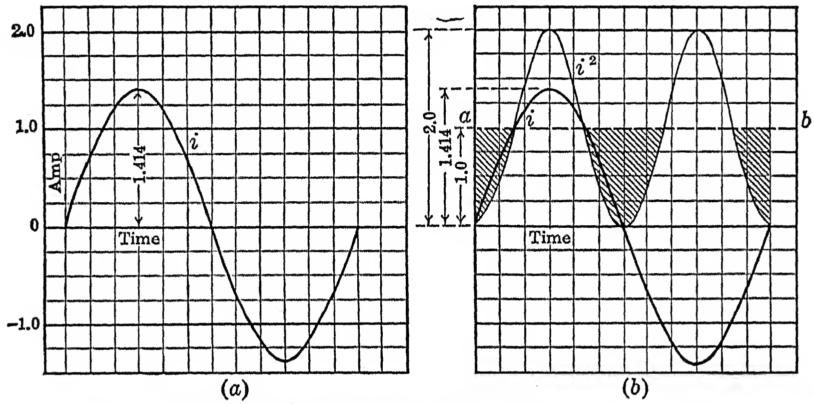


FIG. 7.—Maximum and rms values of sine-wave alternating current.

Figure 7(b) shows the current wave of Fig. 7(a), together with its squared values. That is, each ordinate of the i wave is squared, and these values are plotted to give the i^2 wave shown. The maximum value of this new wave will be 2.0 ($=\overline{1.414^2}$), since the maximum value of the original current wave is 1.414, or $\sqrt{2}$. The squared wave also lies entirely above the zero axis, because the square of a negative value is positive.

This squared wave has a frequency twice that of the original wave (Sec. 7) and has its horizontal axis of symmetry ab at a distance of 1.0 unit above the zero axis, as shown in Fig. 7(b).

The ordinate of the i^2 wave, Fig. 7(b), when multiplied by the resistance gives the instantaneous power. However, in practice, the average power, rather than the instantaneous power, is usually desired. The average value of power will be equal to the *average* value of the i^2 wave multiplied by the resistance. The average value of this squared wave is 1.0 amp, as shown by the dashed line ab, because the areas above the dashed line will just fit into the shaded valleys below the dashed line. If, therefore, an equivalent rectangle were made from this wave, its height would be 1.0 unit. This value, 1.0, is the *average*

of the squares of the current wave. Average heating varies as the average of the squares of the current. The squared current represented by the dashed line, therefore, is equivalent to the square of a direct current that would produce the same heating effect as this alternating current.

Hence, to obtain in amperes the value of the current given by the wave of Fig. 7, the square root of the average square must be taken. That is, I (in amperes) $= \sqrt{1.0} = 1.0$ amp. This value of the current is called the *root-mean-square* (rms) or *effective* value of the current.

An alternating-current-ampere sine wave, which produces heat at the same rate as a direct-current ampere, has therefore a *maximum* value of 1.414 ($= \sqrt{2}$) amp. In fact, for any sine-wave current, the ratio of *maximum* to *rms* value is equal to $\sqrt{2}$, or 1.414. The ratio of rms to maximum value is $1/1.414 = 0.707$.

To obtain the rms value of *any* current wave, not necessarily a sine wave:

a. Plot a wave whose ordinates are equal to the squares of the ordinates of the given current wave.

b. Find the average value of this squared wave by obtaining the area of its loops, as with a planimeter, and dividing this area by the base.

c. Find the square root of the average in (*b*).

The same result may be obtained by erecting equidistant ordinates on the original wave. This divides the area under the wave into small areas having equal bases. The ordinates at the centers of these small areas are measured, their squares are averaged, and the square root of this average then is obtained. This will give the rms value of the wave. The rms value may also be found by integration (Sec. 7).

7. Current-squared Wave; Average Current.—Let the equation of a current wave be

$$i = I_m \sin \omega t. \tag{I}$$

Let it be required to find the equation of the current-squared wave.

$$i^2 = I_m^2 \sin^2 \omega t. \tag{II}$$

$$\cos (x + y) = \cos x \cos y - \sin x \sin y \text{ [(38), p. 605]}.$$

Letting $x = y = \omega t$,

$$\cos (\omega t + \omega t) = \cos \omega t \cos \omega t - \sin \omega t \sin \omega t,$$

$$\cos 2\omega t = \cos^2 \omega t - \sin^2 \omega t, \tag{III}$$

$$\cos^2 \omega t = 1 - \sin^2 \omega t \text{ [(34), p. 605]}. \tag{IV}$$

From (III) and (IV),

$$\cos 2\omega t = 1 - 2 \sin^2 \omega t. \tag{V}$$

Hence,

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}. \tag{VI}$$

Hence, from (II) and (VI),

$$i^2 = I_m^2 \sin^2 \omega t = I_m^2 \frac{1 - \cos 2\omega t}{2}. \tag{7}$$

This is a cosine wave having a frequency $2f$, where f is the frequency of the current given by $f = \omega/2\pi$. When $t = 0$, $i^2 = 0$; when $t = \pi/4\omega = \pi/8\pi f = 1/8f$, $i^2 = I_m^2/2$, Fig. 8. i^2 is a maximum when $2\omega t = \pi$ radians $= 180°$. The corresponding value of time,

$$t = \frac{\pi}{2\omega} = \frac{1}{4f}, \text{ Fig. 8.}$$

Under these conditions, $i^2 = I^2_m$. It follows that the axis of the current-squared wave is at a distance $I_m^2/2$ above the axis of reference.

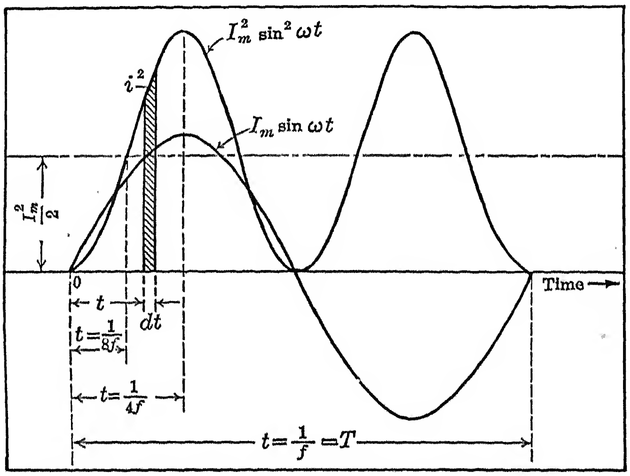


FIG. 8.—Current and current-squared sine waves.

From (7) the rms value is determined readily.

The area of the differential strip at time t, Fig. 8, is $i^2\, dt$, and the total area under the i^2 wave is $\int_0^T i^2\, dt$.

The average of the i^2 wave is its area divided by its base, the time T being chosen as the time of one cycle. That is,

$$\text{Average } i^2 = \frac{1}{T}\int_0^T i^2\, dt = \frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t\, dt. \tag{VII}$$

Substituting (VI) in (VII) and taking the square root,

$$I = \sqrt{\frac{1}{T}\int_0^T I_m^2 \frac{1 - \cos 2\omega t}{2}\, dt} = \sqrt{\frac{I_m^2}{2T}\int_0^T (1 - \cos 2\omega t)\, dt}$$

$$= \sqrt{\frac{I_m^2}{2T}\left[t - \frac{1}{2\omega}\sin 2\omega t\right]_0^T} = \sqrt{\frac{I_m^2}{2T}[(T-0) - (0-0)]}$$

since $\sin 2\omega t = 0$ when $t = 0$, and when $t = T$.

$$I = \sqrt{\frac{I_m^2}{2T}[T]} = \frac{I_m}{\sqrt{2}}. \quad \text{Q.E.D.} \tag{8}$$

It is frequently desirable to know the *average* value of a sine wave for one half-cycle. This average value has limited uses—rectifier-type instruments (p. 105), electroplating, battery charging, where the results are proportional to the number of coulombs flowing in the circuit rather than to the power, being typical applications. Under these conditions the a-c wave must be rectified (Chap. XV). The average value, which is applicable to full-sine-wave rectification, Fig. 9(*a*), is equal to $2/\pi$, or 0.637 times the maximum value. This may be proved as follows:

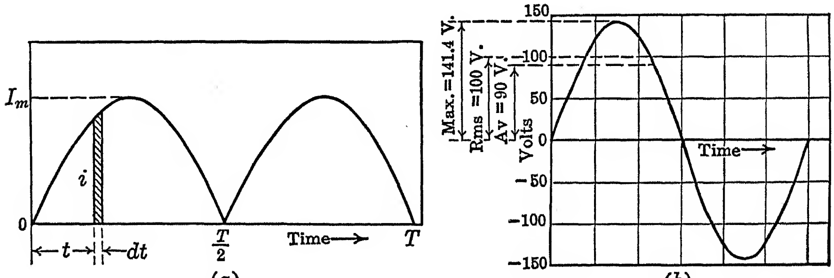


FIG. 9.—Maximum, rms, and average values of sine wave.

The equation of the first positive half-cycle of the current wave, Fig. 9(*a*), is $i = I_m \sin \omega t$ where t varies between 0 and $T/2$ and the area of a differential strip at time t is $i\,dt$. The area under the positive loop is $\int_0^{T/2} i\,dt$, and the average value is given by this area divided by the base $T/2$. Hence,

$$\begin{aligned} I_{av} &= \frac{1}{T/2}\int_0^{T/2} I_m \sin \omega t\,dt \\ &= \frac{I_m}{T/2}\left[-\frac{1}{\omega}\cos \omega t\right]_0^{T/2} = \frac{2I_m}{2\pi f T}\left[-\cos \omega\left(\frac{T}{2}\right) + \cos (0)\right] \\ &= \frac{I_m}{\pi f T}[-(-1) + (1)] = \frac{2}{\pi} I_m = 0.637 I_m. \qquad (9) \\ &\left[\cos \frac{\omega T}{2} = \cos \pi = (-1); fT = 1, \text{ since } T = \frac{1}{f}\cdot\right] \end{aligned}$$

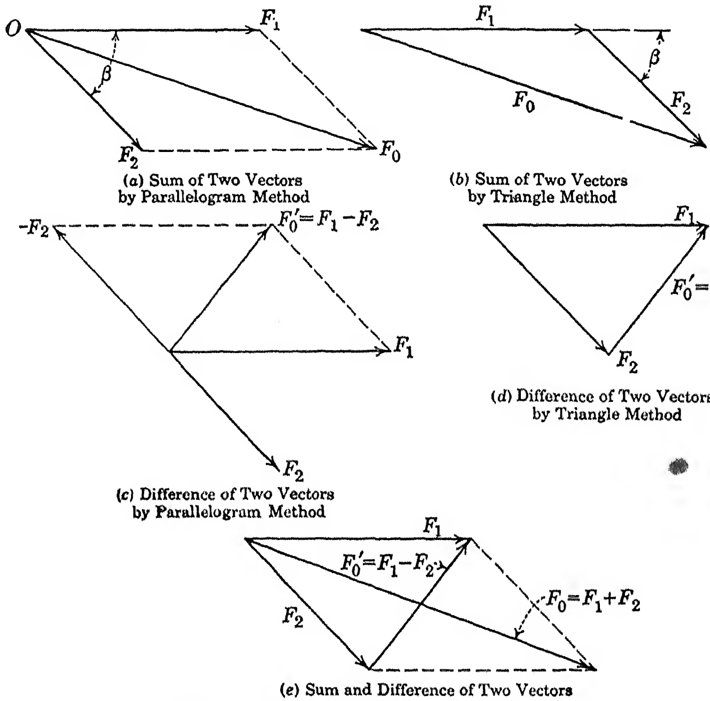
The ratio of *rms* to *average* value is then $0.707/0.637 = 1.11$, and the ratio of average to rms value is 0.9. The ratio of rms to average value enters into computations of induced emfs in alternators, transformers, and other types of alternating-current machinery.

The ratio of rms to average value is called the *form factor* of the wave. The form factor of a sine wave is 1.11. The maximum, rms, and average values for a sine wave of voltage whose rms value is 100 volts are shown in Fig. 9(*b*).

The average values of voltages and currents should not be used in computing power.

8. Scalars and Vectors.—Quantities, in general, are divided into two classes, scalars and vectors.

A scalar is a quantity that is completely determined by its magnitude alone. Examples of scalar quantities are energy, gallons, mass, temperature, etc. Such quantities are added algebraically. For example, 2 gal of water plus 5 gal of water equals 7 gal of water.



(a) Sum of Two Vectors by Parallelogram Method

(b) Sum of Two Vectors by Triangle Method

(c) Difference of Two Vectors by Parallelogram Method

(d) Difference of Two Vectors by Triangle Method

(e) Sum and Difference of Two Vectors

FIG. 10.—Sum and difference of two vectors.

A vector has direction as well as magnitude. A common example of a vector is force. When a force is under consideration, not only its magnitude but its direction as well must be considered. When two or more forces are added, they are not necessarily added algebraically but must be combined in such a way as to take into consideration their directions as well as their magnitudes.

Figure 10(a) shows two forces acting at the point O and represented by the vectors F_1 and F_2. The length of each of these vectors, to scale, is equal to the *magnitude* of the force that it represents. The direction of each of these vectors shows the *direction* in which the force acts. β is the angle between F_1 and F_2. Their sum F_0, or the single force that would have the same effect at their point of application O as

F_1 and F_2 acting in conjunction, is called their *resultant*. F_0 is one diagonal of the parallelogram having F_1 and F_2 as adjacent sides.

Figure 10(b) shows a triangle having F_1 and F_2 as two of its sides, F_1 and F_2 being parallel to and acting in the same directions as F_1 and F_2 of Fig. 10(a). The exterior angle between F_1 and F_2 in Fig. 10(b) is, therefore, equal to β. The third side of the triangle F_0 is equal in magnitude to and in the same direction as F_0 of Fig. 10(a). The resultant of two vectors, therefore, may be found by means of a triangle properly constructed, of which two sides are the two component vectors and the third side is their sum. Such a triangle is called a *triangle of vectors* or a vector polygon. It is usually simpler to use the vector polygon rather than the parallelogram of vectors.

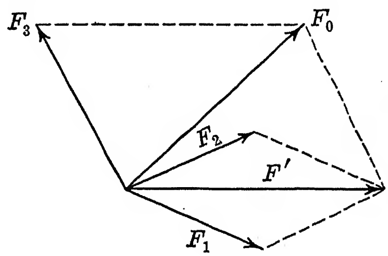


FIG. 11.—Sum of three vectors.

To subtract one vector from another, reverse the first vector and add it vectorially to the second vector. For example, in Fig. 10(c), it is desired to subtract F_2 from F_1. F_2 is reversed, giving $-F_2$. F_0', the vector sum of F_1 and $-F_2$, found by completing the parallelogram, is equal to $F_1 - F_2$. Vectors may be subtracted by the triangle method, as shown in Fig. 10(d). The vector F_0', connecting the ends of the two vectors F_1 and F_2 whose difference is desired, is their vector *difference*.

If a parallelogram, Fig. 10(e), having vectors F_1 and F_2 as adjacent sides, be completed, one diagonal F_0 of the parallelogram is the vector *sum* of F_1 and F_2. The other diagonal F_0' of the parallelogram is the vector *difference* of F_1 and F_2.

A vector is often indicated by placing a dot under its symbol. For example, in Figs. 10(a) and 10(b),

$$\underset{\cdot}{F}_0 = \underset{\cdot}{F}_1 + \underset{\cdot}{F}_2$$

shows that F_0 is the *vector* sum of F_1 and F_2 and not their algebraic sum.

When more than two vectors are added, the resultant of two is first found, and this resultant is combined with a third vector, etc. This is illustrated in Fig. 11, in which three vectors F_1, F_2, F_3, are added.

F_1 and F_2 are first combined, and their resultant F' is found. Then F' is combined with F_3, giving F_0 as the sum of all three vectors, F_1, F_2, F_3. That is,

$$\underset{\cdot}{F}_0 = \underset{\cdot}{F}_1 + \underset{\cdot}{F}_2 + \underset{\cdot}{F}_3. \tag{10}$$

F' is an intermediate vector and does not appear in the ultimate result.

9. Ohm; Volt.—If a resistance of 1 ohm, as measured with direct current, has no inductance or capacitance and is so designed that alternating current in flowing through it does not produce any secondary effects, such as eddy currents or skin effect, it offers a resistance of 1 *ohm* to alternating current.

When an alternating-current ampere flows through such a resistance, the drop across its terminals is equal to 1 alternating-current *volt.*

If the current in a pure resistance R be given by $i = I_m \sin \omega t$, the voltage across the resistance is given by

$$e = iR = I_m R \sin \omega t = E_m \sin \omega t. \tag{6a}$$

This is similar to (6), p. 9 and is a sine function like Eq. (5) (p. 8) for current.

Hence, the relation between *maximum* and *rms* volts is the same as the relation between *maximum* and *rms* amperes. For a sine wave, the maximum voltage is $\sqrt{2}$, or 1.414, times the rms voltage.

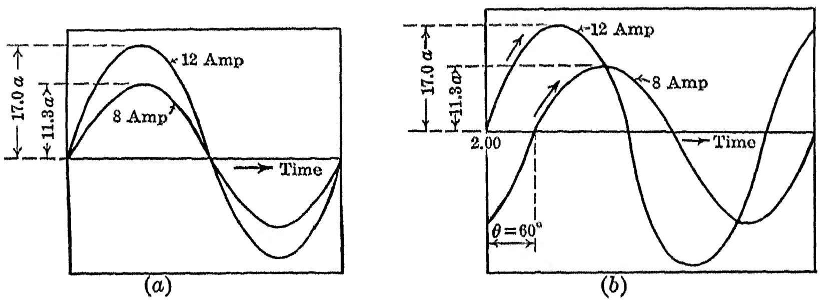


FIG. 12.—Phase relations of alternating currents.

10. Phase Relations.—The current and voltage in the ordinary alternating-current system have the same fundamental frequency under normal operating conditions, although they do not necessarily pass through their corresponding zero values at the same instant of time. Figure 12(a) shows two sine-wave currents, one having the rms value of 8 and the other of 12 amp. Their maximum values are, accordingly, $8\sqrt{2}$, or 11.3, amp and $12\sqrt{2}$, or 17.0, amp. Both currents go through zero, increasing positively, at the same instant and, therefore, are said to be *in phase* with each other.

Figure 12(b) shows two sine-wave currents having *rms* values of 8 and 12 amp, but not passing through zero at the same instant. The 8-amp current passes through zero, increasing positively, later than does the 12-amp current. It must be remembered that time is increasing from left to right. If the 12-amp current is passing through its zero value at 2.00 o'clock, the 8-amp current is passing through its corresponding zero value some time later, for any value of time to the

right of 2.00 is *later* than 2.00 o'clock. The 8-amp current, therefore, *lags* the 12-amp current.

The time of lag shown in Fig. 12(*b*) corresponds to 60° and is represented by the angle θ. The 8-amp current, therefore, *lags* the 12-amp current by an angle θ, or by 60°. Or the 12-amp current *leads* the 8-amp current by an angle θ, or by 60°. If, the frequency is 60 cycles, the time corresponding to 60° is $(60/360)(1/60)$, or $1/360$ sec. Hence in time the 8-amp current lags the 12-amp current by $1/360$ sec.

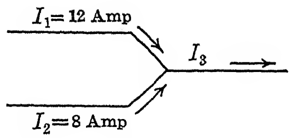


FIG. 13.—Alternating currents meeting at junction.

In Fig. 12(*a*), the two currents are *in phase*. In Fig. 12(*b*), the two currents have a *phase difference* of 60°.

These phase differences may exist between currents and voltages, between two or more voltages, or between two or more currents.

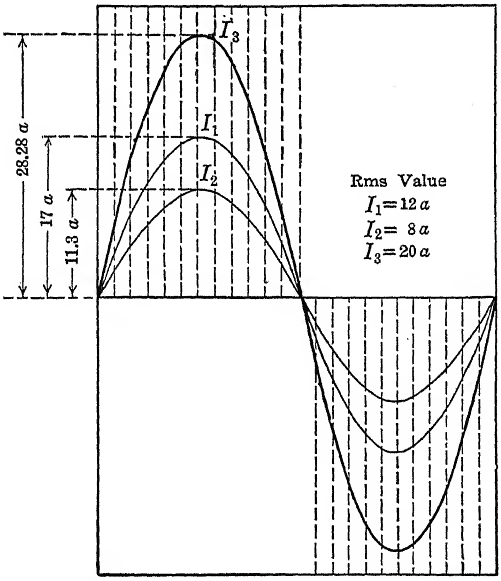


FIG. 14.—Addition of two currents in phase.

11. Addition of Currents.—Figure 13 shows two currents, having rms values of 8 and 12 amp uniting to flow in a common wire. If these two currents were direct currents, then, by Kirchhoff's first law (Vol. I, Chap. III), the current I_3 could have only two possible numerical values, $12 + 8 = 20$ amp, if the two currents flow in the same direction, and $12 - 8 = 4$ amp, if they flow in opposite directions.

If the two currents, Fig. 13, are alternating, their sum I_3 may be

equal numerically to *any* value *from* 20 *amp to* 4 *amp,* depending on the phase relation existing between I_1 and I_2.

Figure 14 shows these two currents plotted *in phase.* Their sum I_3 is found by adding their ordinates at each instant. The resulting current obtained in this manner will be a sine wave and will have a *maximum* value of 28.28 amp corresponding to an *rms* value of

$$\frac{28.28}{\sqrt{2}} = 20 \text{ amp.}$$

That is, when two currents are in phase, their sum is found arithmetically.

Figure 17 (p. 20) corresponds to the condition of Fig. 12(*b*), where the two currents differ in phase by 60°. Their sum is found in the same

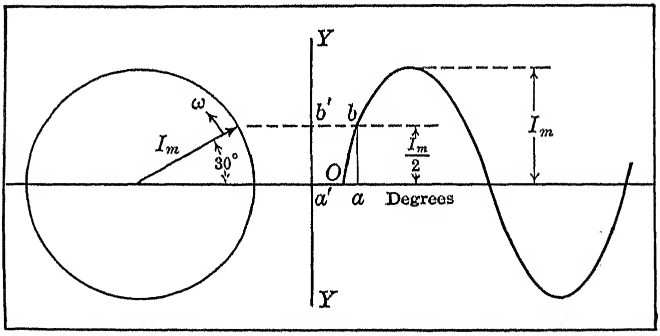


FIG. 15.—Instantaneous values of current from rotating radius-vector.

manner as in Fig. 14 by adding the two, point by point, and obtaining the resulting current I_3. The resultant I_3 will not have a maximum value of 28.28 amp as it did when the currents were in phase, but its maximum value will be less, actually being 24.7 amp. This corresponds to a rms value of 17.45 amp for the sum of the two, rather than of 20 amp as before. *Therefore, the sum of any number of alternating currents or voltages depends on their phase relations as well as on their magnitudes.*

If voltages rather than currents be added, it follows that their sum depends on their phase relations as well as on their magnitudes.

12. Vector Representation of Alternating Quantities.—It is shown in Fig. 4 (p. 5) that a sine wave can be drawn by projecting a rotating radius, in its successive positions, to meet corresponding equally spaced ordinates. The value of the current or voltage may be found at any instant by projecting the radius upon a vertical line.

This is illustrated in Fig. 15. A current has a maximum value I_m. This value I_m is laid off as a radius, and this radius rotates at a

speed in rps equal to the frequency of the current. The angular velocity will be ω radians per sec, where $\omega = 2\pi f$. For example, if the current has a frequency of 60 cycles, the radius I_m must make 60 complete rps or $2\pi 60 = 377$ radians per sec, in a counterclockwise direction. Counterclockwise rotation has been adopted internationally as the positive direction of rotation.

When the radius I_m is at the right-hand horizontal position, the value of the current is zero. When I_m has advanced 30°, the point b on the current wave has been reached. The value of the current at this instant is ab, or, which is the same thing, the current value is given by the distance $a'b'$, the projection of the rotating radius I_m on the *vertical* axis. At this particular instant, the distance

$$ab = a'b' = I_m/2,$$

since $\sin 30° = 0.5$.

Consider two currents I_{1m} and I_{2m}, Fig. 16, having rms values of 12.0 and 8.0 amp. The current I_{2m}, whose maximum value is 11.3

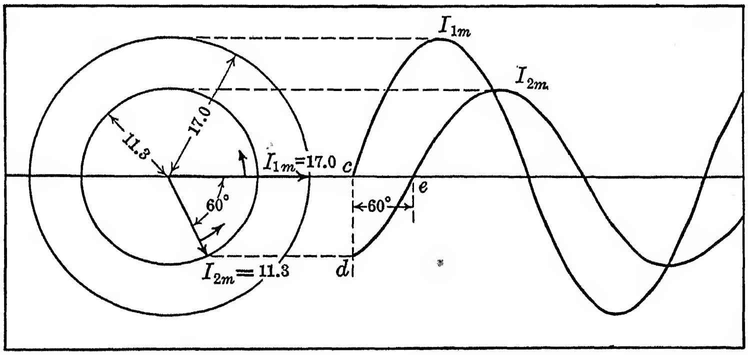


FIG. 16.—Current waves produced by two current radius-vectors differing in phase by 60°.

amp, lags current I_{1m}, whose maximum value is 17.0 amp, by 60°. When the radius I_{1m} is in the horizontal position, the value of I_{1m} is zero at this instant. At this same instant, the radius I_{2m} will not have reached its horizontal position, the value of the current being represented by cd. In fact, the radius I_{2m} does not reach its horizontal or zero position, corresponding to point e on its current wave, until I_{1m} has advanced 60° beyond the horizontal. Further, the horizontal distance ce is 60°, the same as the phase angle between the two rotating radius vectors.

These two current waves, therefore, can be constructed in their proper phase relation by means of two rotating radii, or radius vectors,

having lengths of 17.0 and 11.3 amp, having equal angular velocities, and differing in phase by 60°.

13. Vector Addition of Sine Waves.—Assume that it is desired to add the two currents of Fig. 16. This may be done by adding the ordinates of the two curves at each point, Fig. 17, and plotting a new curve I_3. This new curve is the sum of the two currents whose maximum values are 17.0 and 11.3 amp and rms values 12 and 8 amp,

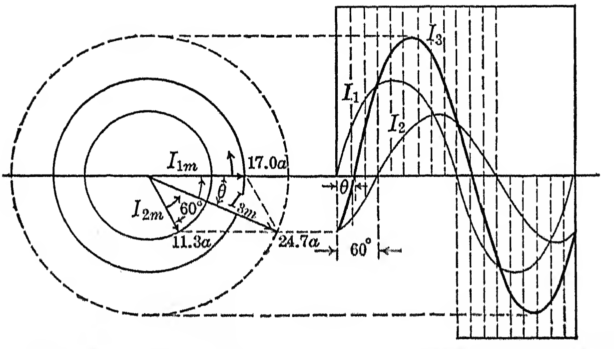


FIG. 17.—Relation of vector addition of vectors to scalar addition of ordinates.

and whose phase difference is 60°. The maximum value of this resultant, if measured accurately, is 24.7 amp. This corresponds to an rms value of 17.45 amp. The sum, therefore, of two sine-wave alternating currents having rms values of 12 and 8 amp, and differing in phase by 60°, is 17.45 rms amp.

If the rotating vectors, Fig. 17, be added vectorially by completing the parallelogram, a third vector I_{3m} results. This vector I_{3m} is found to be 24.7 amp, the value of the maximum of the resultant current wave I_3 as just found. If a sine wave be plotted using I_{3m} as the rotating vector, projecting horizontally as before, it will coincide with I_3 as obtained by the addition of the ordinates for the 12- and 8-amp (rms) waves. The angle θ by which the radius vector I_{1m} leads I_{3m} equals the angle θ by which the current wave I_1 leads the current wave I_3.

Hence, this problem can be solved without going through the somewhat lengthy process of plotting the waves and adding their ordinates. It is necessary merely to lay off the maximum values of the waves 60° apart and add them vectorially, just as forces are combined. The resulting vector will be the maximum value of the wave as obtained also by adding the waves of I_1 and I_2.

In practice, one generally has to do with rms rather than maximum values. If the rms values of the waves be added in this same manner, their vector sum is the sum of the two alternating currents in rms

amperes. This is illustrated in Fig. 18, where the 12- and 8-amp vectors are laid off 60° apart, the 12-amp vector leading. By completing the parallelogram, the resultant current Oc is obtained. This has a value of 17.45 amp. Its value is readily found as follows:

Project ac upon Ob, where $ac = 8$.

$$ab = ac \cos 60° = 4.00.$$
$$bc = ac \sin 60° = 6.93.$$
$$Oc = \sqrt{(12 + 4.00)^2 + (6.93)^2} = 17.45 \text{ amp.} \quad \textit{Ans.}$$

The angle θ can be readily determined.

$$\tan \theta = \frac{6.93}{12 + 4} = 0.433.$$
$$\theta = 23.4°.$$

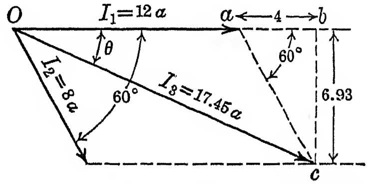


FIG. 18.—Vector addition of currents, using rms values.

Example.—Each of two alternator coils Oa and Ob, Fig. 19(a), is generating an emf of 160 volts. These voltages differ in phase by 90°. Determine the voltage across their open ends if they are connected together at O as shown.

Let E_{oa} and E_{ob}, Fig. 19(b), represent the voltages across coils Oa and Ob. Let the voltage across the open ends a and b be denoted by E_{ab}. To obtain the voltage E_{ab}, it is necessary to use E_{ao}, displaced 180° from E_{oa} (Chap. V). Then, vectorially, $\mathbf{E}_{ab} = \mathbf{E}_{ao} + \mathbf{E}_{ob}$. Combining E_{ao} and E_{ob} vectorially, the voltage E_{ab} is

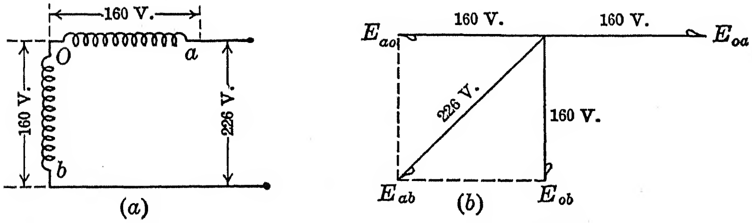


FIG. 19.—Vector addition of two equal voltages having 90° phase difference.

obtained. As E_{ao} and E_{ob} are at right angles, their resultant, which is the hypotenuse of a right triangle, is

$$E_{ab} = \sqrt{E_{ao}^2 + E_{ob}^2} = \sqrt{160^2 + 160^2} = 226 \text{ volts.} \quad \textit{Ans.}$$

It must be kept constantly in mind that alternating voltages and currents must be combined vectorially.

The only occasions when arithmetical addition is permissible are when the voltages or the currents are in phase.

14. Addition of Sine Waves.—Although the resultant of two or more sine waves may be found by the use of vectors, by the method described in Sec. 13, it is often useful to combine sine waves directly.

First consider the addition of a sine and a cosine wave or of two sine waves differing in phase by 90°. Let the waves be given by $A \sin x$ and $B \cos x$, Fig. 20. Their sum, found by adding the ordinates of the two waves, is given by $C \sin (x + \theta)$ having a maximum value C, and the phase with respect to the Y-axis of reference is $\theta°$. In order to

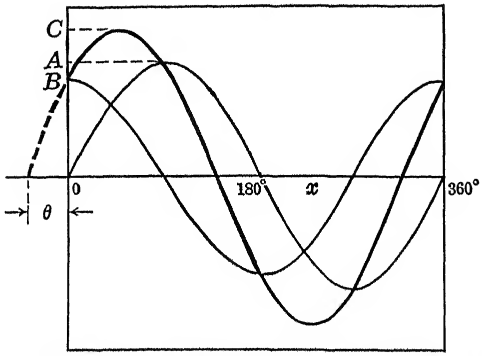


FIG. 20.—Addition of sine and cosine waves.

determine the resultant wave the parameters C and θ may be determined as follows:

$$A \sin x + B \cos x = C \sin (x + \theta). \tag{I}$$

Expanding the right-hand side of the equation by (36) p. 605,

$$A \sin x + B \cos x = C \sin x \cos \theta + C \cos x \sin \theta. \tag{II}$$

Equating the coefficients of $\sin x$ and of $\cos x$ on the two sides of the equation,

$$A = C \cos \theta; \tag{III}$$
$$B = C \sin \theta. \tag{IV}$$

Squaring (III) and (IV) and adding,

$$A^2 + B^2 = C^2(\cos^2 \theta + \sin^2 \theta) \tag{V}$$

Since $\cos^2 \theta + \sin^2 \theta = 1$,

$$C = \sqrt{A^2 + B^2}.$$

Dividing (IV) by (III),

$$\frac{B}{A} = \frac{C \sin \theta}{C \cos \theta} = \tan \theta.$$

Hence,

$$A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin \left(x + \tan^{-1} \frac{B}{A}\right). \tag{11}$$

Example.—A 60-cycle current $9 \sin \omega t$ is added to a 60-cycle current $8 \cos \omega t$, where $\omega = 2\pi 60$.

Determine the resultant current i_3. From (11),

$$i_3 = \sqrt{9^2 + 8^2} \sin (\omega t + \theta),$$
$$\tan \theta = 8/9 = 0.888 \qquad \theta = 46.1°.$$
$$i_3 = 12.05 \sin (\omega t + 46.1°). \quad Ans.$$

Waves Differing in Phase by Angles Other Than 90°.—If waves differ in phase by angles other than 90°, their sum may be found by means of (11), it being necessary first to apply the equation in reverse.

Example.—Two 25-cycle emfs differing in phase by 60° are given by

$$e_1 = 120 \sin (\omega t - 30°) \qquad \text{and} \qquad e_2 = 100 \sin (\omega t - 90°),$$

where $\omega = 2\pi 25$, Fig. 21.

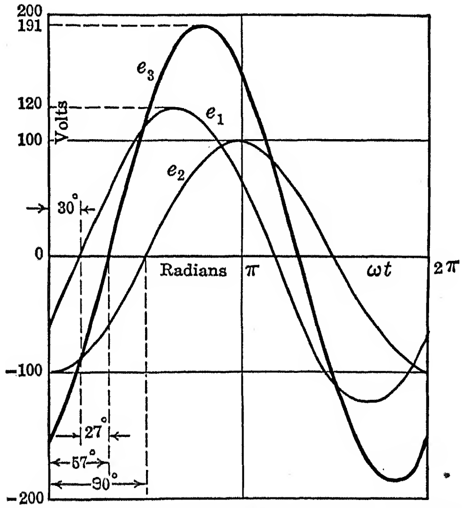


FIG. 21.—Addition of sine waves differing in phase by 60°.

Determine their sum e_3.

$$e_3 = 120 \sin (\omega t - 30°) + 100 \sin (\omega t - 90°). \tag{I}$$

Expanding (I) [(37), p. 605],

$$e_3 = 120 (\sin \omega t \cos 30° - \cos \omega t \sin 30°) + 100 (\sin \omega t \cos 90° - \cos \omega t \sin 90°), \tag{II}$$

$\cos 30° = 0.866$; $\sin 30° = 0.5$; $\cos 90° = 0$; $\sin 90° = 1$,

$$e_3 = 104 \sin \omega t - 60 \cos \omega t + 0 - 100 \cos \omega t \tag{III}$$

$$= 104 \sin \omega t - 160 \cos \omega t. \tag{IV}$$

Using (11),

$$e_3 = \sqrt{104^2 + 160^2} \sin \left(\omega t + \tan^{-1} \frac{-160}{104}\right)$$

$$= 191 \sin (\omega t - 57.0°),$$

$-57.0° = \tan^{-1} (-160/104) = \tan^{-1} (-1.538)$. Since the numerator is negative and the denominator positive, θ must be negative and in the fourth quadrant (see p. 603). The angle between the 120-volt wave e_1 and the resultant wave e_3 is $57.0° - 30.0° = 27.0°$.

CHAPTER II

ALTERNATING-CURRENT CIRCUITS

15. Alternating-current Power; Voltage and Current in Phase.—The power in a direct-current circuit under steady conditions is given by the product of the volts across the circuit and the current in amperes in the circuit. This same rule applies to alternating-current circuits, provided that *instantaneous* values of volts and amperes are considered. The product of volts and amperes at any instant does give the instantaneous power in watts. The *average*

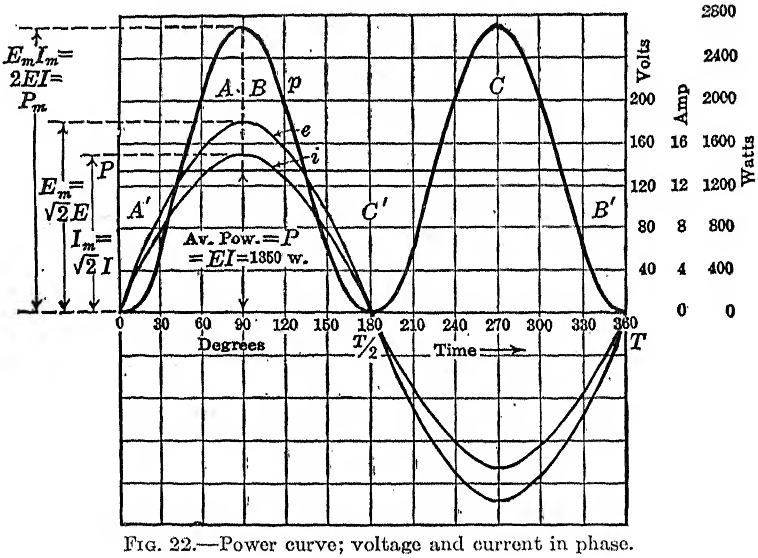


FIG. 22.—Power curve; voltage and current in phase.

power, however, is not necessarily given by the product of the rms volts and rms amperes, the values of which are ordinarily determined with instruments.

Figure 22 shows a voltage wave e and a current wave i in phase. This condition occurs when there is only resistance in the circuit. Thus, in Sec. 9 (p. 16) it is shown that the relation between instantaneous voltage and current is $e = iR$. That is, the voltage at any instant is equal to the current at that instant times a constant, so that the voltage wave must be in phase with the current wave.

The voltage has an rms value of E volts and the current an rms value of I amp; hence their maximum values are $E_m = \sqrt{2}\,E$ and $I_m = \sqrt{2}\,I$. To obtain the instantaneous power p, the amperes and volts at the particular instant are multiplied together. Hence, the ordinates, obtained by multiplying together instantaneous values of e and i, give a power curve p. The curve p gives the power in the circuit *at any instant*. The ordinates of this power curve will *always* be positive when e and i are in phase. During the entire first half-cycle the voltage and current are both positive. During the entire second half-cycle the voltage and current are both negative, and the product of two negative quantities is positive. Quite apart from this mathematical reason, it is true that the sign of the power does not change if both current and voltage are reversed. For example, if a direct-current voltage impressed across a resistance be reversed, the current also reverses. The power dissipated in the resistance does not change, for it is well known that the power dissipated in a constant resistance with fixed voltage is constant, irrespective of the polarity. That is, the power is positive so long as the voltage and current act in the same direction.

Under the conditions shown in Fig. 22 the current and the voltage act in conjunction throughout the cycle, and the ordinates of the power curve are always positive.

It will be noted that this power curve is a sine wave having double the frequency of either the voltage or the current. In fact, this power wave is identical in character with the current-squared waves (i^2) of Figs. 7(b) and 8.

Its equation is

$$\begin{aligned} p &= (E_m \sin \omega t)(I_m \sin \omega t) \\ &= (\sqrt{2}\,E \sin \omega t)(\sqrt{2}\,I \sin \omega t) = 2EI \sin^2 \omega t \\ &= 2EI\,\frac{1 - \cos 2\omega t}{2} \text{ [see Eq. (VI), p. 11].} \end{aligned} \tag{12}$$

For every cycle of either voltage or current, the power wave touches the zero axis twice, so that in such a circuit the power is zero twice during each cycle. Since the maximum values of the voltage and current waves occur at the same instant, the corresponding maximum value of the power curve is

$$(\sqrt{2}\,E)(\sqrt{2}\,I) = 2EI,$$

where E and I are the *rms* values of voltage and current.

In Fig. 22 the maximum value P_m of the power curve occurs when $\cos 2\omega t$ in (12) equals -1 so that $2\omega t = \pi$ and $\omega t = \pi/2$, or 90°. The

maximum value E_m of the voltage is 180 volts, and the maximum value I_m of the current is 15 amp, so that the maximum value of the power is $180 \cdot 15 = 2{,}700$ watts.

Even though the power may vary over wide limits during the cycle, its effect will be determined usually by its *average* value. That is, the energy over a complete cycle is equal to the average power (or average ordinate of the power curve) multiplied by the time required to complete a cycle. The average power is determined as follows:

The horizontal axis of symmetry of the power curve is at a distance $E_mI_m/2 = EI$ above the zero axis, Fig. 22. Consequently, $EI = P$

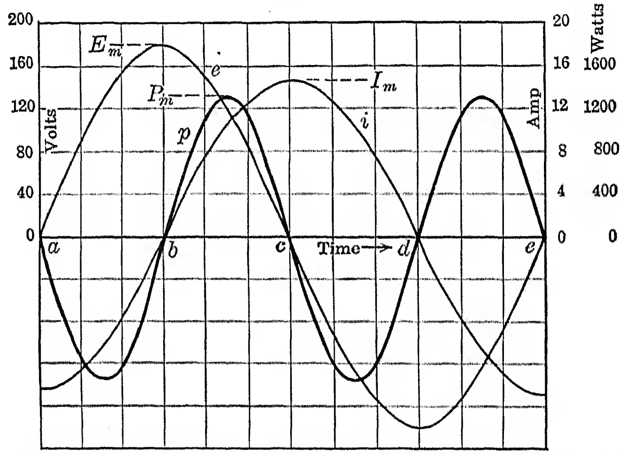


FIG. 23.—Power curve; voltage and current in quadrature, current lagging.

must be the *average* value of the power, since areas A, B, C of the upper half-waves will just fill the corresponding areas A', B', C' in the valleys below the axis of symmetry P of the power curve. Hence, when the current and the voltage are *in phase*, the average power is their product, just as with direct currents.

Example.—An incandescent-lamp load takes 30 amp from 115-volt 60-cycle mains. (In this type of load, the current and voltage are substantially in phase.) Determine (*a*) maximum value P_m of power curve; (*b*) average power P.

(*a*) $P_m = \sqrt{2} \cdot 115 \cdot \sqrt{2} \cdot 30 = 2 \cdot 115 \cdot 30 = 6{,}900$ watts. *Ans.*

(*b*) $P = EI = 115 \cdot 30 = 3{,}450$ watts. *Ans.*

16. Alternating-current Power; Voltage and Current in Quadrature.—Figure 23 shows the voltage wave e and the current wave i 90° out of phase, or in quadrature, the voltage wave leading. Let it be required to determine the power curve for this condition. At points a, b, c, d, e, either the voltage or the current is zero, and the power therefore must be *zero* at each of these points. Between a and b the

voltage is positive, and the current is negative. The product of a positive and a negative quantity is negative. Also, the voltage and the current are acting in *opposition*. Hence the power between points a and b must be *negative*. This means that the circuit is *giving* power to the source of supply. Between points b and c both voltage and current are positive and, therefore, are acting in *conjunction*. Hence the power between these two points must be *positive*. Between c and d the current is positive, but the voltage is *negative*. The power is again negative, therefore, between these two points. Between d and e both the current and the voltage are negative, and the power is positive.

The resulting power curve p is a sine wave having double the frequency of either the voltage or the current. Its axis of symmetry coincides with the axis of voltage and current. Hence, there must be as much of the power curve above the zero axis as there is below that axis, or the positive energy above the axis must be equal to the negative energy below the axis. That is, all the *positive* energy received *from* the source of supply is returned *to* that source. The net power (and energy also), therefore, is *zero*. When voltage and current differ in phase by 90°, or are in quadrature, the average power is zero. If the current *leads* the voltage by 90°, the average power is *zero*, as is shown in Fig. 34 (p. 38).

In Fig. 23, the equations of the voltage and current waves are $E_m \sin \omega t$ and $I_m \sin (\omega t - 90°)$ or $-I_m \cos \omega t$ so that the equation of the power curve

$$p = -E_m I_m \sin \omega t \cos \omega t.$$

But since $\sin 2x = 2 \sin x \cos x$ [(42), p. 603]

$$p = -\frac{E_m I_m}{2} \sin 2\omega t. \tag{13}$$

Thus, Fig. 23, $E_m = 180$ volts, and $I_m = 15$ amp, so that the maximum value P_m of the power curve p is $(180 \cdot 15)/2 = 1{,}350$ watts.

17. Alternating-current Power; Voltage and Current Differ in Phase by Angle θ.—If voltage and current differ in phase by an angle θ that lies between $+90°$ and $-90°$ and is greater than 0, which occurs with resistance together with inductance or capacitance in the circuit (Secs. 21 and 23), the average power is neither EI nor zero but is given by

$$P = EI \cos \theta. \tag{14}$$

In Fig. 24, the current wave i lags the voltage wave e by an angle θ. The power curve p is obtained by multiplying the ordinates of the two

waves at each instant. At points a, b, c, d, e, either the voltage or the current is zero, and the power is zero at each of these points. Between a and b, and between c and d, the current and voltage are in opposition, and the power is negative. Between b and c and between d and e, they are in conjunction and the power is positive. It will be noted that the positive areas AA under the power curve are greater than the negative areas BB shown shaded. Therefore the average power P

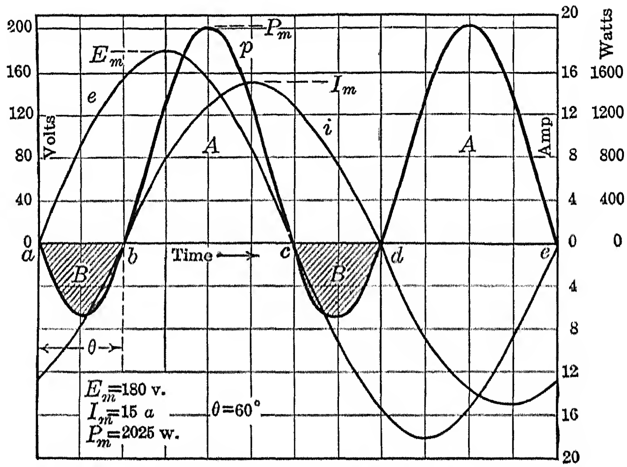


FIG. 24.—Power curve; current lags voltage by angle θ.

is positive, and its value is obtained by dividing the net area under the power curve by its base.

The power curve

$$p = (E_m \sin \omega t)[I_m \sin (\omega t - \theta)],$$

which on expanding becomes

$$p = E_m I_m (\sin^2 \omega t \cos \theta - \sin \omega t \cos \omega t \sin \theta)$$
$$= \frac{E_m I_m}{2} [(1 - \cos 2\omega t) \cos \theta - \sin 2\omega t \sin \theta].$$

The average power

$$P = \frac{1}{T}\int_0^T p\, dt = \frac{E_m I_m}{2T}\left[t \cos \theta - \frac{\sin 2\omega t}{2\omega} \cos \theta + \frac{\cos 2\omega t}{2\omega} \sin \theta\right]\Bigg|_0^T$$
$$P = \frac{E_m I_m}{2T} T \cos \theta = \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta = EI \cos \theta. \quad \text{Q.E.D.}$$

($\sin 2\omega t = 0$ when $t = 0$ and $t = T$; $\cos 2\omega t = 1$ when $t = 0$ and $t = T$)

$\cos \theta$ is the *power factor* of the circuit. P is the *true watts* and EI the *apparent watts*, or *volt-amperes* (va).

The power factor

$$\text{P.F.} = \cos \theta = \frac{\text{watts}}{\text{va}} = \frac{P}{EI}. \qquad (15)$$

The power factor never can be greater than unity. It should be noted that, when θ is zero (current and voltage in phase), (14) reduces to $P = EI \cdot 1 = EI$ as shown in Sec. 15. (This is always the case with *resistance* only in the circuit.) When θ is 90° (current and voltage in quadrature), (14) becomes $P = EI \cdot 0 = 0$ as shown in Sec. 16.

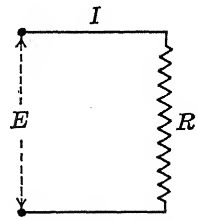


FIG. 25.—Circuit with resistance only.

18. Circuit with Resistance Only.—Figure 25 shows an alternating-current circuit containing resistance only in which is a current

$$i = I_m \sin \omega t,$$

where ω is the angular velocity of the rotating vector in radians per second [see Sec. (5), p. 8]. As one revolution of the rotating vector corresponds to 2π radians, the vector must complete $2\pi f$ radians per sec, where f is the frequency. Hence, $\omega = 2\pi f$. (For 60 cycles, $\omega = 377$; for 25 cycles, $\omega = 157$.)

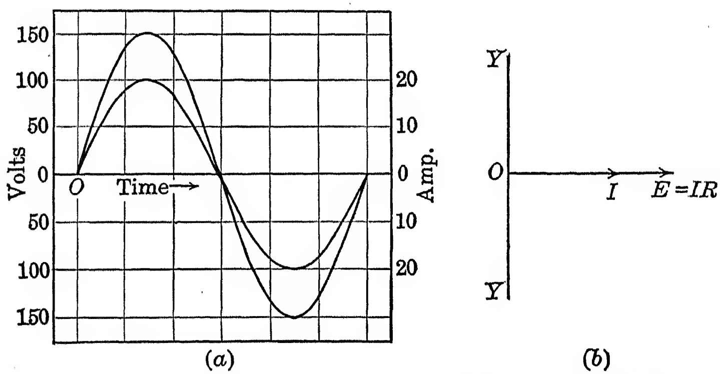


FIG. 26.—Voltage and current waves in phase, with vector diagram.

Let it be required to determine the impressed emf having an instantaneous value e and an rms value E. From the definition of an alternating-current volt (Sec. 9),

$$e = Ri = RI_m \sin \omega t = E_m \sin \omega t, \tag{16}$$

where E_m is the maximum value of the wave.

The current and the voltage have the same frequency $\omega/2\pi$. They are also in phase; for when $t = 0$, $\sin \omega t = 0$ and both the voltage and current waves are crossing the zero axis and increasing positively, as shown in Fig. 26(a). To illustrate with numerical values, the voltage wave has a maximum value of 150 volts, and the current wave a maximum value of 20 amp, so that the rms values are 106.0 volts and 14.14

amp. From (16),

$$E_m = I_m R; \qquad \frac{E_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} R \qquad \text{or} \qquad E = IR. \tag{17}$$

That is, if rms values are used, $E = IR$. Figure 26(b) shows the vector diagram for this circuit, using rms values, the scale being larger than that of (a). The IR drop is in phase with the current I and is equal to the voltage E, since no other voltage exists in the circuit. For convenience, the positions of the voltage and current vectors are taken along the X-axis. They may have any position in the coordinate plane, it being merely necessary that they be in phase and have their proper magnitudes. They may be considered as the rotating vectors,

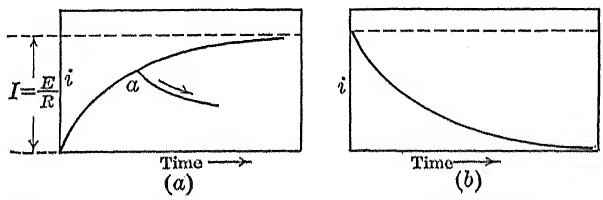


FIG. 27.—Increase and decrease of current in inductive circuit.

Figs. 16 and 17 (pp. 19 and 20), divided by $\sqrt{2}$ and photographed at the desired position by a high-speed camera.

As voltage and current are in phase, the power

$$P = EI \tag{18}$$

as is shown in Fig. 22. Also,

$$P = I^2R. \tag{19}$$

Note that, with resistance only, the alternating-current circuit follows the same laws as the direct-current circuit in regard to the relations existing among voltage, current, resistance, and power.

19. Circuit with Inductance.—It is shown in Vol. I, Chap. VIII, that inductance always *opposes* any *change* in the current. For example, when the current starts to increase in a circuit with inductance, the emf of self-induction opposes this increase. This is illustrated in Fig. 27(a), which shows the rise of current in a direct-current circuit containing resistance and inductance, when a steady voltage is impressed. The current rises *gradually* to its ultimate value.

On the other hand, when the current starts to decrease in the circuit, the inductance tends to prevent this decrease, as is shown in Fig. 27(b). In other words, if inductance is present in a circuit, it always *opposes* any change in the current. With a *steady* direct current, however, the inductance has no effect.

If, in Fig. 27(a), the voltage across the circuit be lowered when the current reaches point a, the current will not reach its Ohm's-law value.

An effect similar to the foregoing occurs in an a-c circuit containing resistance R and inductance L in series. For example, when the voltage is increasing positively, the current tends to increase also in the same direction. However, the emf of self-induction $-L\ di/dt$ causes the current to lag; and before the current can attain its Ohm's-law value of E/R, the voltage begins to decrease in a manner similar to that at a, Fig. 27. Hence the current cannot reach the value E/R.

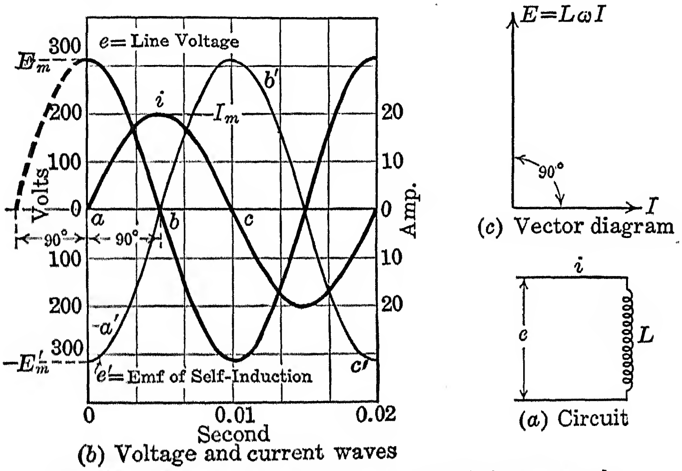


FIG. 28.—Waves and vector diagram with inductance only.

Consider a circuit with inductance only, such as is shown in Fig. 28(a), in which there is a sinusoidal current i. Starting at a, Fig. 28(b), the current is *changing* at its maximum rate in a positive direction. At this instant, therefore, the emf of self-induction e' must be at its negative maximum. At instant b, the current is at its maximum value so that its rate of change is zero. Hence, at this instant the emf of self-induction is zero. At c the current is changing at its maximum rate negatively, and the emf of self-induction must be at its positive maximum because of the negative sign in $-L\ di/dt$. Continuing in this way, the induced-emf curve $a'b'c'$ is obtained. This curve is a sine wave and lags the current by 90°.

This is the only emf in the circuit, and it *opposes* change in the current. It is somewhat similar in effect to the counter emf in a motor. The line, in the case of the motor, must supply a voltage opposite and equal to the counter emf before any current can flow to the armature. The same condition exists in the alternating-current circuit. Before any current can flow to a circuit containing inductance only,

a voltage opposite and equal to the emf of self-induction must be supplied.

In Fig. 28(*b*), therefore, the voltage e, which is the line voltage, is opposite and equal to the emf of self-induction e'. In Fig. 28(*c*) is shown the vector diagram in which the voltage vector E leads the current vector I by 90°, the vectors representing rms values, the scale being different from that in (*a*).

Note that the impressed voltage *leads* the current by 90°, or the current *lags* this voltage by 90°. With inductance only in the circuit, the current *lags* the impressed voltage by 90°. (In practice, it is impossible to obtain a pure inductance, as inductance must necessarily be accompanied by a certain amount of resistance.)

The foregoing also may be proved as follows: Let the current be given by $i = I_m \sin \omega t$. The emf of self-induction

$$e' = -L\frac{di}{dt} = -L\omega I_m \cos \omega t \qquad \text{(I)}$$
$$= L\omega I_m \sin (\omega t - 90°) \qquad \text{(II)}$$

is a cosine or sine wave lagging 90° with respect to $I_m \sin \omega t$ and is shown by $-e'$, Fig. 28(*b*).

The line voltage that balances this emf,

$$e = L\omega I_m \sin (\omega t + 90°) = E_m \sin (\omega t + 90°) \qquad \text{(III)}$$
$$= L\omega I_m \cos \omega t = E_m \cos \omega t \qquad \text{(IV)}$$

is a cosine or sine wave *leading* the current $I_m \sin \omega t$ by 90° and is shown by e.

Example.—In a circuit of pure inductance of 0.05 henry the current has a maximum instantaneous value of 20.0 amp; and the frequency is 50 cycles per sec, Fig. 28(*b*). Determine (*a*) equation of current wave i; (*b*) equation of induced-emf wave e'; (*c*) equation of impressed emf e; (*d*) maximum rate at which current in (*a*) changes; (*e*) maximum value E'_m of induced emf e' as computed from (*d*).

(*a*) $I_m = 20.0$ amp.

$i = 20 \sin 2\pi 50t = 20 \sin 314t$. *Ans.*

(*b*) $e' = -L\dfrac{di}{dt} = -0.05\dfrac{d}{dt}(20 \sin 314t)$

$= -0.05 \cdot 20 \cdot 314 \cos 314t$

$= -314 \cos 314t$. *Ans.*

(*c*) $e = -e' = 314 \cos 314t$

$= 314 \sin (314t + 90°)$. *Ans.*

(*d*) Rate of change of current

$$\frac{di}{dt} = 20 \cdot 314 \cos 314t = 6{,}280 \cos 314t,$$

which is a maximum when $\cos 314t$ is unity. Hence,

6,280 amp per sec. *Ans.*

(e) $E'_m = -L\left(\frac{di}{dt}\right)_{\max} = -0.05 \cdot 6{,}280 = -314$ volts. *Ans.*

These values are shown in Fig. 28(*b*).

The quantitative effect of self-inductance on the current is determined as follows:

In (IV) the maximum value of e occurs when $\cos \omega t$ is unity. Hence,

$$E_m = L\omega I_m,$$
$$I_m = \frac{E_m}{L\omega}. \tag{V}$$

Dividing both sides of (V) by $\sqrt{2}$,

$$\frac{I_m}{\sqrt{2}} = \frac{E_m}{\sqrt{2}} \frac{1}{L\omega},$$
$$I = \frac{E}{L\omega} = \frac{E}{2\pi f L}, \tag{20}$$

where I and E are rms values of current and voltage.

Hence, in a circuit having inductance only, the current is *directly* proportional to the impressed voltage and is *inversely* proportional to the frequency and the self-inductance.

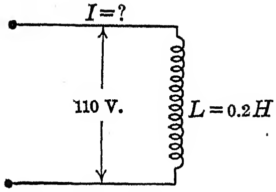


FIG. 29.—Circuit with inductance only.

That is, $2\pi f L$ is the choking effect, or the resistance to the flow of current, offered by inductance and is called the *inductive reactance* of the circuit. It is denoted by X_L and is expressed in *ohms*.

The impressed voltage is

$$E = 2\pi f L I = I X_L \quad \text{volts.} \tag{21}$$

Example.—Figure 29 shows a pure inductance of 0.2 henry connected across 110-volt 60-cycle mains. What is the current?

$$X_L = 2\pi 60 \cdot 0.2 = 377 \cdot 0.2 = 75.4 \text{ ohms.}$$
$$I = \frac{110}{75.4} = 1.46 \text{ amp.} \quad \textit{Ans.}$$

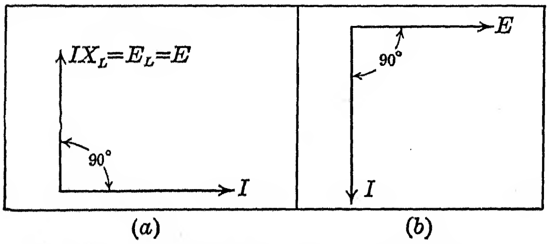


FIG. 30.—Vector diagrams with inductance only.

Figure 30 shows vector diagrams for a pure inductive circuit. In (*a*) the positive horizontal axis is chosen as the reference position for

the current vector. In (*b*) the same axis is chosen as the reference position for the voltage vector. In each case the impressed voltage leads the current by 90°, and either convention may be used. In fact, it is not necessary to confine either vector to one of the coordinate axes, but the diagram may have any position in the coordinate plane so long as the voltage leads the current by 90° and the vectors have the correct magnitudes. The impressed voltage *leads* the current by 90°.

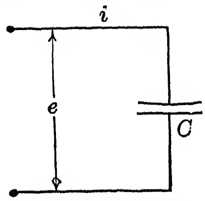


FIG. 31.—Circuit with capacitance only.

20. Circuit with Capacitance Only.—When a direct-current voltage is impressed across the plates of a perfect capacitor (Vol. I, Chap. X), there is an initial rush of current that charges the capacitor to the impressed voltage. After this there is no further current if the impressed voltage remains constant. If the capacitor plates now are short-circuited, making the capacitor voltage zero, current flows from the positive plate of the capacitor.

Figure 31 shows an alternating emf e impressed across the plates of a capacitor C. When the voltage starts from its zero value at a, Fig. 32, and increases positively, current flows into the capacitor from

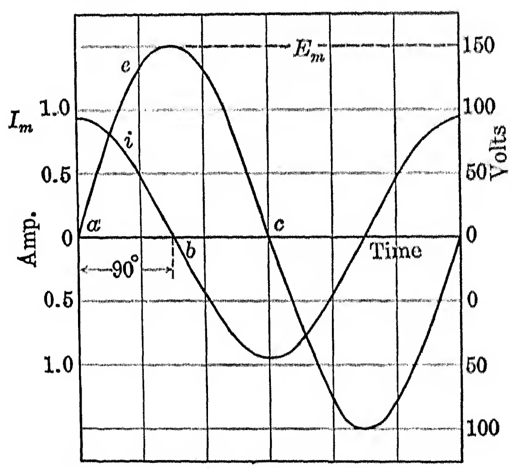


FIG. 32.—Current and emf waves with capacitance only.

the positive wire. This current, therefore, is positive. As long as the emf across the capacitor plates continues to increase, current must flow into the capacitor from the positive wire and this current will be positive in sign. When time b is reached, the increase of emf ceases and the current decreases to zero. Between b and c the emf is decreasing, and current is flowing *out* of the capacitor into the positive wire;

and as the current has reversed its direction, the sign of the current is now negative. This reversal of current is shown by the current wave i in Fig. 32. After e goes through zero at c, the emf is negative and the charge in the capacitor is reversed, so that the current still remains negative. This continues until the emf reaches its negative maximum. At this instant, the current reverses and again becomes positive.

An examination of Fig. 32 shows that, when an alternating emf is impressed across a capacitor, the current to the capacitor *leads* the

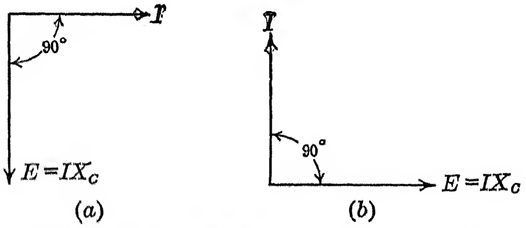


FIG. 33.—Vector diagrams with capacitance only.

impressed emf by 90°. This is illustrated by Fig. 33, in which the relation is shown vectorially. As in Fig. 30, in (*a*) the positive horizontal axis is chosen as the reference position for the current vector, and in (*b*) the same axis is chosen as the reference position for the voltage vector. In each case the current vector leads the voltage vector by 90°.

These relations of current and voltage in a capacitive circuit also may be proved as follows:

Let e be the instantaneous emf across the capacitor, C the capacitance in farads, and q the charge in coulombs at any instant.

Let $e = E_m \sin \omega t$ be the equation of the emf. Then $q = Ce$.

$$i = \frac{dq}{dt} = C\frac{de}{dt} = C\omega E_m \cos \omega t = C\omega E_m \sin(\omega t + 90°). \tag{I}$$

On the other hand, if the current $i = I_m \sin \omega t$ be given,

$$q = \int i\,dt = \int I_m \sin \omega t\,dt,$$

$$e = \frac{q}{C} = \frac{I_m}{C}\int \sin \omega t\,dt = \frac{I_m}{C\omega}(-\cos \omega t)$$

$$= \frac{I_m}{C\omega}\sin(\omega t - 90°). \tag{II}$$

(I) shows that the sine wave of current *leads* the impressed voltage wave by 90°, and (II) shows that the voltage *lags* the current by 90°, Fig. 32.

It will be seen from the foregoing that alternating current does not actually flow conductively through the insulation of the capacitor. A perfect capacitor offers an *infinite resistance* to alternating as well as to direct current. With alternating current, however, the capacitor is alternately charged and discharged, so that a quantity of electricity flows into the positive plate and then out again, etc. It is this quantity of electricity that flows to charge and to discharge the capacitor that constitutes the alternating current. An ammeter placed in the line to such a capacitor indicates a current. The more rapidly the voltage alternates, the greater the quantity of electricity charged and discharged per second and the greater the current. Hence, the current must be proportional to the frequency. This is further shown by (22).

In (I), the current reaches its maximum when $\cos \omega t = 1.0$ or, in (II), when $\sin (\omega t - 90°) = 1.0$.

Hence, in either case,

$$E_m = \frac{I_m}{C\omega}, \tag{III}$$

and

$$I_m = E_m C\omega. \tag{IV}$$

Dividing both sides of (IV) by $\sqrt{2}$,

$$\frac{I_m}{\sqrt{2}} = \frac{E_m}{\sqrt{2}} C\omega, \tag{V}$$

$$I = EC\omega = EC(2\pi f) = 2\pi fCE, \tag{22}$$

where I and E are rms values and C is in *farads*. The current is directly proportional to the voltage and capacitance as well as to the frequency.

In Fig. 32 the maximum instantaneous value of the emf is 150 volts, the frequency is 50 cycles, and the capacitance is 20 μf. Hence from (IV) the maximum instantaneous current is

$$I_m = 150 \cdot 20 \cdot 10^{-6} \cdot 2\pi 50 = 0.942$$

amp as shown. The rms value of the emf is $150/\sqrt{2} = 106.0$ volts, and the rms value of the current is $0.942/\sqrt{2} = 0.666$ amp.

Capacitance in an alternating-current circuit is somewhat analogous to conductance in the direct-current circuit. For example, in the direct-current circuit the current $I = EG$, where G is the conductance; likewise, with capacitance in the alternating-current circuit, the current $I = E(C\omega)$. The quantity $C\omega$ corresponds to G and is called *capacitive susceptance.*

Equation (22) may also be written

$$I = \frac{E}{1/2\pi fC} = \frac{E}{X_C}. \tag{23}$$

X_C is the *capacitive reactance* of the circuit, is expressed in ohms, and is equal to $1/2\pi fC$. Also,

$$E = \frac{I}{2\pi fC} = IX_C. \tag{24}$$

If the capacitance C is expressed in microfarads, X_C may be readily determined as follows:

$$X_C = \frac{1}{2\pi fC \cdot 10^{-6}} = \frac{10^6}{2\pi fC} \quad \text{ohms.} \tag{25}$$

Example.—What is the capacitive reactance of a 10-μf capacitor at 60 cycles per sec, and how much current will it take from 110-volt 60-cycle mains?

$$10\ \mu\text{f} = 0.00001 \text{ farad.}$$

$$X_C = \frac{1}{2\pi 60 \cdot 0.00001} = \frac{100{,}000}{2\pi 60} = 265 \text{ ohms.} \quad \textit{Ans.}$$

Also, using Eq. (25),

$$X_C = \frac{10^6}{2\pi 60 \cdot 10} = 265 \text{ ohms.} \quad \textit{Ans.}$$

$$I = {}^{110}\!/_{265} = 0.415 \text{ amp.} \quad \textit{Ans.}$$

The average power in a circuit containing capacitance only is zero.

This may be shown by plotting the power curve from the current and voltage curves, Fig. 34, as is done for a circuit with inductance only, Fig. 23.

In Fig. 34, as in Figs. 22, 23, 24, the maximum value of the emf is 180 volts, and the maximum value of the current is 15 amp. If the equations of the emf and current waves are $e = E_m \sin \omega t$ and $i = I_m \cos \omega t$, the equation of the power curve is

$$p = E_m I_m \sin \omega t \cos \omega t = \frac{E_m I_m}{2} \sin 2\omega t$$

(see p. 27). This is a double-frequency sine wave with the zero axis as its axis of symmetry, so that, as with inductance only, the average power is zero. In Fig. 34,

$$p = \frac{180 \cdot 15}{2} \sin 2\pi 120t = 1{,}350 \sin 754t.$$

Also at instants a, b, c, d, e, Fig. 34, either the emf or the current is zero so that the power is zero at each of these instants. Between a

and b, both emf and current are positive, the power is positive, and the capacitor is taking and storing energy from the source; between b and c the emf is positive and the current negative, so that the power is

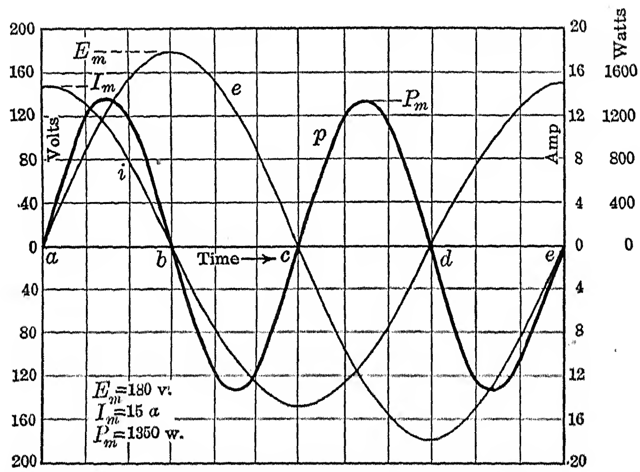


FIG. 34.—Voltage, current, and power waves with capacitance only.

negative and the capacitor is returning energy to the source. Between c and d both emf and current are negative, the power is positive, and the capacitor again is taking energy; between d and e, where the power is negative, the capacitor returns this energy to the source. Although the average power is zero, there is a transfer of energy alternately from the source to the capacitance and from the capacitance to the source.

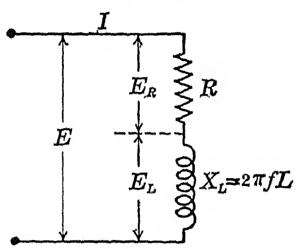


FIG. 35.—Circuit with resistance and inductance in series.

21. Resistance and Inductance in Series. Figure 35 shows a circuit of resistance R and inductive reactance X_L connected in series across an alternating-current circuit whose frequency is f cycles per sec. The voltage impressed across the circuit is E, and the current is I. Let it be required to determine the relations among I, E, R, X_L.

Figure 36(a) shows a vector diagram for this circuit. As the current I is the same in both R and X_L, it is laid off horizontally to scale. The position of the current vector I is arbitrary (it is given the position shown merely for convenience). From Fig. 26(b) (p. 29), the voltage E_R across the resistance R is *in phase* with the current. It is laid off, therefore, along the current vector, Fig. 36(a).

From Fig. 30(*a*) (p. 33), the voltage E_L across the inductance *leads* the current I by 90° and is equal to IX_L, Fig. 36(*a*).

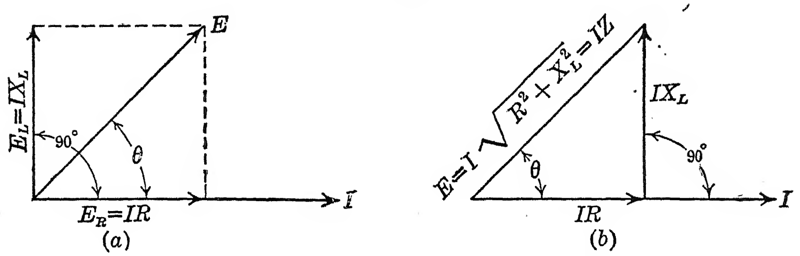


FIG. 36.—Vector diagram for circuit with resistance and inductance in series.

The line voltage E must be the vector sum of the voltages E_R and E_L. Hence, the parallelogram is completed, and the diagonal, which is the vector sum of E_R and E_L, gives the line voltage E. The same result is obtained if IX_L is laid off perpendicular to I at the end of the vector IR, using a triangle rather than a parallelogram, Fig. 36(*b*).

In the right triangle formed by these three voltages, the hypotenuse is

$$E = \sqrt{(IR)^2 + (IX_L)^2}$$
$$= \sqrt{I^2(R^2 + X_L^2)} = I\sqrt{R^2 + X_L^2},$$

and

$$I = \frac{E}{\sqrt{R^2 + X_L^2}} = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} = \frac{E}{Z}. \tag{26}$$

$Z = \sqrt{R^2 + X_L^2}$ is the *impedance* of the circuit, is expressed in ohms, and ordinarily is denoted by Z. Equation (26) corresponds to Ohm's law for the direct-current circuit. The current in an alternating-current circuit is *directly* proportional to the *voltage* across the circuit and *inversely* proportional to the *impedance* of the circuit. That is, if the voltage in volts be divided by the impedance in ohms, the value of the current in amperes is obtained.

Also, the voltage

$$E = IZ. \tag{27}$$

An inspection of Fig. 36 shows that the angle θ by which the current lags the voltage may be determined by

$$\tan\theta = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi fL}{R}, \tag{28}$$

$$\cos\theta = \frac{IR}{\sqrt{(IR)^2 + (IX_L)^2}} = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{Z}. \tag{29}$$

Example.—A circuit with 0.1 henry inductance and 20 ohms resistance in series is connected across 100-volt 25-cycle mains. Determine (*a*) impedance;

(*b*) current; (*c*) voltage across the resistance; (*d*) voltage across the inductance; (*e*) angle by which voltage leads current.

$X_L = 2\pi 25 \cdot 0.1 = 157 \cdot 0.1 = 15.7$ ohms.

(*a*) $Z = \sqrt{(20)^2 + (15.7)^2} = \sqrt{646} = 25.4$ ohms. *Ans.*

(*b*) $I = \frac{E}{Z} = \frac{100}{25.4} = 3.94$ amp. *Ans.*

(*c*) $E_R = IR = 3.94 \cdot 20 = 78.8$ volts. *Ans.*

(*d*) $E_L = IX_L = 3.94 \cdot 15.7 = 61.8$ volts *Ans.*

As a check, $\sqrt{(78.2)^2 + (61.8)^2} = 100$ volts.

(*e*) $\tan \theta = \frac{X_L}{R} = \frac{15.7}{20} = 0.785.$

From p. 608, $\theta = 38.1°$. *Ans.*

22. Power.—It has been shown already that a pure inductance consumes no power. During those periods when the current is increasing from zero to its maximum value, the energy received from the source is stored in the magnetic field of the inductance; during those periods when the current is decreasing from its maximum value to zero, all the energy stored by the inductance is returned to the source. Therefore, over a cycle the net energy taken by a pure inductance is zero. Hence, the inductance of Fig. 35 consumes no power. All the power expended in the circuit must be accounted for in the resistance. That is,

$$P = I^2R = I(IR).$$

IR is obviously equal to $E \cos \theta$, Fig. 36.

Therefore, the power

$$P = I(IR) = IE \cos \theta = EI \cos \theta, \tag{30}$$

which is the same as Eq. (14), p. 27.

As is shown in Sec. 17, $\cos \theta$ is the *power factor* of the circuit and is equal to the power divided by the volt-amperes.

$$\cos \theta = \text{P.F.} = \frac{P}{EI}.$$

The power factor $\cos \theta$ can never exceed 1.0. It is usually less than 1.0.

Example.—Determine the power and the power factor in the circuit (Sec. 21).

$$P = I^2R = (3.94)^2 \cdot 20 = 310 \text{ watts. } \textit{Ans.}$$

$$\text{P.F.} = \frac{P}{EI} = \frac{310}{100 \cdot 3.94} = 0.787. \quad \textit{Ans.}$$

Also,

$$\cos \theta = \text{P.F.} = \frac{R}{Z} = \frac{20}{25.4} = 0.787. \quad \textit{Ans.}$$

$$P = EI \cos \theta = 100 \cdot 3.94 \cdot 0.787 = 310 \text{ watts. } \textit{Ans.}$$

23. Resistance and Capacitance in Series.—Figure 37 shows a circuit with resistance R and capacitive reactance X_C in series. An alternating voltage E, of frequency f cycles per sec, is impressed across the circuit, and a current I results. It is required to determine the relation existing among E, I, R, X_C.

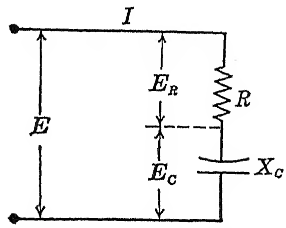


FIG. 37.—Circuit with resistance and capacitance in series.

The current I is the same in both R and X_C and is laid off horizontally in the vector diagram, Fig. 38. The voltage $E_R = IR$ across the resistance is *in phase* with the current. The voltage $E_C = IX_C$ across the capacitive reactance *lags* the current I by 90° [see Fig. 33(*a*) p. 35]. The line voltage E is the vector sum of IR and IX_C and is, therefore, the hypotenuse of the right triangle having these two voltages as sides. Hence,

$$E = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2} = IZ, \tag{31}$$

where Z is the *impedance* of the circuit.

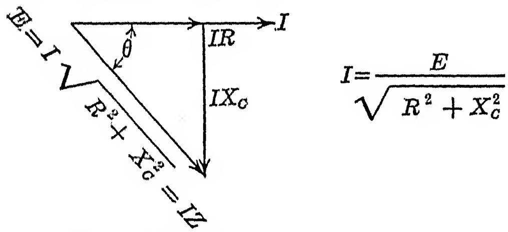


FIG. 38.—Vector diagram with resistance and capacitance in series.

Solving for the current I,

$$I = \frac{E}{\sqrt{R^2 + X_C^2}} = \frac{E}{\sqrt{R^2 + (1/2\pi fC)^2}} = \frac{E}{Z}. \tag{32}$$

The power taken by the circuit is

$$P = I^2R = I(IR)$$

as the power taken by the capacitive reactance is zero.

$$IR = E\cos\theta.$$

Therefore, $P = EI\cos\theta$, which is the same expression for power as with inductance and resistance in series.

The angle θ may be determined by

$$\tan\theta = \frac{-IX_C}{IR} = \frac{-X_C}{R} = \frac{-1}{2\pi fCR}. \tag{33}$$

Hence θ is negative, being in the fourth quadrant (p. 603).

$$\cos\theta = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + (1/2\pi fC)^2}} = \frac{R}{Z} = \text{P.F.}$$

C must be expressed in farads.

Example.—A capacitance of 20 μf and a resistance of 100 ohms are connected in series across 120-volt 60-cycle mains. Determine (*a*) impedance of the circuit; (*b*) current; (*c*) voltage across resistance; (*d*) voltage across capacitance; (*e*) angle between voltage and current; (*f*) power; (*g*) power factor.

$$20\ \mu\text{f} = 0.000020 \text{ farad.}$$

$$X_C = \frac{1}{2\pi 60 \cdot 0.000020} = \frac{10^6}{2\pi 60 \cdot 20} = 132.6 \text{ ohms.}$$

(*a*) $Z = \sqrt{(100)^2 + (132.6)^2} = \sqrt{27{,}600} = 166.0$ ohms. *Ans.*

(*b*) $I = \dfrac{120}{166.0} = 0.723$ amp. *Ans.*

(*c*) $E_R = IR = 0.723 \cdot 100 = 72.3$ volts. *Ans.*

(*d*) $E_C = IX_C = 0.723 \cdot 132.6 = 95.9$ volts. *Ans.*

$$\sqrt{(72.3)^2 + (95.9)^2} = 120 \text{ volts } (\textit{check}).$$

(*e*) $\tan\theta = \dfrac{-X_C}{R} = \dfrac{-132.6}{100} = -1.326.$

$\theta = -53.0°$. *Ans.*

(*f*) $P = I^2R = (0.723)^2 \cdot 100 = 52.2$ watts. *Ans.*

(*g*) $\cos\theta = \dfrac{R}{Z} = \dfrac{100}{166.0} = 0.602.$ *Ans.*

Also,

$$P = 120 \cdot 0.723 \cdot \cos(-53.0°) = 52.2 \text{ watts. } \textit{Ans.}$$

Also,

$$\text{P.F.} = \frac{P}{EI} = \frac{52.2}{120 \cdot 0.723} = 0.602 \ (\textit{check}).$$

24. Resistance, Inductance, and Capacitance in Series.—Figure 39 shows resistance R, inductive reactance X_L, and capacitive reactance X_C, connected in series. The voltage across the circuit is E volts, the frequency is f cycles per sec, and the current is I amp.

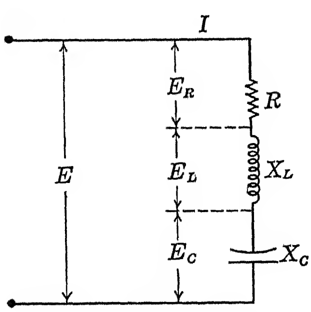


FIG. 39.—Circuit with resistance, inductance, and capacitance in series.

As this is a series circuit, the current is the same in all parts of the circuit, and for convenience the current vector I is laid off horizontal in the circuit vector diagram, Fig. 40. [In (*a*) the parallelogram of vectors is used, in (*b*) the polygon of vectors is used.] The voltage $E_R = IR$ across the resistance is *in phase* with the current and is laid off to scale along the current vector. The voltage $E_L = IX_L$ across the inductance is laid off at right angles to the current and *leading*. The voltage

$E_C = IX_C$ across the capacitance is laid off at right angles to the current and *lagging*.

The voltage across the inductance and that across the capacitance are in *opposition*, Fig. 40, so that the resultant voltage of these two is their arithmetical difference. In Fig. 40, IX_L is shown greater than IX_C. IX_C, therefore, is subtracted directly from IX_L, or added

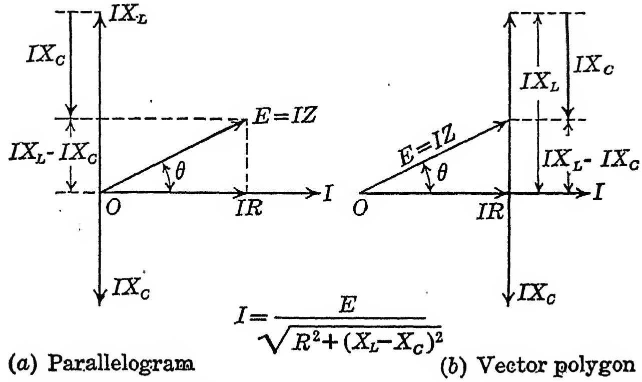


FIG. 40.—Vector diagrams with resistance, inductance, and capacitance in series.

vectorially to IX_L. The line voltage must be the vector sum of the three voltages and is the hypotenuse of a right triangle of which IR and $IX_L - IX_C$ are the sides. Therefore,

$$\begin{aligned} E &= \sqrt{(IR)^2 + (IX_L - IX_C)^2} \\ &= I\sqrt{R^2 + (X_L - X_C)^2} = IZ. \end{aligned} \tag{34}$$

Solving for I,

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E}{Z}, \tag{35}$$

which is the equation for the series alternating-current circuit in the *steady* state.

The values of X_L and X_C may be substituted in (35), which becomes

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}. \tag{36}$$

The phase angle θ is found by

$$\tan\theta = \frac{X_L - X_C}{R}. \tag{37}$$

If X_L is greater than X_C, the tangent is positive and θ is positive, as in Fig. 40. This shows that the current lags. If X_C is greater

than X_L, the tangent is negative and the angle θ is negative. This shows that the current leads.

The power factor of the circuit

$$\text{P.F.} = \cos\theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}. \tag{38}$$

Example.—A series circuit with a resistance of 50 ohms, a capacitance of 25 μf, and an inductance of 0.15 henry is connected across 120-volt 60-cycle mains.

Determine (*a*) impedance of the circuit; (*b*) current; (*c*) voltage across resistance; (*d*) voltage across inductance; (*e*) voltage across capacitance; (*f*) phase angle of circuit; (*g*) power factor of circuit; (*h*) power given to circuit.

$$X_L = 2\pi 60 \cdot 0.15 = 377 \cdot 0.15 = 56.6 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi 60 \cdot 0.000025} = 106 \text{ ohms.}$$

(*a*) $Z = \sqrt{(50)^2 + (56.6 - 106)^2} = \sqrt{(50)^2 + (-49.4)^2} = 70.2$ ohms. *Ans.*

(*b*) $I = \dfrac{120}{70.2} = 1.71$ amp. *Ans.*

(*c*) $E_R = IR = 1.71 \cdot 50 = 85.5$ volts. *Ans.*

(*d*) $E_L = IX_L = 1.71 \cdot 56.6 = 96.8$ volts. *Ans.*

(*e*) $E_C = IX_C = 1.71 \cdot 106 = 181.1$ volts. *Ans.*

(*f*) $\tan\theta = \dfrac{X_L - X_C}{R} = \dfrac{56.6 - 106}{50} = \dfrac{-49.4}{50} = -0.988.$

$\theta = -44.7°$. Therefore, the current leads. *Ans.*

(*g*) $\cos\theta = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{50}{70.2} = 0.711.$ *Ans.*

$\cos\theta = \dfrac{P}{EI} = \dfrac{146}{120 \cdot 1.71} = 0.711$ (*check*).

(*h*) $P = 120 \cdot 1.71 \cdot 0.711 = 146$ watts. *Ans.*

Also,

$$P = I^2R = (1.71)^2 \cdot 50 = 146 \text{ watts (}\textit{check}\text{).}$$

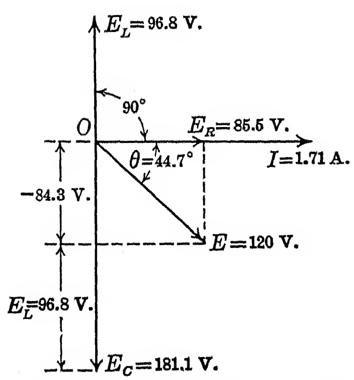


FIG. 41.—Vector diagram for series circuit, with numerical values.

Figure 41 gives the vector diagram for the circuit conditions of this example.

It will be noted that the magnitude of the voltage across the capacitance is greater than the line voltage by a considerable amount. This would be impossible in a direct-current circuit, for the voltage across any part of the circuit cannot exceed the line voltage. This condition can exist in an alternating-current circuit, because the capacitance voltage and the

inductance voltage are in phase opposition. Both may be large, provided that their difference is less than the line voltage.

25. Resonance in Series Circuit.—The general equation (36) for the current in a series circuit in the steady state shows that for fixed values of resistance and impressed voltage the current is a maximum when the expression in the parentheses under the square-root sign is equal to zero.

That is, in the equation

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi Cf}\right)^2}}$$

the current is a maximum when

$$2\pi fL - \frac{1}{2\pi fC} = 0$$

and then is

$$I = \frac{E}{\sqrt{R^2 + (0)}} = \frac{E}{R},$$

its Ohm's-law value.

Under these conditions,

$$2\pi fL = \frac{1}{2\pi fC}, \tag{39}$$

and

$$2\pi fLI = \frac{I}{2\pi fC}. \tag{40}$$

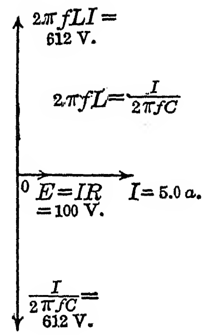


Fig. 42.—Vector diagram for series circuit in resonance.

That is, the voltage across the inductance is equal to the voltage across the capacitance. As these two voltages are in phase opposition, they balance each other, so that the IR drop is equal to the line voltage. This is illustrated in Fig. 42.

When the foregoing conditions exist, the circuit is said to be in *resonance.* The current is then in phase with the line voltage, and the power $P = EI$.

Solving Eq. (39) for the resonant frequency f_r,

$$4\pi^2 LCf_r^2 = 1,$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \tag{41}$$

It follows from (41) that

$$LC\omega_r^2 = 1, \tag{42}$$

where $\omega_r = 2\pi f_r$.

As the voltage across the inductance equals the voltage across the capacitance when the circuit is in resonance and the two are in opposition, each may reach a high value, even with moderate line voltage. This is illustrated by the following example:

Example.—A circuit has a resistance of 20 ohms, an inductance of 0.3 henry, a capacitance of 20 μf, and the current is 5.0 amp. Determine (*a*) frequency at which circuit will be in resonance; (*b*) line voltage; (*c*) voltage across inductance; (*d*) voltage across capacitance; (*e*) power to circuit. (*f*) Draw vector diagram.

(*a*) $f_r = \dfrac{1}{2\pi\sqrt{0.3 \cdot 0.000020}} = 65$ cycles. *Ans.*

(*b*) $E = IR = 5 \cdot 20 = 100$ volts. *Ans.*

(*c*) $E_L = 2\pi f_r LI = 6.28 \cdot 65 \cdot 0.3 \cdot 5 = 612$ volts. *Ans.*

(*d*) $E_C = \dfrac{I}{(2\pi f_r C)} = 612$ volts. *Ans.*

(*e*) $P = EI = 100 \cdot 5 = 500$ watts. *Ans.*

(*f*) The vector diagram is shown in Fig. 42.

It is to be noted that the voltage across the inductance and that across the capacitance are equal, each being 612 volts, or more than six times the line voltage.

It should be noted also that the current is a *maximum* when a series circuit is in resonance.

26. Resonance Characteristics of Series Circuits.—In any circuit whose frequency is fixed, there is an infinite number of combinations of inductance and capacitance that will give resonance. This may be seen from an examination of Eq. (41). It is merely necessary that the product LC remain constant. For example, after the circuit has been adjusted to resonance, if the inductance be halved and the capacitance be doubled, the resonant condition still exists. But the manner in which the current alters as the frequency changes depends on the relation of the inductance to the capacitance. This is illustrated in Fig. 43. The voltage across a circuit having 10 ohms resistance is maintained constant at 100 volts. The circuit is first tuned to 60 cycles by adjusting the inductance and capacitance to 0.02 henry and 352 μf. The variation of current with frequency under these conditions is shown by curve I. The current is zero at zero frequency, since the capacitor gives an open circuit for direct current. The current reaches its maximum when the frequency becomes 60 cycles per sec. The current is zero at infinite frequency, since, with inductance, the inductive reactance is infinite at infinite frequency. Curve II shows the variation of current with frequency when the inductance is 0.05 henry and the capacitance is 140.8 μf. The values of current, except at the resonant frequency, are now considerably less than those

given by curve I. Curve III shows the variation of current with frequency when the inductance is 0.1 henry and the capacitance is 70.4 μf ($LC = 7.04 \cdot 10^{-6}$), curve IV shows the variation of current with frequency when the inductance is 0.4 henry and the capacitance is 17.6 μf ($LC = 7.04 \cdot 10^{-6}$).

It is to be noted that, as the inductance is increased and the capacitance is correspondingly decreased, the tuning of the circuit

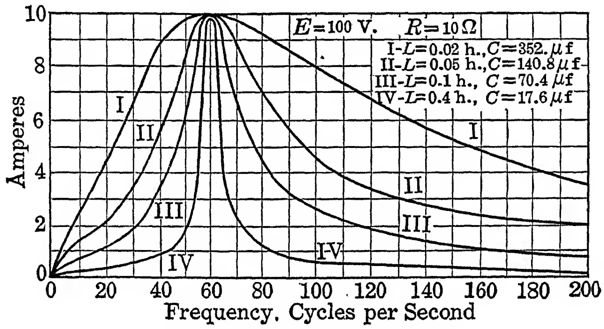


FIG. 43.—Resonance curves.

becomes *sharper;* that is, a small variation of frequency on either side of the resonant frequency causes a large decrease in current. The tuning with curve IV is very sharp.

This relation is particularly useful in communication circuits, for example, in radio receiving sets, where sharp tuning is often essential.

It is to be noted that for all the curves I to IV the product of L and C is constant and is equal to $7.04 \cdot 10^{-6}$.

27. Selectivity of Resonant Circuit.—In Fig. 43 it is shown that for a given value of resistance the sharpness of tuning or the selectivity of an a-c circuit depends on the relative values of L and C. To have some means for comparing the selectivity of different circuits, a value of current equal to the maximum or Ohm's-law value divided by $\sqrt{2}$ is chosen arbitrarily, and the frequency range $f_2 - f_1$, Fig. 44, over which the current will exceed this value is determined. If the resonant frequency is f_r, the measure of selectivity of the circuit, given by Q, is defined as

$$Q = \frac{f_r}{f_2 - f_1}. \qquad (43)$$

In Fig. 44, let the maximum rms value of the current be I_M where $I_M = E/R$. E is the impressed voltage and R is the effective[1] resistance of the circuit. f_1 and f_2 are the frequencies corresponding to $I_M/\sqrt{2}$.

[1] See Sec. 31 p. 55.

Then,

$$\frac{I_M}{\sqrt{2}} = \frac{E}{R}\frac{1}{\sqrt{2}} = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}, \tag{I}$$

where $\omega = 2\pi f$ has two values, $2\pi f_1$ and $2\pi f_2$, f_1 and f_2 being the two roots of the

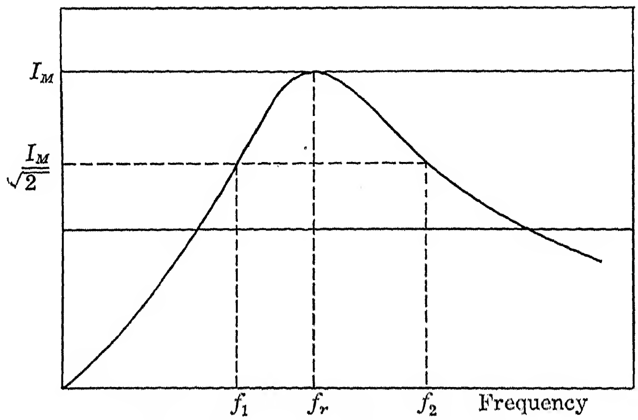


FIG. 44.—Selectivity of tuned circuit.

quadratic equation. Squaring the two right-hand terms of (I) and equating the denominators, since the numerators E^2 are the same,

$$2R^2 = R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2. \tag{II}$$

Rearranging (II) and taking the square root,

$$2\pi fL - \frac{1}{2\pi fC} = \pm R. \tag{III}$$

Multiplying (III) by $2\pi fC$,

$$4\pi^2 f^2 LC - 1 = \pm 2\pi fCR, \tag{IV}$$

$$4\pi^2 f^2 LC \mp 2\pi fCR = 1. \tag{V}$$

Dividing (V) by $4\pi^2 LC$ and completing the square,

$$f^2 \mp \frac{R}{2\pi L}f + \left(\frac{R}{4\pi L}\right)^2 = \left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}, \tag{VI}$$

$$f = \pm\frac{R}{4\pi L} \pm \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}. \tag{VII}$$

The term under the radical is obviously greater than $R/4\pi L$ so that if the negative sign before the radical were used a negative value of frequency would be obtained, which is physically impossible. Hence only the positive sign can be used.

$$f_2 = +\frac{R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}. \tag{VIII}$$

$$f_1 = -\frac{R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}. \tag{IX}$$

$$f_2 - f_1 = \frac{R}{2\pi L}.$$

Hence, from (43),

$$Q = \frac{f_r}{f_2 - f_1} = \frac{f_r}{R/2\pi L} = \frac{2\pi f_r L}{R}. \tag{44}$$

Reference is frequently made to the Q of a circuit, meaning its selectivity as measured by (44). Usually in such tuned circuits, there is no appreciable resistance except that of the wire in the inductance coil. Hence, from Eq. (28) (p. 39), Q is equal to the tangent of the phase angle θ of the coil. (Also see Sec. 31). At radio frequencies particularly, there may be appreciable losses in the capacitor, and R

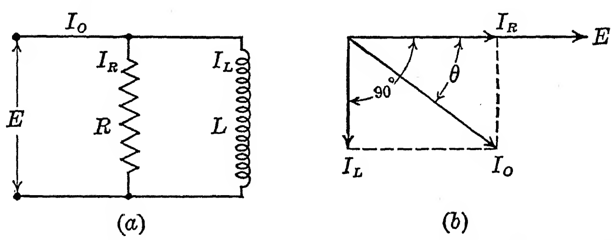


FIG. 45.—Resistance and inductance in parallel, with vector diagram.

may not be constant but will change with frequency owing to skin effect and other similar effects.

The dissipation factor D of a circuit is defined as $1/Q$, so that $D = \cot\theta$.

28. Parallel Circuits.—In practice, parallel circuits are more common than series circuits, because of the extended use of the multiple system of transmission and distribution. The solution of problems with two or more loads in parallel involves finding the current in each branch of the circuit and then combining these currents *vectorially* to give the resultant current.

This is illustrated in Fig. 45, which shows resistance and inductance in parallel, and the vector diagram. The voltage E is common to both branches so that its position is taken along the positive axis of abscissas. The resistance current I_R is in phase with E, and the inductance current I_L lags E by 90°. The resultant current I_0 is their vector sum.

$$I_0 = \sqrt{I_R^2 + I_L^2}. \tag{45}$$

$$\tan\theta = \frac{-I_L}{I_R} = \frac{-E/X_L}{E/R} = \frac{-R}{X_L}. \tag{46}$$

Similarly, Fig. 46 shows resistance and capacitance in parallel, together with the vector diagram. The capacitive current I_C leads E by 90°.

$$I_0 = \sqrt{I_R^2 + I_C^2} \tag{47}$$

$$\tan\theta = \frac{I_C}{I_R} = \frac{E/X_C}{E/R} = \frac{R}{X_C}. \tag{48}$$

Equations (46) and (48) should be compared with Eqs. (28) and (33), (pp. 39 and 41) for the series circuit. Also, see (50), (52), (54) for $\cos\theta$.

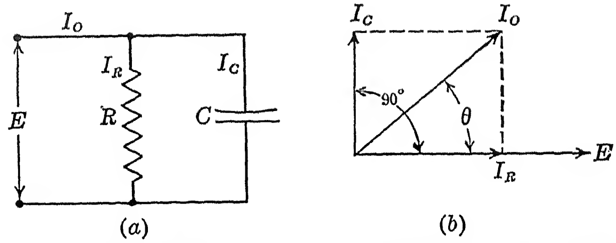


FIG. 46.—Resistance and capacitance in parallel, with vector diagram.

The following example illustrates the method for finding the currents with resistance, inductance, and capacitance in parallel:

Example.—A resistance of 10 ohms, an inductive reactance of 8 ohms, and a capacitive reactance of 15 ohms are connected in parallel across 120-volt 60-cycle

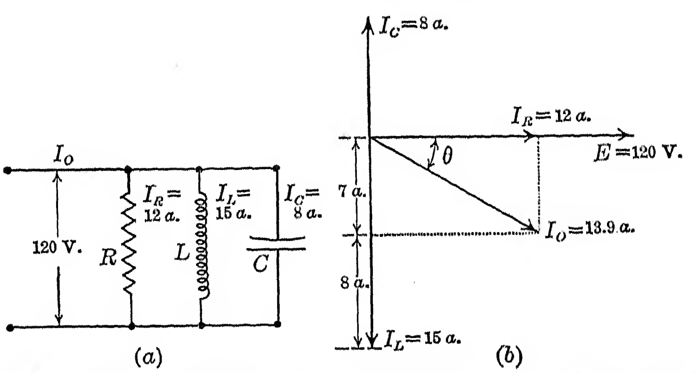


FIG. 47.—Resistance, inductance, and capacitance in parallel, with vector diagram.

mains, Fig. 47(*a*). Determine (*a*) total current; (*b*) circuit power factor; (*c*) power.

The currents taken by the resistance, inductive reactance, and capacitive reactance are

$$I_R = {}^{120}\!/_{10} = 12 \text{ amp in phase with } E.$$
$$I_L = {}^{120}\!/_{8} = 15 \text{ amp in quadrature with } E \text{ and lagging.}$$
$$I_C = {}^{120}\!/_{15} = 8 \text{ amp in quadrature with } E \text{ and leading.}$$

These currents are shown vectorially in Fig. 47(*b*).

The voltage E is the same for all three branches of the circuit and is laid off as a horizontal vector. The resistance current I_R is in phase with the voltage E. The inductive current I_L *lags* the voltage by 90°, and the capacitive current I_C *leads* the voltage by 90°. As the inductive current and capacitive current are in phase opposition, they subtract arithmetically from each other, giving 7 amp lagging by 90°. The resultant current I_0 is the vector sum of the 7 amp and the 12 amp.

(a) $I_0 = \sqrt{12^2 + 7^2} = 13.9$ amp. lagging. *Ans.*
From Fig. 47(b),

(b) The cosine of the angle θ between the voltage and the current is

$$\cos\theta = \frac{I_R}{I_0} = \frac{12}{13.9} = 0.864 = \text{P.F.} \quad \textit{Ans.}$$

$$\theta = -30.2°.$$

(c) $P = EI_R = 120 \cdot 12 = 1{,}440$ watts. *Ans.*
Also,

$$P = EI_0 \cos\theta = 120 \cdot 13.9 \cdot 0.864 = 1{,}440 \text{ watts.} \quad \textit{Ans.}$$

For convenience, the following equations are given for parallel circuits:

R and L in parallel:

$$Z = \frac{1}{\sqrt{(1/R)^2 + (1/X_L)^2}} = \frac{RX_L}{\sqrt{R^2 + X_L^2}}. \tag{49}$$

$$I_0 = \frac{E}{Z}; \qquad \cos\theta = \frac{I_R}{I_0} = \frac{E/R}{E/Z} = \frac{Z}{R}. \tag{50}$$

R and C in parallel:

$$Z = \frac{1}{\sqrt{(1/R)^2 + (1/X_C)^2}} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}. \tag{51}$$

$$I_0 = \frac{E}{Z}; \qquad \cos\theta = \frac{Z}{R}. \tag{52}$$

R, L, and C in parallel:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{RX_LX_C}{\sqrt{X_L^2X_C^2 + R^2(X_C - X_L)^2}}. \tag{53}$$

$$I_0 = \frac{E}{Z}; \qquad \cos\theta = \frac{Z}{R}, \tag{54}$$

where I_0 is the total current and E is the circuit voltage.

In (49) to (54), R is the value in ohms of the resistance element R of the circuits in Figs. 45 to 47 and is not the equivalent resistance of the entire circuit. (See Sec. 59, p. 85.)

29. Resonance in Parallel Circuit.—Resonance (or antiresonance)[1] in a parallel circuit occurs when the resultant current and the line voltage are in phase. Under these conditions, the capacitive current must be equal to the inductive current. These two, being opposite and equal, will balance each other, leaving only the resistance current. This is illustrated in Fig. 48(a). e is the voltage wave; i_r is the current in the resistance; i_l is the current in the inductance; i_c is the current in the capacitance and is equal and opposite to i_l. As the inductive current *lags* the voltage by 90° and the capacitive current *leads* the

[1] This is frequently called the *antiresonance*, to distinguish it from the resonance, which, in a generalized network, occurs when the current is a maximum.

voltage by 90°, they are in phase opposition; and, being equal, they balance.

Figure 48(*b*) illustrates vectorially these circuit conditions, rms values being used and the scale in (*b*) being different from that in (*a*). E is the line voltage, I_R the current in the resistance, I_L the current in the inductance, and I_C the current in the capacitance.

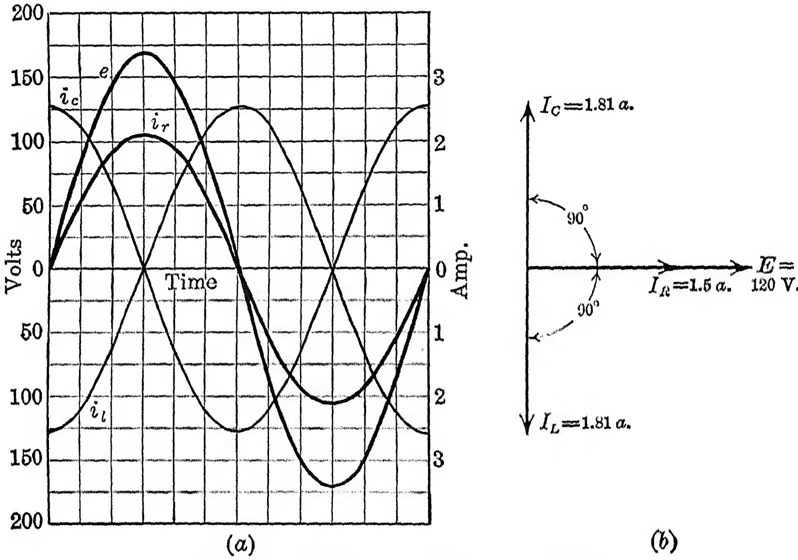


FIG. 48.—Antiresonance in parallel circuit.

It is to be noted that the total current is a *minimum* when the *parallel* circuit is in resonance (or antiresonance), whereas in the *series* circuit the current is a *maximum* at resonance. (Compare Figs. 49 and 43.) In the *parallel* circuit the inductive and capacitive *currents* are opposite and equal at resonance; in the *series* circuit the inductive and capacitive *voltages* are opposite and equal at resonance. If a pure capacitance and a pure inductance were connected in parallel and adjusted for resonance, the line current would be zero, even though the inductance and capacitance were each taking current. Since at resonance the inductive current is $E/2\pi f_r L$ and the capacitive current is $2\pi f_r CE$,

$$\frac{E}{2\pi f_r L} = 2\pi f_r CE,$$

and

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \text{cycles per sec.} \qquad (55)$$

Also,

$$LC\omega_r^2 = 1, \tag{56}$$

where f_r and ω_r are the resonant frequency and the resonant angular velocity.

These relations are the same as those for resonance in the series circuit [Eqs. (41), (42), p. 45)]. Equations (55) and (56) are valid when the inductive and capacitive branches contain only pure inductance and pure capacitance. When there is resistance in either the inductive or the capacitive branch, this relationship *does not hold* (see Sec. 62, p. 88).

Example.—A resistance of 80 ohms, an inductance of 0.176 henry, and a capacitance are connected in parallel across 120-volt 60-cycle mains. Determine (*a*) value of capacitance for antiresonance; (*b*) total current; (*c*) power.

(*a*) I_C must be equal to I_L.

$$I_L = \frac{120}{2\pi 60 \cdot 0.176} = 1.81 \text{ amp.}$$
$$I_C = 120 \cdot 2\pi 60 \cdot C = 1.81 \text{ amp.}$$
$$C = \frac{1.81}{120 \cdot 2\pi 60} = 0.0000400 \text{ farad}$$
$$= 40.0\ \mu\text{f.}\quad \textit{Ans.}$$

(*b*) Since I_L and I_C are opposite and equal, they cancel, leaving only I_R. Hence,

$$I = I_R = {}^{120}\!/_{80} = 1.50 \text{ amp.}\quad \textit{Ans.}$$

(*c*) The inductance and capacitance take no total power, and all the power is accounted for by the resistance. Hence,

$$P = 120 \cdot 1.5 = 180 \text{ watts.}\quad \textit{Ans.}$$

The instantaneous values of these quantities are shown in Fig. 48(*a*), where $E_m = 120\sqrt{2} = 170$ volts; $I_{mR} = 1.5\sqrt{2} = 2.11$ amp;

$$I_{mL} = I_{mC} = 1.81\sqrt{2} = 2.56 \text{ amp.}$$

The corresponding vector diagram is shown in (*b*).

30. Resonance Characteristics of Parallel Circuits.—In Fig. 43 the current in a circuit with resistance, inductance, and capacitance in series is shown as a function of the frequency. The current is zero at zero and at infinite frequency and is a maximum at the resonant frequency. In Fig. 49 the current in a circuit with resistance, inductance, and capacitance in parallel is shown as a function of the frequency. The applied voltage is 50 volts. The inductance is 0.00531 henry, and the capacitance is 4.78 μf so that the resonant frequency is 1,000 cycles [Eq. (55)]. There are two current curves, one for the circuit when the parallel resistance R, Fig. 47 (*a*), is 100 ohms and the other for the circuit when the parallel resistance is infinite, that is, R, Fig. 47(*a*),

is open-circuited. There are also two impedance curves shown by dashed lines, one corresponding to 100 ohms parallel resistance and the other corresponding to infinite parallel resistance. The data for these curves were computed from Eq. (53). At zero frequency the reactance of the inductance is zero so that the impedance of the circuit is zero and the current is infinite. At infinite frequency the reactance of the capacitance is zero so that the impedance of the circuit again is

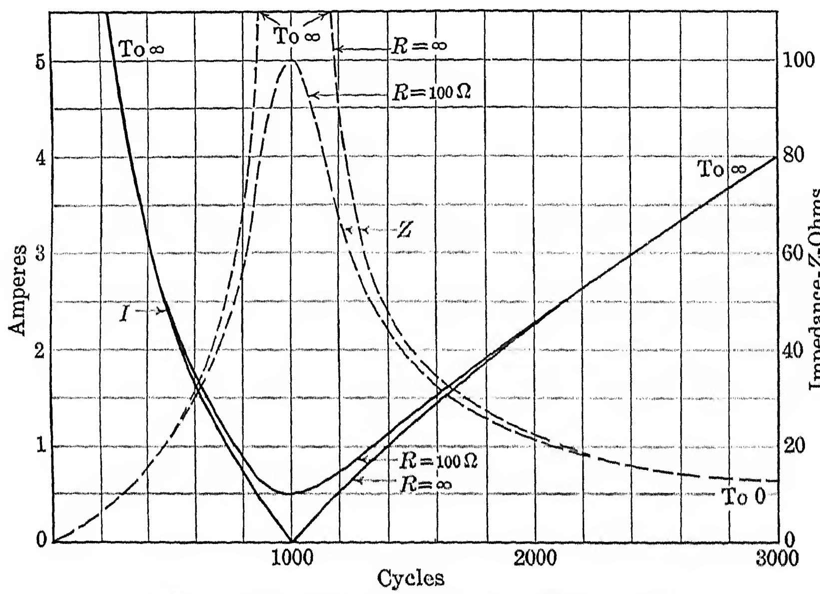


FIG. 49.—Resonance characteristics of parallel circuit.

zero and the current is infinite. At the resonant frequency of 1,000 cycles the current is a minimum, rather than a maximum as it is with the series circuit. With the 100-ohm resistance, the current at the resonant frequency is 0.5 amp; with the infinite resistance, the impedance is infinite and the current is zero. This latter condition is represented by the vector diagram in Fig. 48 (*b*) if the current I_R to the resistance is assumed to be zero. Under these conditions the inductive and capacitive current are opposite and equal, and the resultant current is zero. In practice, these conditions cannot be attained. There must be losses in the inductance and in the capacitance so that always there will be a resultant current, although it may be small.

The resemblance of the 100-ohm impedance curve, Fig. 49, to the current curves of Fig. 43 should be noted. Similarily, if impedance curves were drawn, Fig. 43, they would resemble the current curve, Fig. 49.

31. Effective Resistance.—A coil of copper wire with an air core is connected across a direct-current source, and its resistance is measured. The voltage across the coil is 22 volts when the current is 4.6 amp. This makes its resistance 4.78 ohms. This same coil is connected across 110 volts, 60 cycles. It then takes 1.2 amp, and a wattmeter in circuit shows that the coil is taking 7.3 watts. If the direct-current resistance were used, the power should be only

$$(1.2)^2 \cdot 4.78 = 6.89 \text{ watts.}$$

The greater loss with alternating current is due to the fact that the alternating current is not distributed uniformly over the cross section of the wire (skin effect); also, the resulting flux induces eddy currents in the conductor.

If an iron core be inserted in this coil, the voltage and frequency being maintained constant, the current drops to 0.20 amp and the power becomes 0.26 watt. The power calculated on the basis of the direct-current resistance would be $(0.20)^2 \cdot 4.78 = 0.191$ watt. The excess power over the calculated direct-current power is accounted for not only by the effects just mentioned but also by the eddy-current and hysteresis losses in the iron caused by the alternating flux. It is seen that, with a given value of current, the losses with alternating current may be greater than with direct current. Under such conditions the apparent resistance of the circuit with alternating current is greater than with direct current. The apparent resistance with alternating current is called *effective* resistance.

If R_e be the *effective* resistance of a circuit, the power loss P for a current I is

$$P = I^2R_e$$

and

$$R_e = \frac{P}{I^2}. \tag{57}$$

For example, in the illustration just given the *effective* resistance of the coil *without* iron is $7.3/(1.2)^2 = 5.07$ ohms, which is 6 per cent greater than the direct-current resistance. *With* iron, the effective resistance is $0.26/(0.20)^2 = 6.5$ ohms, or 36 per cent greater than the direct-current resistance.

32. Polygon of Voltages; Three Voltages.—The inductances and capacitances so far considered have been assumed as perfect, that is, as having no losses, so that the phase angle of current with respect to voltage is exactly 90°. In practice, this condition is impossible of realization. It is shown in Sec. 31 that because of the resistance of the wire and because of iron losses, if an iron core is used, there must

be losses in any inductor or impedance coil. With a moderately careful design the phase angle of impedance coils may be made as great as 87°, but coils having larger angles than this are very difficult to design, and the expense of construction becomes relatively large.

Capacitors, as a rule, have very small losses, and their phase angles are nearly 90°; but even such capacitors are not pure. Carefully constructed air capacitors may have an angle that differs from 90° by only 2 or 3 minutes.

Losses in inductors and capacitors may be taken into consideration by assuming a pure inductance or capacitance and then adding series resistance, called the *effective resistance* (Sec. 31) and sometimes the

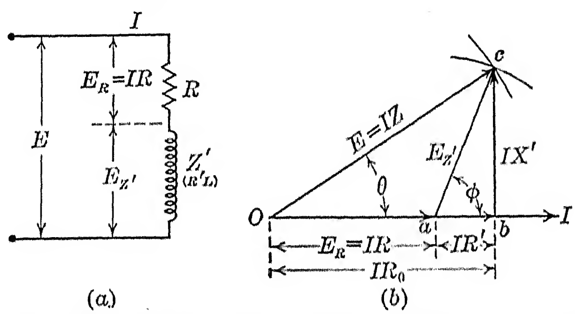


FIG. 50.—Circuit with resistance and impedance in series, and vector diagram.

equivalent resistance, to account for the losses. The equivalent resistance may be combined with other resistances in the circuit to obtain the total resistance of the circuit.

Figure 50 (*a*) shows a series circuit connected across an alternating voltage E, of frequency f. This circuit has a resistance R and an impedance coil Z', of an effective resistance R' and inductance L. The reactance X' of the impedance coil is $2\pi fL$. Figure 50(*b*) shows the vector diagram for this circuit. The voltage IR is in phase with the current I. The voltage $E_{Z'}$ across the impedance coil leads the current by an angle ϕ that is less than 90°, owing to the effective resistance R' of the impedance coil. The circuit voltage E is the vector sum of IR and $E_{Z'}$. The impedance voltage $E_{Z'}$ consists of two components, IR' in phase with the current and IX' in quadrature with the current. The impedance coil itself may be considered as a simple series circuit consisting of a resistance R' and a reactance X', Fig. 36 (p. 39). Therefore the projection on the current vector of the voltage $E_{Z'}$ across the impedance is the voltage drop due to the resistance of this impedance. Divide this projected voltage by the current and the effective resistance R' of the impedance coil is obtained. The circuit may be considered as consisting of an equivalent pure

resistance $R_0 = R + R'$ and a pure reactance X' in series, Fig. 50(*b*).

A voltmeter across the resistance R measures the voltage E_R; across the impedance, it measures the voltage $E_{Z'}$; across the line, it measures the voltage E.

To construct the vector diagram for this circuit, the current vector I is laid off horizontally, as shown in (*b*). The voltage $E_R = IR = Oa$ is laid off to scale in phase with the current I; from the outer end a of E_R an arc ac is swung upward having $E_{Z'}$ for its radius. Then with O, the origin, as a center, another arc Oc having

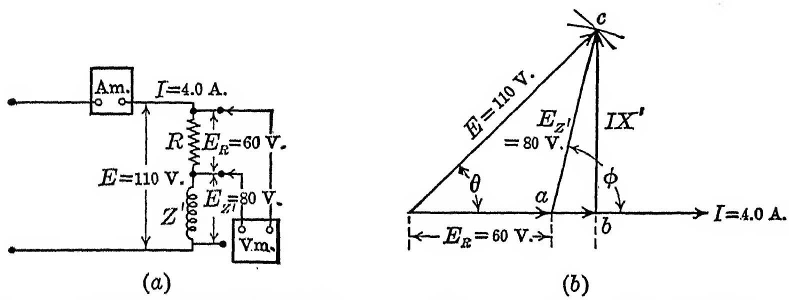


FIG. 51.—Circuit with resistance and inductive impedance in series and polygon of voltages.

E for its radius is swung to intersect the arc ac at c. Lines drawn from the end of E_R and from O to the intersection c of the arcs ac and Oc complete the vector diagram. Thus the line voltage E is made to equal the vector sum of the two component voltages E_R and $E_{Z'}$; θ, the circuit power-factor angle, and ϕ, the impedance-coil power-factor angle, can both be found by trigonometry, as is illustrated in the following example. The line voltage E and the current I are known. Hence, after θ is determined, it is a simple matter to find the power and the power factor of the circuit.

Example.—A resistance and an impedance coil are connected in series across a 60-cycle alternating-current circuit, Fig. 51(*a*); the current is 4.0 amp. The voltage across the resistance is found to be 60 volts; that across the impedance coil 80 volts; and the line voltage 110 volts. Determine (*a*) resistance R; (*b*) circuit power-factor angle θ and power factor; (*c*) impedance-coil power-factor angle ϕ and the corresponding coil power factor; (*d*) circuit power; (*e*) impedance-coil power; (*f*) impedance-coil effective resistance; (*g*) impedance-coil reactance; (*h*) equivalent resistance R_0 of the circuit.

The vector diagram, Fig. 51(*b*), is constructed in the same manner as Fig. 50(*b*).

(*a*) $R = \dfrac{E_R}{I} = \dfrac{60}{4} = 15.0$ ohms. *Ans.*

(*b*) Applying the law of cosines (p. 605) to Fig. 51(*b*),

$$\overline{80}^2 = \overline{110}^2 + \overline{60}^2 - 2 \cdot 110 \cdot 60 \cos \theta.$$

$$\cos \theta = \frac{9{,}300}{13{,}200} = 0.704.$$

$$\theta = 45.2°. \quad \textit{Ans.}$$

$$\text{P.F.} = \cos \theta = 0.704. \quad \textit{Ans.}$$

(c) $bc = IX' = E \sin \theta = 110 \cdot 0.7096 = 78.05$ volts.

$$\sin \phi = \frac{bc}{ac} = \frac{78.05}{80} = 0.9757.$$

$$\phi = 77.35°. \quad \textit{Ans.}$$

$$\cos \phi = 0.219. \quad \textit{Ans.}$$

(d) Circuit power

$$P = 110 \cdot 4 \cdot \cos \theta = 440 \cdot 0.704 = 310 \text{ watts.} \quad \textit{Ans.}$$

(e) Impedance-coil power

$$P' = E_Z' \cdot I \cdot \cos \phi$$
$$= 80 \cdot 4 \cdot 0.219 = 70.0 \text{ watts.} \quad \textit{Ans.}$$

Power in the resistance $P_r = 60 \cdot 4 = 240$ watts.

$$P_r + P' = 310 \text{ watts} = P \text{ (check).}$$

(f) $ab = IR' = ac \cos \phi = 80 \cdot 0\ 219 = 17.52$ volts.

$$R' = \frac{17.52}{4} = 4.38 \text{ ohms.} \quad \textit{Ans.}$$

From (c),

(g) the reactance voltage in the impedance coil,

$$IX' = bc = 78.05 \text{ volts.}$$

$$\frac{78.05}{4} = 19.5 \text{ ohms reactance.} \quad \textit{Ans.}$$

(h) $R_0 = R + R' = 15.0 + 4.38 = 19.38$ ohms. *Ans.*

33. Capacitive Impedance.—As is stated in Sec. 32 the phase angle of capacitors ordinarily does not depart very much from 90°;

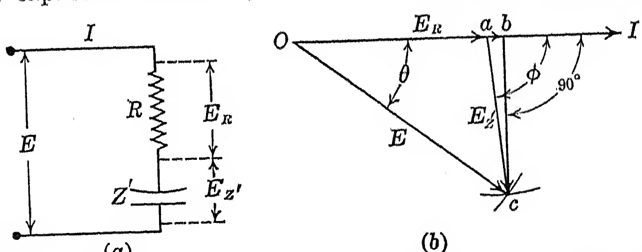


FIG. 52.—Resistance and capacitive impedance in series with vector polygon.

and under many conditions where low-loss dielectrics are used, the angle may be considered to be 90°. However, the method of Sec. 32 may be applied to such capacitors. In Fig. 52(a) a capacitive impedance Z' is shown in series with a resistance R. The impressed emf is

E volts and the current is I amp. The vector diagram is shown in (b). The reference vector I is laid off horizontally, and the voltage E_R ($= Oa$) across the resistance is laid off to scale in phase with I. From a an arc ac having $E_{Z'}$, the voltage across the capacitor, for its radius, is swung downward. From the origin O another arc Oc having E as its radius is swung to intersect the arc ac at c. The vectors $ac = E_{Z'}$ and $Oc = E$ complete the vector diagram, which may be solved in the same manner as Fig. 51(b). The angle ϕ is usually so nearly 90° that considerable care must be taken in making the measurements if reasonable precision is to be obtained.

34. Polygon of Voltages; Four Voltages.—If the three sides of a triangle are fixed, the triangle itself is fixed as regards both its area

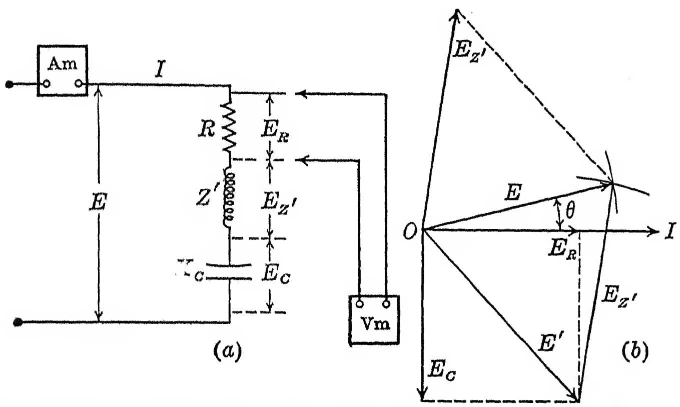


FIG. 53.—Circuit with resistance, inductive impedance, and capacitance in series, and polygon of voltages.

and its angles. If the four sides of a polygon are given, however, the polygon itself is not determined. In order to determine the polygon, some other factor, such as the angle included between two of its sides, must be known. The indeterminate condition exists in the vector diagram with resistance, inductive impedance, and capacitive impedance in series. These give three voltages, which together with the line voltage make four voltages. These four voltages in themselves would constitute an indeterminate polygon. If, however, the angle between two of these voltages is known, the polygon and its angles are uniquely determined.

This is illustrated in Fig. 53, in which resistance R, inductive impedance Z', and capacitive impedance X_C are connected in series, and the current is I amp. Assume that the capacitive power-factor angle is 90°, which is practically the case with most commercial capacitors. This constitutes the angle that makes the polygon of

voltages determinate. Along I lay off E_R to scale, Fig. 53(b). Lagging I by 90°, lay off E_C to scale. Add these two vectorially, giving $\dot{E}' = \dot{E}_R + \dot{E}_C$. From the end of E' swing upward an arc of radius $E_{Z'}$, and from O swing an arc of radius equal to the line voltage E. Complete the polygon where these two arcs intersect. Then from O draw $E_{Z'}$ parallel to the $E_{Z'}$ swung from the end of E'.

It is seen that

$$\dot{E}_{Z'} + (\dot{E}_R + \dot{E}_C) = \dot{E}.$$

That is, the vector sum of the three component voltages is equal to the line voltage, which verifies the method.

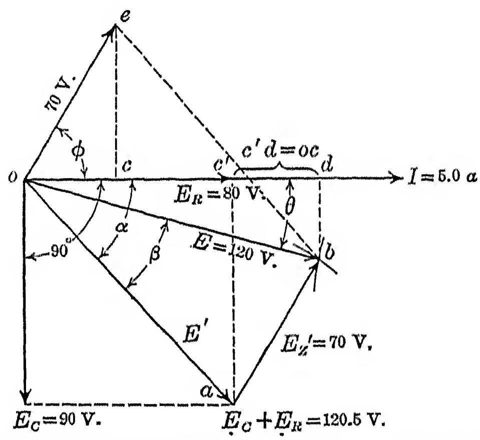


FIG. 54.—Polygon of voltages for alternating-current series circuit.

It is not necessary to assume that the angle of the capacitance is 90°, provided that this angle is known. For example, the angle may be determined by the triangle of voltages as in Fig. 52 or by other means; the angle between E_C and I can then be made equal to this known value. Likewise, the angle between the impedance voltage $E_{Z'}$ and I may be the known angle.

Example.—A resistor, an impedance coil, and a capacitor are connected in series. The voltage E_R across the resistor is 80 volts; that across the impedance coil $E_{Z'}$ is 70 volts; that across the capacitor E_C is 90 volts; and the line voltage E is 120 volts. The current to the circuit is 5 amp, and the capacitor current leads its voltage by 90°. Determine (a) circuit power-factor angle θ; (b) power of circuit; (c) effective resistance of impedance coil; (d) power factor and power-factor angle of impedance coil; (e) reactance of impedance coil.

The voltage polygon is shown in Fig. 54.

(a) $E' = \sqrt{90^2 + 80^2} = \sqrt{14{,}500} = 120.5$ volts;

$$\tan \alpha = {}^{90}\!/\!_{80} = 1.125; \qquad \alpha = 48.4°.$$

Applying the law of cosines (see p. 605) to triangle oab,

$$\overline{70}^2 = \overline{120.5}^2 + \overline{120}^2 - 2 \cdot 120.5 \cdot 120 \cos \beta,$$

$$\cos \beta = \frac{24{,}000}{28{,}900} = 0.8305,$$

$$\beta = 33.8°.$$

$$\theta = \alpha - \beta = 48.4° - 33.8° = 14.6°. \quad \textit{Ans.}$$

$$\cos 14.6° = 0.968 \text{ (current leads).}$$

(b) $P = 120 \cdot 5 \cdot 0.968 = 580.8$ watts. *Ans.*

(c) The distance $od = 120 \cos \theta = 120 \cdot 0.968 = 116.2$ volts. $oc = c'd$. since oc is the projection of oe on od, and cd is the projection of ab on od, and ab is equal and parallel to oe.

Therefore,

$$oc = od - 80 = 116.2 - 80 = 36.2 \text{ volts.}$$

$$\frac{36.2}{5} = 7.24 \text{ ohms effective resistance in impedance coil.} \quad \textit{Ans.}$$

(d) $\cos \phi = \dfrac{oc}{oe} = \dfrac{36.2}{70.0} = 0.517 = \text{P.F.}$ *Ans.*

$\phi = 58.8°$. *Ans.*

(e) $ce = IX' = 70 \sin \phi = 70 \cdot 0.856 = 59\ 92$ volts.

$X' = \dfrac{59.92}{5} = 11.98$ ohms. *Ans.*

In making these circuit measurements it must be remembered that the ordinary voltmeter takes appreciable current; unless this

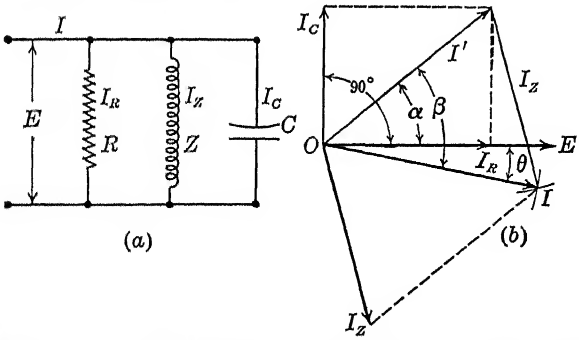


FIG. 55.—Parallel circuit with resistance, inductive impedance, and capacitance all in parallel, with vector diagram.

current is small compared with the circuit current, considerable error may result. Hence a high-resistance voltmeter should be used when the impedances of the circuit elements are relatively high.

35. Polygon of Currents.—If the resistances, impedances, etc., are in parallel, the voltage is the same for each branch of the circuit, but the currents may differ. The polygon is composed of currents, therefore, rather than of voltages. Figure 55(a) shows a circuit

with resistance R, inductive impedance Z, and capacitance C in parallel. Assume that the capacitance current is in quadrature with its voltage. Figure 55(*b*) represents the polygon of currents. The voltage E, being common to all branches, is laid off horizontally. The current I_R is laid off in phase with E, and the current I_C leads E by 90°. These two are combined, giving I'. From the end of I', I_Z is swung downward to meet I, which is swung from O. This completes the polygon, which is similar to those shown in Figs. 53 and 54, except that the vectors represent currents rather than voltages. With only three currents, the diagrams are analogous to those of Figs. 50 to 52, except that the polygons are of currents rather than of voltages.

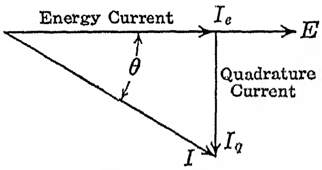


FIG. 56.—Energy and quadrature currents.

36. Energy and Quadrature Currents. Figure 56 shows the vector diagram for a load connected across an alternating-current supply. This load is typical of most commercial loads, except incandescent lamp loads. It takes a current I agging the voltage E by the angle θ. The current I may be resolved, into two components, I_e in phase with the voltage and I_q in quadrature with the voltage. I is the vector sum of I_e and I_q.

The power taken by the load is

$$P = EI \cos \theta,$$

where

$$I \cos \theta = I_e.$$

Therefore

$$P = EI_e. \tag{58}$$

I_e is the *energy component* of the current, because this component multiplied by the voltage gives the circuit power.

The component I_q in quadrature with the voltage can contribute no power. I_q is the *quadrature*, or wattless, component of the current.

If this load is being supplied over a transmission line, the line loss is proportional to

$$I^2R = (I_e^2 + I_q^2)R = I_e^2R + I_q^2R, \tag{59}$$

where R is the transmission-line resistance.

It will be noted that the quadrature component produces line loss yet contributes no power to the load. It is ordinarily desirable, therefore, to make I_q as small as possible, in other words, to have the system operate at high power factor. For example, when $\theta = 45°$, P.F. $= 0.707$, the energy and quadrature currents are equal. The

quadrature current contributes as much to the line loss, therefore, as the energy current does, but it contributes nothing to the power supplied to the load.

Example.—A transmission line Fig. 57(*a*) supplies 50 kw at 220 volts, single phase, to a load having a power factor of 0.60, lagging current. Each wire has a resistance of 0.02 ohm. Determine (*a*) energy current; (*b*) quadrature current; (*c*) line loss due to energy current; (*d*) line loss due to quadrature current; (*e*) total line loss; (*f*) line loss that would exist if the load power factor were unity.

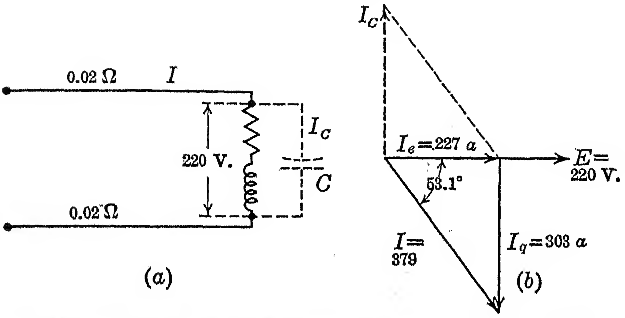


FIG. 57.—Energy and quadrature currents in transmission line.

The total current

$$I = \frac{50{,}000}{220 \cdot 0.6} = 379 \text{ amp.}$$

(*a*) $I_e = 379 \cos \theta = 379 \cdot 0.6 = 227$ amp. *Ans.*
(*b*) $I_q = 379 \sin \theta = 379 \cdot 0.8 = 303$ amp. *Ans.*
(*c*) $I_e^2 \cdot 0.04 = 2{,}070$ watts. *Ans.*
(*d*) $I_q^2 \cdot 0.04 = 3{,}680$ watts. *Ans.*
(*e*) $I^2 \cdot 0.04 = 5{,}750$ watts. *Ans.*
(*f*) If the power factor of the load were unity, the quadrature current I_q would be zero and the line current $I = I_e$.

Therefore, the loss would be

$$I^2 \cdot 0.04 = 2{,}070 \text{ watts.} \quad \textit{Ans.}$$

In this particular case, the line loss due to the quadrature current is much greater than that due to the energy current, yet the quadrature current contributes no power to the load. The quadrature current in the line may be reduced or eliminated by connecting a capacitor C, shown dotted, in parallel with the load as indicated in Fig. 57(*a*). If the capacitor current I_C is equal to the quadrature current I_q, the resultant current in the line is the energy current, as is indicated in (*b*). Ordinarily it is satisfactory if the power factor is raised to 0.8 or 0.9 by the capacitor current (see p. 410).

From the foregoing it must not be inferred that the energy and quadrature currents exist separately. Only one current actually flows, but this current may be resolved into two components, which

produce different effects in the circuit. The effect of each component then can be studied, resulting in a much better understanding of the circuit relations than if an attempt is made to consider the current as a whole.

37. Reactive Volt-amperes.—It is shown in Sec. 36 that the average power in an alternating-current circuit is given by the product of the circuit voltage and the *energy* current. This is the power which is actually delivered, such as the power to some motor or to a lamp load, or the power lost as I^2R. Also, during each cycle energy may be exchanged between a part of the circuit and the source, as, for example, between an electromagnetic field and the source (see Sec. 16, p. 26). This energy does not leave the system and so does not appear in the average power delivered from the source to the circuit. It does have important effects on the system, such as causing a loss of energy in flowing through resistance; in Sec. 36 it is shown that this power loss is given by the product of the quadrature current squared and the resistance. The effects on the system of this exchange of energy may be attributed to a quantity called *reactive volt-amperes.* The reactive volt-amperes are equal to the product of the voltage and the quadrature current, or the product of the current and the quadrature voltage. The quadrature voltage is the voltage represented by the length of the perpendicular to the current vector from the end of the voltage vector. The *var* (volt-ampere reactive) has been standardized as the unit of reactive volt-amperes. The kilovar (kvar) is equal to 1,000 vars. It follows that the watts and the vars may be added in quadrature to give the total volt-amperes; that is,

$$\text{Va} = \sqrt{(\text{watts})^2 + (\text{vars})^2}. \tag{60}$$

The performance of power-transmission systems frequently can be analyzed much more readily if the total volt-amperes be resolved into the two components watts and vars (see Sec. 99, p. 150, and Sec. 232, p. 414).

Example.—Determine the reactive volt-amperes, or vars, in the example, Sec. 36.

$$220 \cdot 303 = 66{,}660 \text{ vars} = 66.66 \text{ kvars.} \quad \textit{Ans.}$$

Also,

$$\sqrt{(220 \cdot 379)^2 - (50{,}000)^2} = 66{,}660 \text{ vars.} \quad \textit{Ans.}$$

38. Impedances in Parallel.—A method of solving circuits in which there are two or more impedances in parallel is to determine the current in each impedance, and resolve each current into an energy and a quadrature component. All the energy components are added, and all the quadrature components are added, thus giving the total

energy current and the total quadrature current. The total current then is the resultant of the total energy and total quadrature currents.

Example.—In Fig. 58(a) are shown three impedances Z_1, Z_2, and Z_3 in parallel and connected across a 120-volt 60-cycle supply. Determine (a) each impedance;

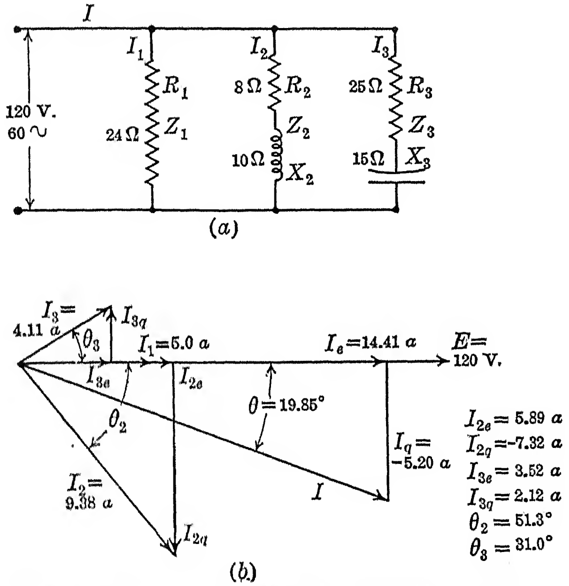


FIG. 58.—Impedances in parallel, with vector diagram.

(b) each current; (c) energy and quadrature components of each current; (d) resultant energy and quadrature currents; (e) total current; (f) power factor and power-factor angle; (g) total watts; (h) total vars.

(a) $Z_1 = 24$ ohms. *Ans.*

$$Z_2 = \sqrt{8^2 + 10^2} = \sqrt{64 + 100} = 12.80 \text{ ohms. } \textit{Ans.}$$

$$Z_3 = \sqrt{25^2 + 15^2} = \sqrt{625 + 225} = 29.15 \text{ ohms. } \textit{Ans.}$$

(b) $I_1 = {}^{120}\!/_{24} = 5.0$ amp. *Ans.*

$$I_2 = \frac{120}{12.80} = 9.38 \text{ amp. } \textit{Ans.}$$

$$I_3 = \frac{120}{29.15} = 4.11 \text{ amp. } \textit{Ans.}$$

(c) $\cos\theta_1 = 1$; $I_{1e} = I_1 = 5.0$ amp; $I_{1q} = 0$. *Ans.*

$$\cos\theta_2 = \frac{8.0}{12.80} = 0.6250;\ I_{2e} = 9.38 \cdot 0.6250 = 5.89 \text{ amp. } \textit{Ans.}$$

$$\sin\theta_2 = \frac{-10}{12.80} = -0.7804;\ I_{2q} = 9.38 \cdot (-0.7804) = -7.32 \text{ amp. } \textit{Ans.}$$

$$\cos\theta_3 = \frac{25}{29.15} = 0.857;\ I_{3e} = 4.11 \cdot 0.857 = 3.52 \text{ amp. } \textit{Ans.}$$

$$\sin\theta_3 = \frac{15}{29.15} = 0.5145;\ I_{3q} = 4.11 \cdot 0.5145 = 2.12 \text{ amp. } \textit{Ans.}$$

(d) $I_e = 5.0 + 5.89 + 3.52 = 14.41$ amp. *Ans.*
$I_q = -7.32 + 2.12 = -5.20$ amp. *Ans.*

(e) $I = \sqrt{14.41^2 + (-5.20)^2} = \sqrt{207.4 + 27.0} = 15.33$ amp. *Ans.*

(f) $\tan\theta = \dfrac{-5.20}{14.41} = -0.361$; $\theta = -19.85°$. *Ans.*

$\cos(-19.85°) = 0.9406 = \text{P.F.}$ *Ans.*

(g) $P = 120 \cdot 14.41 = 1{,}730$ watts. *Ans.*

(h) $Q = 120 \cdot (-5.20) = -624$ vars. *Ans.*

The vector diagram for the circuit is shown in Fig. 58(b).

39. Maximum Power in a Series Circuit.—If a series circuit across constant voltage has a variable resistance R and a fixed reactance X, the power taken by the circuit will vary as R is varied. When $R = 0$, the power is zero; when $R = \infty$, the power is zero. With a finite value of voltage, the power between these two values of R is not zero but must be finite. If the power is plotted as a function of R, it is zero when $R = 0$, increases to a maximum when $R = X$, and decreases to zero when $R = \infty$. The fact that the maximum power occurs when $R = X$ is shown by the following example.

Example.—A circuit having a fixed reactance of 12 ohms (either inductive or capacitive) in series with a variable resistance R is connected across 100-volt 60-cycle mains. Determine (a) value of R for maximum power; (b) maximum power.

(a) The current

$$I = \frac{100}{\sqrt{R^2 + (12)^2}},$$

$$P = I^2R = \frac{(100)^2R}{R^2 + 144}. \tag{I}$$

Differentiating (I) with respect to R and equating to zero,

$$\frac{dP}{dR} = (100)^2\,\frac{(R^2 + 144) - R(2R)}{(R^2 + 144)^2} = 0. \tag{II}$$

$$R^2 = 144; \qquad R = 12 \text{ ohms} = X. \quad \textit{Ans.}$$

$$R = X.$$

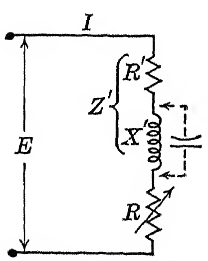


FIG. 59.—Maximum power in a-c circuit.

Thus, the maximum power taken by such a circuit occurs *when the resistance is equal to the reactance.*

(b) From (I), the power,

$$P = (100)^2\,\frac{12}{144 + 144} = 10{,}000\,\frac{12}{288} = 417 \text{ watts.} \quad \textit{Ans.}$$

If the impressed voltage is constant and the resistance R is in series with an impedance Z' whose resistance is R' and reactance is X' (either inductive or capacitive), Fig. 59, R takes the maximum power when $R = Z'$.

The current

$$I = \frac{E}{\sqrt{(R + R')^2 + (X')^2}}, \tag{I}$$

$$P = I^2R = E^2 \frac{R}{(R + R')^2 + (X')^2}, \tag{II}$$

$$\frac{dP}{dR} = E^2 \frac{(R + R')^2 + (X')^2 - R \cdot 2(R + R')}{[(R + R')^2 + (X')^2]^2} = 0, \tag{III}$$

$$R^2 + 2RR' + R'^2 + X'^2 - 2R^2 - 2RR' = 0, \tag{IV}$$

$$R^2 = R'^2 + X'^2 \tag{V}$$

$$R = \sqrt{R'^2 + X'^2} = Z'. \quad \text{Q.E.D.}$$

40. Harmonics.—Thus far, only sine or cosine waves have been considered. In practice, nonsinusoidal waves frequently occur. For example, the flux distribution along the air gaps of alternators usually is nonsinusoidal so that the emf in the individual armature conductor likewise is nonsinusoidal (p. 179). Also, with a sinusoidal emf wave the current wave may be nonsinusoidal owing to a saturated core in an inductance or a transformer, Fig. 104, p. 118. Fourier showed that any periodic wave may be expressed as the sum of a d-c component (zero frequency) and sine (or cosine) waves having fundamental and multiple or higher frequencies, the higher frequencies being called *harmonics*. The d-c component or any of the other frequencies may be absent. In the usual a-c power circuit only odd harmonics occur since the circuit conditions are such that the positive and negative loops of both the voltage and the current waves are ordinarily similar. With even harmonics, the positive and negative loops of the waves are dissimilar since the phase of any even harmonic with respect to the fundamental will be opposite in the positive and negative loops of the wave. This is illustrated in Fig. 60(a) and (b), where an emf wave e is shown as being composed of a fundamental e_1 and a second harmonic e_2. The resultant wave e is found by adding the ordinates of e_1 and e_2. In (a) the second harmonic e_2 is in phase with the fundamental e_1, and in (b) it lags the fundamental e_1 by 90° in terms of its own scale of angles.

It is to be noted that in each case the positive and negative loops of the resultant wave e differ from each other. In (a), for example, the peak of the wave is at the left-hand side of the positive and the right-hand side of the negative loop. Such dissymmetrical loops do occur occasionally in a-c power circuits when, for example, d-c and a-c magnetization of a saturated iron core occur simultaneously. However, such circumstances are rare so that for the most part only odd harmonics occur in *power* circuits.

In Fig. 60(c) is shown an emf wave e consisting of a fundamental component e_1 having a maximum value of E_{1m} volts and a third harmonic e_3 having a maximum value of E_{3m} volts, the third harmonic lagging the fundamental by $\alpha°$ in terms of its own scale of angles.

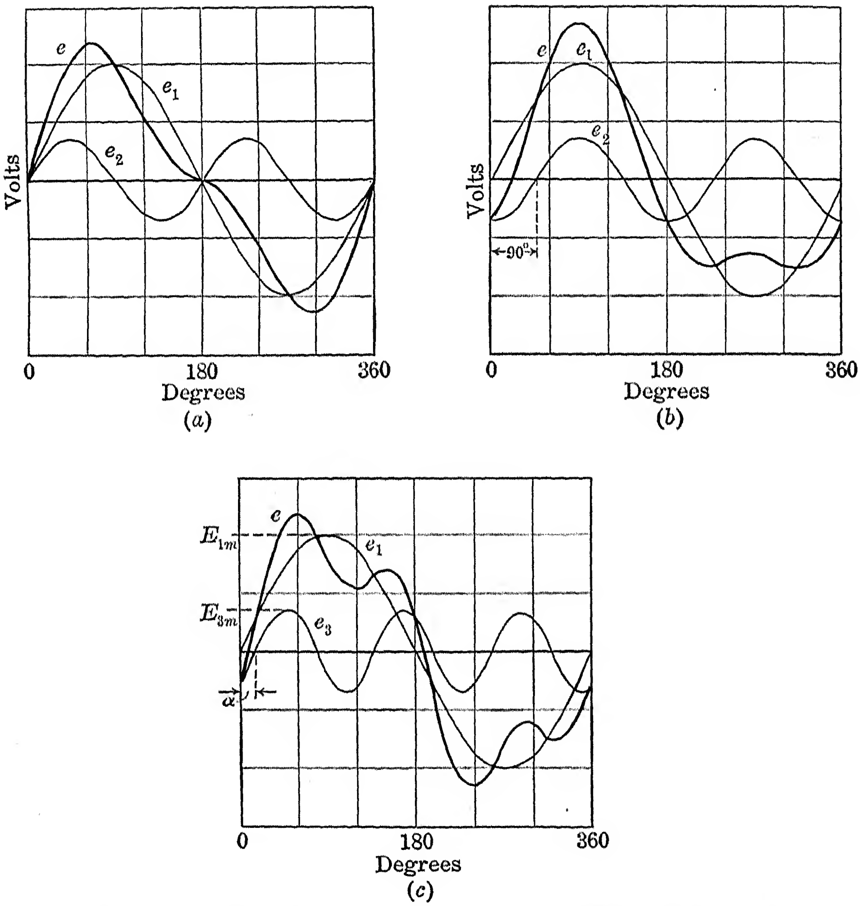


FIG. 60.—Harmonics in alternating-voltage waves: (a) and (b) illustrate waves with fundamental and second harmonic; (c) illustrates wave with fundamental and third harmonic.

The equation of the resultant emf is

$$e = E_{1m} \sin \omega t + E_{3m} \sin 3(\omega t - \alpha) \qquad \text{volts.} \quad (61)$$

The shape and the ratio of maximum to rms value of such non-sinusoidal waves depend on the phase relation of harmonic and fundamental, as well as on their amplitudes.

It can be shown that the rms value or the value of voltage measured

on an a-c instrument that measures rms values is

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2 \cdots} \qquad \text{volts,} \qquad (62)$$

where E_1, E_3, E_5 are the rms values of the fundamental, the third, and the fifth harmonics.

Example.—In a nonsinusoidal emf wave having a fundamental frequency of 60 cycles the rms values of the fundamental, third harmonic, and fifth harmonic are 120 volts, 30 volts, and 15 volts. What will an a-c voltmeter that measures rms values indicate when connected across the circuit?

$$E = \sqrt{120^2 + 30^2 + 15^2} = \sqrt{15{,}525} = 124.6 \text{ volts.} \quad \textit{Ans.}$$

Since d-c voltage and current have a frequency zero, a d-c voltage may also be included in (62).

Example.—A d-c battery having an emf of 100 volts is connected in series with a 120-volt 60-cycle power supply. What will an a-c voltmeter that measures rms values indicate when connected across the two voltages in series?

$$E = \sqrt{100^2 + 120^2} = \sqrt{24{,}400} = 156.2 \text{ volts.} \quad \textit{Ans.}$$

Nonsinusoidal currents may be treated in the same manner as nonsinusoidal voltages.

A study of (*a*), (*b*), and (*c*), Fig. 60, shows that with nonsinusoidal waves the ratio of maximum to rms value usually is *not* $\sqrt{2}$. (For a more comprehensive treatment, see "Principles of Alternating Currents" by R. R. Lawrence.)

CHAPTER III

COMPLEX QUANTITIES

From the two preceding chapters, it is apparent that alternating-current problems cannot be solved ordinarily by the use of simple algebra, since geometrical relations must be taken into consideration. That is, the solutions of alternating-current circuits involve vector rather than scalar operations, and simple algebra is not adequate to obtain the desired results. By means of *complex algebra*, however, it is possible to solve alternating-current circuits by algebraic

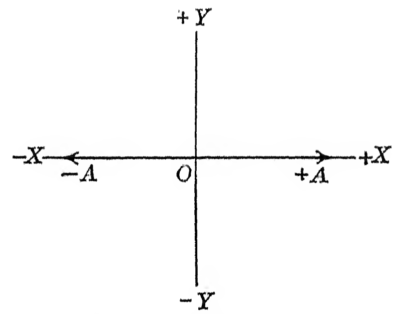


FIG. 61.—Operating on vector with (−1).

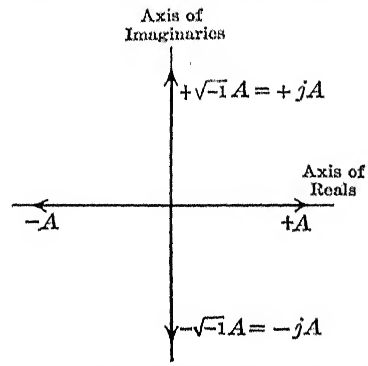


FIG. 62.—Operating on vector with $\sqrt{-1}$.

operations alone. It is not necessary to employ directly the usual trigonometric operations on vector quantities. Furthermore, without complex algebra, many problems would be difficult to solve.

41. Rectangular Notation of Complex Quantities.—In Fig. 61 the usual coordinate axes XX and YY are shown. Consider a vector $+A$ lying along the X-axis in the positive direction. If this vector is operated upon by the factor (-1), it becomes $-A$ and its position is now along the X-axis in the negative direction. That is, by operating on $+A$ with the factor (-1), A is caused to rotate through an angle of 180°. Since (-1) is equal to $(\sqrt{-1}\ \sqrt{-1})$, this same result may be obtained by operating on $+A$ with the operator $(\sqrt{-1}\ \sqrt{-1})$. That is by operating on $+A$ twice with the operator $\sqrt{-1}$, the vector $+A$ is caused to rotate through 180°. Hence, if the vector $+A$ is operated on but once by the operator $\sqrt{-1}$, it is caused to rotate

through 90°. It has been agreed that $\sqrt{-1}$ causes rotation in a positive, or counterclockwise, direction. That is, the vector $+A$ when operated on once by $\sqrt{-1}$ takes a position along the Y-axis in a positive direction, Fig. 62.

It is well known that the square root of a negative quantity as it is used in simple algebra does not denote a physical entity. No *real* quantity squared, whether positive or negative, can be equal to a negative quantity. Because in simple algebra $\sqrt{-1}$ does not represent a physical quantity, it is known as a pure imaginary. Since all vectors that lie along the Y-axis are designated by this operator $\pm\sqrt{-1}$, the Y-axis is called the *axis of imaginaries*. The X-axis is called the *axis of reals*. The term *axis of imaginaries* is somewhat unfortunate, for it implies a nonexistent quantity. In complex algebra, however, quantities along the axis of imaginaries are just as much physical entities as quantities along the axis of reals. Hence with the usual rectangular coordinate axes $\sqrt{-1}$ as a coefficient indicates that the quantity to which it is applied as a coefficient lies along the positive Y-axis, or axis of imaginaries.

In electrical engineering, the operator $+\sqrt{-1}$ is represented by $+j$.[1] The factor $+j$ therefore is an operator which causes the vector on which it operates to be rotated through an angle of 90° in a counterclockwise direction.

The operator $(-\sqrt{-1})(-\sqrt{-1})$ also rotates the vector $+A$ through an angle of 180°, as does the operator $(\sqrt{-1}\sqrt{-1})$. If $+\sqrt{-1}$ causes positive, or counterclockwise, rotation through 90°, $-\sqrt{-1}$ causes negative, or clockwise, rotation through 90°. As a coefficient $-\sqrt{-1}$, or $-j$, indicates that any real quantity to which it is applied as a coefficient lies along the negative Y-axis, or negative axis of imaginaries, as shown in Fig. 62. Also $-j$ causes any vector on which it operates to be rotated through an angle of 90° in a clockwise direction.

Since complex algebra deals with points in a plane rather than with points on a line, the plane represented by the coordinate axes, Figs. 61, 62, is called the *complex plane*.

42. Rectangular Vectors.—A vector can be resolved into two or more components, and ordinarily each component may be operated on independently. If the two components are at right angles to each other, it is usually more convenient to take the direction of one along the X-axis and the other along the Y-axis. In complex algebra,

[1] In mathematics, $\sqrt{-1}$ is usually represented by the symbol i. The fact that in electrical engineering i stands for current has caused the adoption of the symbol j for $\sqrt{-1}$.

each vector is resolved into two components at right angles to each other. The component along the Y-axis is designated by $\pm j$. For example, Fig. 63, the vector A lying in the first quadrant is resolved into two components, a_1 along the axis of reals and $+ja_2$ along the axis

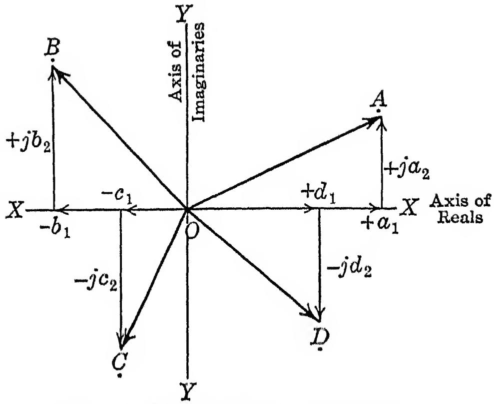


Fig. 63.—Rectangular, complex vectors.

of imaginaries. That is $\dot{A} = a_1 + ja_2$. For vector B in the second quadrant, $\dot{B} = -b_1 + jb_2$; vector C in the third quadrant, $\dot{C} = -c_1 - jc_2$; vector D in the fourth quadrant, $\dot{D} = d_1 - jd_2$. Vectors defined by their components along the axis of reals and axis of imaginaries will be termed *rectangular vectors.*

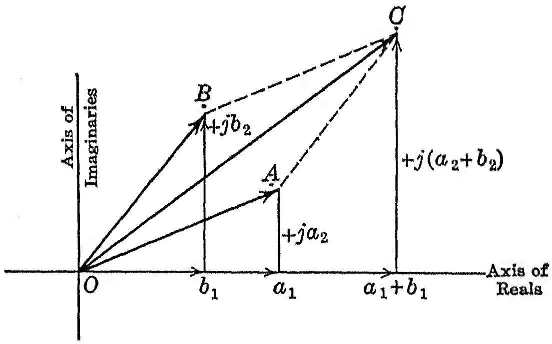


Fig. 64.—Addition of rectangular vectors.

In the algebra of complex quantities, ordinary algebraic operations are followed. The operator j is treated as a coefficient and is given its algebraic value of $\sqrt{-1}$ (for example, $j^2 = -1$).

43. Addition and Subtraction of Rectangular Vectors.—Let it be required to add the vectors $\dot{A}$ and $\dot{B}$, Fig. 64, where $\dot{A} = a_1 + ja_2$ and $\dot{B} = b_1 + jb_2$. The addition involves merely the adding together

of the real components and of the imaginary components of these two vectors. For example,

$$\dot{C} = \dot{A} + \dot{B} = (a_1 + b_1) + j(a_2 + b_2). \tag{63}$$

If any of the quantities a_1, b_1, a_2, b_2 are negative, they are given the negative sign.

Example.—Add $8 - j10$ to $6 + j4$. $(8 - j10) + (6 + j4) = 14 - j6$. *Ans.* This vector has a magnitude of $\sqrt{(14)^2 + (6)^2} = 15.23$ and lies in the fourth quadrant. It makes an angle $\tan^{-1}(-6/14) = -23.2°$ with the positive direction of the axis of reals.

Subtraction is accomplished in the same manner as addition.

Example.—Subtract $12 - j10$ from $7 + j4$.

$$(7 + j4) - (12 - j10) = 7 + j4 - 12 + j10 = -5 + j14. \quad \textit{Ans.}$$

This vector lies in the second quadrant and makes an angle with the positive direction of the axis of reals of $\tan^{-1}(14/-5) = \tan^{-1}(-2.80) = 109.7°$.

44. Multiplication of Rectangular Vectors.—Let it be required to multiply vector $\dot{A} = a_1 + ja_2$ by vector $\dot{B} = b_1 + jb_2$. Ordinary algebraic procedure is followed. That is,

$$\begin{aligned} \dot{A}\dot{B} &= (a_1 + ja_2)(b_1 + jb_2) = a_1b_1 + ja_1b_2 + ja_2b_1 + j^2a_2b_2 \\ &= (a_1b_1 - a_2b_2) + j(a_1b_2 + a_2b_1). \end{aligned} \tag{64}$$

If $\dot{B} = b_1 - jb_2$,

$$\dot{A}\dot{B} = (a_1b_1 + a_2b_2) - j(a_1b_2 - a_2b_1). \tag{65}$$

(Also see Sec. 50 for the geometrical relations among two vectors and their product.)

Example.—Determine the product of $8 - j10$ and $6 + j4$.

$$(8 - j10)(6 + j4) = 48 + j32 - j60 - j^240 = 88 - j28. \quad \textit{Ans.}$$

This vector has a magnitude of $\sqrt{(88)^2 + (28)^2} = 92.3$ and lies in the fourth quadrant. It makes an angle $\tan^{-1}(-28/88) = -17.6°$ with the positive direction of the axis of reals.

45. Reciprocals of Rectangular Vectors.—Let it be required to determine

$$\frac{1}{\dot{A}} = \frac{1}{a_1 + ja_2}. \tag{I}$$

(I) is *rationalized* by multiplying numerator and denominator by $a_1 - ja_2$. That is,

$$\begin{aligned} \frac{1}{\dot{A}} &= \frac{1}{a_1 + ja_2} \cdot \frac{a_1 - ja_2}{a_1 - ja_2} = \frac{a_1 - ja_2}{a_1^2 - ja_1a_2 + ja_1a_2 - j^2a_2^2} \\ &= \frac{a_1}{a_1^2 + a_2^2} - j\frac{a_2}{a_1^2 + a_2^2}. \end{aligned} \tag{66}$$

Example.—Find $\dfrac{1}{8 - j10}$.

$$\frac{1}{8 - j10} \cdot \frac{8 + j10}{8 + j10} = \frac{8}{64 + 100} + j\frac{10}{64 + 100} = 0.0488 + j0.0610. \quad \textit{Ans.}$$

46. Division of Rectangular Vectors.—Let it be required to determine

$$\frac{\dot{A}}{\dot{B}} = \frac{a_1 + ja_2}{b_1 + jb_2}. \tag{I}$$

The denominator of (I) is rationalized, as was done in Sec. 45.

$$\begin{aligned}\frac{\dot{A}}{\dot{B}} &= \frac{a_1 + ja_2}{b_1 + jb_2} \cdot \frac{b_1 - jb_2}{b_1 - jb_2} = \frac{a_1b_1 - ja_1b_2 + ja_2b_1 + a_2b_2}{b_1^2 + b_2^2} \\ &= \frac{a_1b_1 + a_2b_2}{b_1^2 + b_2^2} - j\frac{a_1b_2 - a_2b_1}{b_1^2 + b_2^2}.\end{aligned} \tag{67}$$

Example.—Divide the quantity $8 - j10$ by $6 + j4$,

$$\frac{8 - j10}{6 + j4} \cdot \frac{6 - j4}{6 - j4} = \frac{48 - j32 - j60 - 40}{36 + 16} = \frac{8 - j92}{52} = 0.154 - j1.77. \quad \textit{Ans.}$$

This is a vector whose magnitude is

$$\sqrt{(0.154)^2 + (1.77)^2} = 1.78$$

and lies in the fourth quadrant.

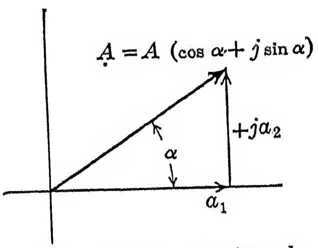


FIG. 65.—Relation of rectangular, exponential, and polar vectors.

47. Exponential Vectors.—A vector such as $\dot{A} = a_1 + ja_2$, Fig. 65, may be expressed by $\dot{A} = A\ (\cos\alpha + j\sin\alpha)$, known as DeMoivre's theorem. The quantity A is called the *modulus* or *magnitude*, and the parenthesis term the *amplitude* or *argument*.

In the argument $\cos\alpha$ and $j\sin\alpha$ may be expanded by Maclaurin's theorem as follows:

$$\cos\alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \cdots, \tag{68}$$

$$j\sin\alpha = j\alpha - \frac{j\alpha^3}{3!} + \frac{j\alpha^5}{5!} - \cdots \tag{69}$$

where $4! = 1 \cdot 2 \cdot 3 \cdot 4.$

Similarly,

$$\epsilon^{j\alpha} = 1 + j\alpha - \frac{\alpha^2}{2!} - \frac{j\alpha^3}{3!} + \frac{\alpha^4}{4!} + \frac{j\alpha^5}{5!} - \frac{\alpha^6}{6!} \cdots, \tag{70}$$

where ϵ is the Napierian logarithmic base $= 2.718$.

Hence,

$$A\epsilon^{j\alpha} = A(\cos\alpha + j\sin\alpha). \tag{71}$$

Also,

$$A\epsilon^{-j\alpha} = A(\cos\alpha - j\sin\alpha). \tag{72}$$

Thus the vector $A\epsilon^{j\alpha} = A\underline{/\alpha}$ and is defined as an *exponential vector.*

$$(A\epsilon^{j\alpha})(B\epsilon^{j\beta}) = AB\epsilon^{j(\alpha+\beta)}; \frac{1}{A} = \frac{1}{A\epsilon^{j\alpha}} = \frac{1}{A}\epsilon^{-j\alpha}; \frac{A}{B} = \frac{A\epsilon^{j\alpha}}{B\epsilon^{j\beta}} = \frac{A}{B}\epsilon^{j(\alpha-\beta)};$$
$$A^n = (A\epsilon^{j\alpha})^n = A^n\epsilon^{jn\alpha}; \sqrt[n]{A} = \sqrt[n]{A\epsilon^{j\alpha}} = \sqrt[n]{A}\,\epsilon^{j(\alpha/n)}.$$

48. Polar Notation.—A vector in the complex plane may also be defined by its magnitude and direction angle with respect to the X-axis. For example, the vector A, Fig. 66, is defined as $A\underline{/\alpha}$; vector B is defined as $B\underline{\backslash\beta}$; vector C is defined as $C\overline{\backslash\gamma}$. Vector C may also be defined as $C\underline{/-\gamma}$. It follows that $\overline{\backslash\gamma} = \underline{/-\gamma}$, etc.

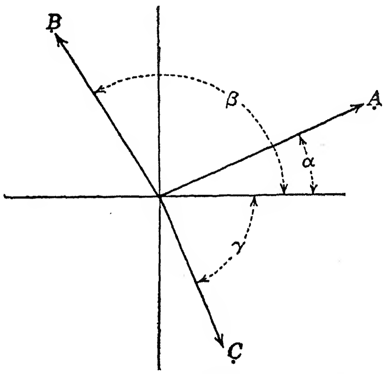


FIG. 66.—Polar notation for vectors.

Vectors defined by the foregoing notation will be termed *polar vectors.* The method of designating a polar vector is in reality a shorthand method of designating an exponential vector, the angular notation such as $\underline{/\alpha}$ being equivalent to $\epsilon^{j\alpha}$. As will be shown, the methods of operating on the two types of vectors are identical. Also, the magnitudes such as A or B are called the *modulus* or absolute value of the vector; the quantities $\underline{/\alpha}$ or $\underline{/\beta}$ are called the *argument* of the vector.

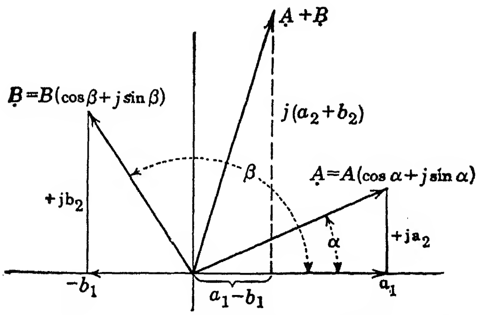


FIG. 67.—Polar vectors expressed as rectangular vectors.

In the foregoing notation, $A\underline{/\alpha}$ is *not a product* and cannot be treated as such.

49. Addition of Exponential and of Polar Vectors.—Exponential and polar vectors cannot be added or subtracted without first converting them into rectangular vectors. For example, Fig. 66, let it be required to add vectors A and B. Referring to Fig. 67,

$$\dot{A} = a_1 + ja_2 = A(\cos\alpha + j\sin\alpha), \tag{73}$$
$$\dot{B} = -b_1 + jb_2 = B(\cos\beta + j\sin\beta), \tag{74}$$
$$\dot{A} + \dot{B} = (A\cos\alpha + B\cos\beta) + j(A\sin\alpha + B\sin\beta). \tag{75}$$

Example.—In Fig. 67, add $\dot{A} = 12\epsilon^{j27^\circ} = 12\underline{/27^\circ}$ and $\dot{B} = 10\epsilon^{j124^\circ} = 10\underline{\backslash 124^\circ}$.

$$\dot{A} = 12(\cos 27^\circ + j\sin 27^\circ) = 12(0.891 + j0.454) = 10.69 + j5.45,$$
$$\dot{B} = 10(\cos 124^\circ + j\sin 124^\circ) = 10(-0.559 + j0.829) = -5.59 + j8.29,$$
$$\dot{A} + \dot{B} = (10.69 - 5.59) + j(5.45 + 8.29) = 5.10 + j13.74. \quad \textit{Ans.}$$
$$\dot{A} + \dot{B} = \sqrt{(5.10)^2 + (13.74)^2}\ \underline{/\tan^{-1}\frac{13.74}{5.10}} = 14.65\underline{/69.6^\circ}. \quad \textit{Ans.}$$

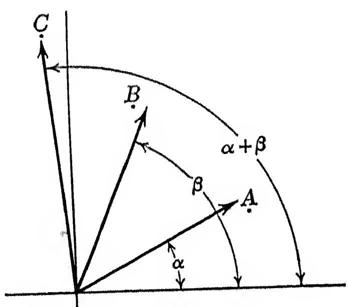
50. Multiplication of Polar Vectors.—The product of two polar vectors is found by taking the *product* of their magnitudes and the *sum* of their angles (see Sec. 47). Thus, Fig. 68,

$$\dot{C} = \dot{A}\dot{B} = A\underline{/\alpha} \cdot B\underline{/\beta} = AB\underline{/\alpha + \beta}. \tag{76}$$

Example.—Determine the product of $16\underline{/32^\circ}$ and $20\underline{\backslash 72^\circ}$.

$$16\underline{/32^\circ} \cdot 20\underline{\backslash 72^\circ} = 320\underline{/32^\circ - 72^\circ} = 320\underline{/-40^\circ} = 320\underline{\backslash 40^\circ}. \quad \textit{Ans.}$$

This vector lies in the fourth quadrant.



$\dot{C} = AB\ \underline{/\alpha+\beta}$

FIG. 68.—Multiplication of polar vectors.

51. Reciprocals of Polar Vectors.—The reciprocal of $A\underline{/\alpha}$ is

$$\frac{1}{\dot{A}} = \frac{1}{A\underline{/\alpha}} = \frac{1}{A}\underline{/-\alpha} = \frac{1}{A}\underline{\backslash\alpha}. \tag{77}$$

Example.—Determine the reciprocal of $25\underline{/32^\circ}$. This is $1/(25\underline{/32^\circ}) = 0.04\underline{\backslash 32^\circ}$, which lies in the fourth quadrant.

It follows that an angle may be transferred from denominator to numerator and from numerator to denominator, *if its sign be reversed.*

52. Division of Polar Vectors.—The quotient of two polar vectors is found by taking the quotient of their magnitudes and the difference of their angles, thus:

$$\frac{\dot{A}}{\dot{B}} = \frac{A\underline{/\alpha}}{B\underline{/\beta}} = \frac{A}{B}\underline{/\alpha - \beta}. \tag{78}$$

Example.—Divide $30\underline{\backslash 115^\circ}$ by $6\underline{\backslash 50^\circ}$, Fig. 69.

$$\frac{30\underline{\backslash 115^\circ}}{6\underline{\backslash 50^\circ}} = 5\underline{\backslash 115^\circ + 50^\circ} = 5\underline{\backslash 165^\circ}. \quad \textit{Ans.}$$

53. Powers and Roots of Polar Vectors.—To find the *n*th power of a polar vector, take the *n*th power of its magnitude and *n* times its

angle. For example

$$\dot{A}^n = (A\underline{/\alpha})^n = A^n\underline{/n\alpha}. \tag{79}$$

Example.—Find $(4\overline{\backslash 64^\circ})^3$.

$$(4\overline{\backslash 64^\circ})^3 = 4^3\overline{\backslash 3 \cdot 64^\circ} = 64\overline{/192^\circ}$$
$$= 64\underline{\backslash 168^\circ}. \quad \textit{Ans.}$$

To find the nth root of a polar vector, take the nth root of its magnitude and $1/n$th of its angle.

$$\sqrt[n]{\dot{A}} = \sqrt[n]{A\underline{/\alpha}} = \sqrt[n]{A}\ \underline{\Big/\frac{\alpha}{n}}. \tag{80}$$

Example.—Determine $\sqrt{4\overline{\backslash 64^\circ}}$.

$$\sqrt{4\overline{\backslash 64^\circ}} = 2\overline{\backslash 32^\circ}. \quad \textit{Ans.}$$

54. Operators for Rotation of Vectors.—To rotate a polar vector $A\underline{/\alpha}$ through an angle $\pm\beta$, the angle $\pm\beta$ is merely added to the angle α. Thus,

$$(A\underline{/\alpha})(1\underline{/\beta}) = A\underline{/\alpha + \beta}, \tag{81a}$$
$$(A\underline{/\alpha})(1\underline{/-\beta}) = A\underline{/\alpha - \beta}. \tag{81b}$$

Hence, $\pm\underline{/\beta}$ is a rotational operator.

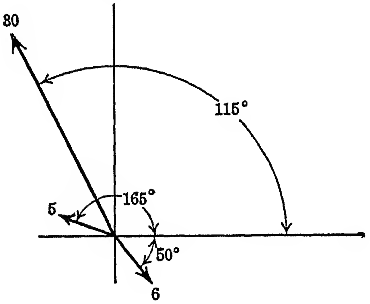


Fig. 69.—Division of polar vectors.

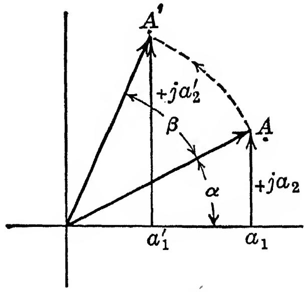


Fig. 70.—Rotation of rectangular vector through angle.

To rotate a rectangular vector through a positive angle β, without changing its magnitude, it is multiplied by $\cos\beta + j\sin\beta$. This may be proved as follows:

In Fig. 70, the vector $\dot{A} = a_1 + ja_2$ is shown making an angle α with the axis of abscissas. The vector $\dot{A}'$ is of the same magnitude as vector $\dot{A}$ but rotated in a counterclockwise direction through the angle β from $\dot{A}$.

$$\begin{aligned}\dot{A}' &= A'[\cos(\alpha+\beta) + j\sin(\alpha+\beta)] \quad \text{[Eq. (73), p. 76]}\\ &= A'(\cos\alpha\cos\beta - \sin\alpha\sin\beta + j\sin\alpha\cos\beta + j\cos\alpha\sin\beta)\\ &\qquad\qquad\qquad\qquad \text{[see (38) and (36), p. 605]}\\ &= A'[\cos\beta(\cos\alpha + j\sin\alpha) + j\sin\beta(\cos\alpha + j\sin\alpha)]\\ &= A'(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta). \end{aligned} \tag{82}$$

Since $\dot{A}' = A$ numerically,

$$\dot{A}' = \dot{A}(\cos\beta + j\sin\beta) \text{ Q.E.D.}$$

Also, from Eqs. (73) and (81*a*),

$$(A/\underline{\alpha})(1/\underline{\beta}) = A(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta).$$

In a similar manner, $\cos\beta - j\sin\beta$ rotates any vector $\dot{A}$ through an angle β in a clockwise direction with no alteration in magnitude. Hence, $\cos\beta \pm j\sin\beta$ is a rotational operator.

Example.—Rotate the vector $-6.0 - j8.5$ in a clockwise direction through an angle of 72°.

$$\cos 72^\circ = 0.309;\ \sin 72^\circ = 0.951.$$

$$\begin{aligned}(-6.0 - j8.5)(0.309 - j0.951) &= -1.854 + j5.706 - j2.63 - 8.08 \\ &= -9.93 + j3.08 = 10.41\backslash\underline{162.8^\circ}.\end{aligned}$$

Since the vector $-6.0 - j8.5 = 10.41/\underline{125.2^\circ}$, it has been rotated from the third into the second quadrant through an angle of 72° without changing its magnitude.

Summary.—To add or subtract vectors, they must be expressed first as rectangular vectors, that is, vectors having their components along the axes of reals and imaginaries. This is the more convenient method of expressing vectors when addition or subtraction is to be performed.

It is possible with rectangular vectors to multiply, to take the reciprocal, to divide, to rotate, and to raise to a power. It is very much simpler to perform these operations with the vectors expressed either in exponential or in polar form. It is necessary to use either polar or exponential notation in finding roots of vector quantities.

Sometimes it is simpler to multiply and divide quantities when expressed in rectangular vectors than to convert them into polar quantities and back again, etc.

APPLICATION OF COMPLEX QUANTITIES TO ALTERNATING CURRENTS

55. Simple Series Circuits.—In Fig. 71(*a*) is shown a simple series circuit consisting of a noninductive resistance R and an inductive reactance X_L. This circuit is identical with that shown in Fig. 35 (p. 38). Let the direction of I be so chosen that I lies along the axis of reals, Fig. 71(*b*). That is, $\dot{I} = I + j0$. Since the IR-vector is in phase with I, its complex expression is $IR + j0$. The IX_L-vector is at right angles to I and leads; therefore, its complex expression is $+jIX_L$. Hence, the line voltage is

$$\dot{E} = IR + jIX_L = I(R + jX) = I\dot{Z}. \qquad (83)$$

It is seen from (83) that the impedance Z is expressed as a complex quantity. Although expressed as a complex quantity, impedance itself is *not* a vector quantity. Impedance, however, does resolve the resistance and reactance voltage drops into two voltage vectors at right angles to each other so that it is actually a *complex operator*.

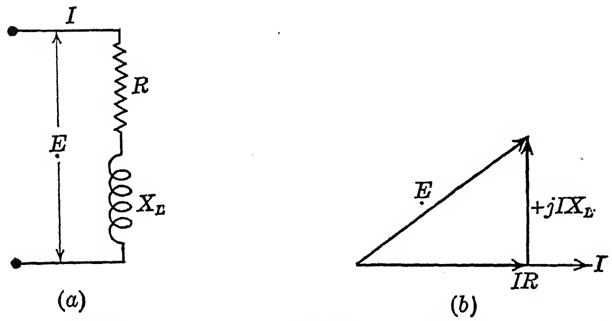


FIG. 71.—Series inductive circuit and its complex-vector diagram.

Because it is expressed algebraically as a complex quantity and treated as a complex quantity, impedance is defined as *vector impedance*[1] or *complex impedance.* To be certain that impedance will be treated as a complex quantity a dot is placed beneath its symbol ($\underset{\cdot}{Z}$).

In the foregoing example, the direction of the current is chosen along the axis of reals. Let the direction of the voltage now be taken along the axis of reals, that is, $\underset{\cdot}{E} = E + j0$ volts.

By using the impedance as a complex operator, the current may be found as follows:

$$\underset{\cdot}{I} = \frac{\underset{\cdot}{E}}{R + jX_L} = \frac{E + j0}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L} = \frac{E(R - jX_L)}{R^2 + X_L^2}. \qquad (84)$$

Example.—In a series circuit, the impressed voltage is 110 volts, 60 cycles, the resistance is 15 ohms, and the inductive reactance is 18 ohms. With the voltage vector taken along the axis of reals, determine the current.

$$\begin{aligned} I &= \frac{110}{15 + j18} = \frac{110}{15 + j18} \cdot \frac{15 - j18}{15 - j18} \\ &= \frac{110(15 - j18)}{225 + 324} = \frac{1{,}650 - j1{,}980}{549} \\ &= 3.01 - j3.61 \text{ amp.} \quad \textit{Ans.} \\ |I| &= \sqrt{3.01^2 + 3.61^2} = 4.70 \text{ amp.} \quad \textit{Ans.} \end{aligned}$$

[1] American Standard Definitions of Electrical Terms, C42 (1941), Definition 05.20.196. "The vector impedance of a portion of an electric circuit for simple sinusoidal current and potential difference is the ratio of the corresponding complex harmonic potential difference to the corresponding complex-current." (A dash over the symbol $\bar{Z}$ is also frequently used.)

The vector diagram is shown in Fig. 72.

$$\tan\theta = \frac{-3.61}{3.01} = -1.20; \qquad \theta = -50.2°.$$

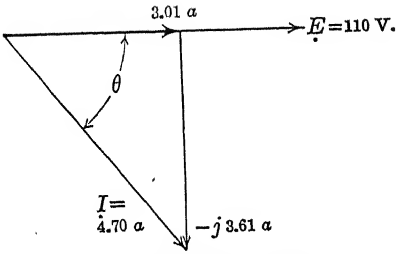


FIG. 72.—Current as a complex vector.

In Fig. 73(*a*) is shown a resistance R and a capacitive reactance X_C in series. If the current I be taken along the axis of reals, IX_C will be lagging 90°, Fig. 73(*b*), and the emf

$$\begin{aligned} \dot{E} &= (I + j0)(R - jX_C) \\ &= \dot{I}R - j\dot{I}X_C = \dot{I}(R - jX_C) \\ &= I(R - jX_C), \quad (85) \end{aligned}$$

since $\dot{I} = I + j0$.

Hence, with capacitance the impedance operator $\dot{Z} = R - jX_C$. If, however, the emf E be taken along the axis of reals,

$$\dot{I} = \frac{E}{R - jX_C} = \frac{E + j0}{R - jX_C}\,\frac{R + jX_C}{R + jX_C} = \frac{E(R + jX_C)}{R^2 + X_C^2} \qquad (86)$$

The current will be given by

$$\dot{I} = E\left(\frac{R}{R^2 + X_C^2} + j\,\frac{X_C}{R^2 + X_C^2}\right) = I_1 + jI_2 \qquad \text{amp} \qquad (87)$$

and will be leading Fig. 73(*c*).

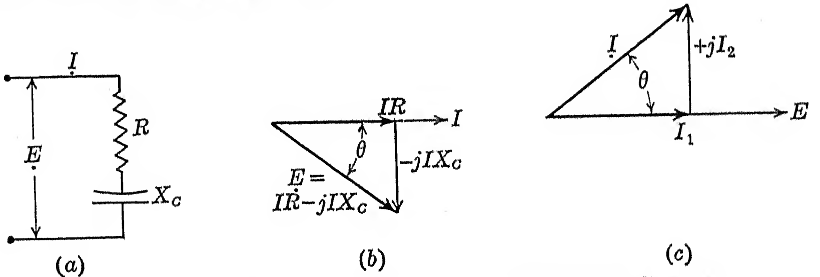


FIG. 73.—Series capacitive circuit and its complex-vector diagram.

It is to be noted that inductive reactance is denoted by $+jX_L$ and capacitive reactance by $-jX_C$.

Impedance also may be expressed as a polar operator, a positive angle ($\angle$) being used for inductive reactance and a negative angle ($\overline{\angle}$) for capacitive reactance.

Example.—A capacitance of 20 μf and a resistance of 100 ohms are connected in series across 120-volt 60-cycle mains. Determine the current, choosing the

position of the voltage vector so that it lies along the positive axis of reals (see example, p. 42).

$$X_C = \frac{1}{20 \cdot 377 \cdot 10^{-6}} = 132.6 \text{ ohms.}$$

$$I = \frac{120}{100 - j132.6} = \frac{120}{100 - j132.6} \cdot \frac{100 + j132.6}{100 + j132.6}$$

$$= \frac{12,000}{10,000 + 17,600} + j\frac{15,910}{10,000 + 17,600}$$

$$= \frac{12,000}{27,600} + j\frac{15,910}{27,600}$$

$$= 0.435 + j0.577 \text{ amp.} \quad \textit{Ans.}$$

The absolute value of I is

$$|I| = \sqrt{(0.435)^2 + (0.577)^2} = \sqrt{0.523} = 0.723 \text{ amp.} \quad \textit{Ans.}$$

The phase angle is

$$\tan^{-1}\frac{0.577}{0.435} = \tan^{-1} 1.326 = 53.0°. \quad \textit{Ans.}$$

The impedance may be expressed in the polar form as follows:

$$100 - j132.6 = \sqrt{27,600}\,\overline{\backslash 53°} = 166\overline{\backslash 53°}.$$

$$I = \frac{120}{166\overline{\backslash 53°}} = 0.723\underline{/53°} \text{ amp } (\textit{check}). \quad \textit{Ans.}$$

In the application of the complex method, it is not necessary that either current or voltage be taken along the axis of reals. This is illustrated by the intermediate voltages and currents, (*d*) and (*e*) in the example, Sec. 60, p. 87.

56. Power Determination.—If the voltage and current of a circuit are expressed as rectangular vectors, it is a simple matter to determine the power.

For example, in Fig. 72, 3.01 amp is the *energy* current, and the power $P = 110 \cdot 3.01 = 331$ watts. In Fig. 73(*c*), I_1 is the energy current, and the power $P = EI_1$ watts. However, if voltage and current are not along the axis of reals, each may be resolved into real and imaginary components and the power readily determined. Consider Fig. 74, in which the voltage $\dot{E} = E_1 + jE_2$ lies in the first quadrant and the current $\dot{I} = I_1 - jI_2$ lies in the fourth quadrant. A study of Fig. 74 shows that the component voltage E_1 and the component current I_1 are *in phase*. Since component voltages and currents may be treated as if they were acting alone, the power contributed by E_1 and I_1 is

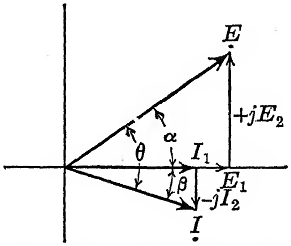


FIG. 74.—Power $= EI \cos \theta = E_1I_1 - E_2I_2$.

$$P_1 = E_1I_1.$$

The coefficient of E_2 is $+j$, and the coefficient of I_2 is $-j$. Hence, these two components are *in phase opposition*, and their product gives negative power. That is,

$$P_2 = -E_2I_2.$$

Components E_1 and I_2, and likewise components E_2 and I_1, contribute no power, since they are in quadrature. The total power under the foregoing conditions is, therefore,

$$P = P_1 + P_2 = E_1I_1 - E_2I_2. \tag{88}$$

If the complex expressions for voltage and current be written as

$$\dot{E} = E_1 + jE_2,$$
$$\dot{I} = I_1 + jI_2,$$

the total power is the sum of the product of the real quantities (E_1 and I_1) and the product of the imaginary quantities (E_2 and I_2), the signs being determined in the ordinary algebraic manner. The operator j, however, *must not be included* in the multiplication of the imaginary quantities. For example, in the expressions

$$\dot{E} = E_1 + jE_2$$

and $\dot{I} = I_1 - jI_2$, the plus sign before jE_2 and the minus sign before jI_2 show that E_2 and I_2 are in *opposition*. Yet their algebraic product, including the operator, is plus. That is, $jE_2(-jI_2) = +E_2I_2$, which is *incorrect* when used to determine the power.

A study of Fig. 74 shows that the phase angle between E and I

$$\theta = \alpha + \beta = \tan^{-1}\frac{E_2}{E_1} + \tan^{-1}\frac{I_2}{I_1}. \tag{89}$$

With polar vectors, the power is obtained in the usual manner, that is, by taking the product of the voltage and current magnitudes and multiplying this result by the cosine of the angle between them. Thus, if $\dot{E} = E\underline{/\alpha}$ and $\dot{I} = I\underline{/\beta}$,

$$P = EI\cos(\alpha - \beta).$$

Example.—When a voltage $60 + j80$ is acting on a circuit, the current is $-3 + j5$. Determine (*a*) complex expression for impedance of circuit; (*b*) whether impedance is capacitive or inductive; (*c*) power; (*d*) phase angle between current and voltage.

$$(a)\ \dot{Z} = \frac{60 + j80}{-3 + j5} = \frac{60 + j80}{-3 + j5}\cdot\frac{-3 - j5}{-3 - j5}$$
$$= \frac{-180 - j240 - j300 + 400}{9 + 25} = \frac{220 - j540}{34} = 6.47 - j15.88 \text{ ohms.}$$
$$= 17.15\overline{\backslash 67.9^\circ}\text{ ohms} \quad \textit{Ans.}$$

(*b*) Capacitive, since imaginary term is minus. *Ans.*
(*c*) $P = [60 \cdot (-3)] + [80 \cdot 5] = 220$ watts. *Ans.*
(*d*) $\tan \alpha = {}^{80}\!/_{60} = 1.333, \qquad \alpha = 53.1°.$

$$\tan \beta = \frac{5}{-3} = -1.667, \qquad \beta = 121°.$$

$$\theta = 121° - 53.1° = 67.9°. \quad \textit{Ans.}$$

Example.—Express the voltage and current in the foregoing example as polar vectors, and repeat (*a*) and (*c*).

$$\dot{E} = \sqrt{(60)^2 + (80)^2}\,\underline{/\tan^{-1} {}^{80}\!/_{60}} = 100\underline{/53.1°} \text{ volts.}$$

$$\dot{I} = \sqrt{(3)^2 + (5)^2}\,\underline{\backslash \tan^{-1} \frac{5}{-3}} = 5.83\underline{\backslash 121°} \text{ amp.}$$

(*a*) $$\dot{Z} = \frac{\dot{E}}{\dot{I}} = \frac{100\underline{/53.1°}}{5.83\underline{\backslash 121°}} = 17.15\underline{/53.1° - 121°} = 17.15\underline{/-67.9°} \text{ ohms.} \quad \textit{Ans.}$$

(*c*) $$\begin{aligned} P &= 100 \cdot 5.83 \cos(-67.9°) \\ &= 583 \cdot 0.376 = 220 \text{ watts.} \quad \textit{Ans. (check).} \end{aligned}$$

57. Conjugate Method for Power.—Conjugate complex quantities are complex quantities with the same real and imaginary components but with the imaginary components of opposite sign. Thus, $E_1 + jE_2$ and $E_1 - jE_2$ are conjugate complex quantities. Likewise, conjugate polar quantities are polar quantities with the angles differing in sign, such as $E\underline{/\alpha}$ and $E\underline{\backslash\alpha}$. The difficulty in Sec. 56 of not being able to multiply together the complex expressions for voltage and current to obtain the power is eliminated if the conjugate of either quantity is used, preferably the voltage. Moreover, the imaginary term in the product gives the *vars* of the circuit.

Thus, in the example, Sec. 56, $\dot{E} = 60 + j80$ volts. The conjugate of $\dot{E}$ is represented by $\mathring{\dot{E}} = 60 - j80$ volts.

$$\begin{aligned} \mathring{\dot{E}}\dot{I} &= (60 - j80)(-3 + j5) \\ &= (-180 + 400) + j(300 + 240) \\ &= 220 + j540 = 220 \text{ watts} + 540 \text{ vars.} \quad \textit{Ans.} \end{aligned}$$

The value of vars may be verified since

$$\begin{aligned} \text{vars} &= EI \sin \theta = 100 \cdot 5.83 \sin 67.9° \cdot \\ &= 583 \cdot 0.9265 = 540 \text{ vars } (\textit{check}). \end{aligned}$$

When the conjugate of the voltage is used, capacitive vars have a positive sign and inductive vars a negative sign. These signs are in accordance with the ASA[1] C42 definitions, the accepted American electrical standards. If the conjugate of the current rather than of the voltage is used, capacitive vars have a negative sign and inductive vars a positive sign.

[1] American Standard Definitions of Electrical Terms, 1941; Definition 05.21.050.

58. Parallel Circuits.—When two or more circuits are in parallel, the current in each may be found in complex. The total current in complex then is found by adding all real components and all imaginary components. Consider Fig. 75, in which is shown a parallel circuit of two branches connected across voltage E. One branch consists of a resistance R_1 in series with an inductive reactance X_L; the second branch consists of a resistance R_2 in series with a capacitive reactance X_C. The currents are I_1 and I_2; the line current is I. Let the voltage E be along the axis of reals, or $\dot{E} = E + j0$.

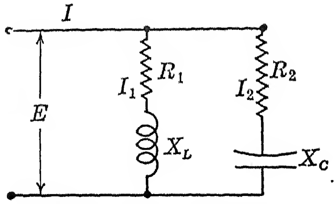


FIG. 75.—Circuits in parallel.

$$\dot{I}_1 = \frac{E}{R_1 + jX_L} = \frac{E}{R_1 + jX_L} \cdot \frac{R_1 - jX_L}{R_1 - jX_L}$$
$$= E\left(\frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}\right) = I_{1_e} - jI_{1_q} \quad (90a)$$

(see p. 79).

$$\dot{I}_2 = \frac{E}{R_2 - jX_C} = \frac{E}{R_2 - jX_C} \cdot \frac{R_2 + jX_C}{R_2 + jX_C}$$
$$= E\left(\frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}\right) = I_{2_e} + jI_{2_q}. \quad (90b)$$

The total current

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = (I_{1_e} + I_{2_e}) + j(-I_{1_q} + I_{2_q})$$
$$= I_e + jI_q.$$
$$\tan\theta = \frac{I_q}{I_e}.$$

Example.—In Fig. 75, let $R_1 = 8$ ohms, $X_1 = 12$ ohms, $R_2 = 15$ ohms, $X_2 = 20$ ohms, $E = 120$ volts, 60 cycles. Determine (*a*) current in each branch; (*b*) total current; (*c*) equivalent impedance of circuit; (*d*) power in each branch; (*e*) total power; (*f*) total power factor.

(*a*) $\dot{I}_1 = \frac{120 + j0}{8 + j12} = \frac{120 + j0}{8 + j12} \cdot \frac{8 - j12}{8 - j12} = 4.61 - j6.92$ amp. *Ans.*

$|I_1| = \sqrt{(4.61)^2 + (6.92)^2} = 8.32$ amp. *Ans.*

$\dot{I}_2 = \frac{120 + j0}{15 - j20} = \frac{120(15 + j20)}{225 + 400} = 2.88 + j3.84$ amp. *Ans.*

$|I_2| = \sqrt{(2.88)^2 + (3.84)^2} = 4.80$ amp. *Ans.*

(*b*) $\dot{I} = \dot{I}_1 + \dot{I}_2 = (4.61 - j6.92) + (2.88 + j3.84) = 7.49 - j3.08$ amp. *Ans.*

$|I| = \sqrt{(7.49)^2 + (3.08)^2} = 8.10$ amp. *Ans.*

(*c*) $\dot{Z} = \frac{\dot{E}}{\dot{I}} = \frac{120 + j0}{7.49 - j3.08} = 13.68 + j5.64$ ohms. *Ans.*

$|Z| = \sqrt{(13.68)^2 + (5.64)^2} = 14.80$ ohms. *Ans.*

(*d*) $P_1 = 120 \cdot 4.61 = (8.32)^2 8 = 553$ watts. *Ans.*

$P_2 = 120 \cdot 2.88 = (4.80)^2 15 = 345$ watts. *Ans.*

(*e*) $P = P_1 + P_2 = 553 + 345 = 898$ watts. *Ans.*

(*f*) $\cos\theta = \dfrac{R}{Z} = \dfrac{13.68}{14.80} = 0.924$. *Ans.*

59. Equivalent Parallel Impedance.—The solution of parallel circuits may be accomplished in the same manner as with d-c parallel circuits (see Vol. I, Chap. II), except that complex impedances rather than simple resistances are involved. With resistances in parallel, the reciprocal of the equivalent resistance is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots.$$

With two resistances in parallel,

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Likewise, with impedances in parallel,

$$\frac{1}{\dot{Z}} = \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} + \frac{1}{\dot{Z}_3} + \cdots; \tag{91}$$

With two impedances in parallel,

$$\dot{Z} = \frac{\dot{Z}_1 \dot{Z}_2}{\dot{Z}_1 + \dot{Z}_2}. \tag{92}$$

Each of the foregoing impedances must be expressed in complex (in rectangular, polar or exponential form).

In Fig. 75, let $R_1 = 8\Omega$; $X_L = 12\Omega$; $R_2 = 15\Omega$; $X_C = 20\Omega$; $E = 120$ volts, 60 cycles. Determine (*a*) impedance in complex and magnitude; (*b*) current; (*c*) power factor; (*d*) power-factor angle.

(*a*) $\dfrac{1}{\dot{Z}} = \dfrac{1}{\dot{Z}_1} + \dfrac{1}{\dot{Z}_2} = \dfrac{1}{8 + j12} + \dfrac{1}{15 - j20} = \dfrac{8 - j12}{(8)^2 + (12)^2} + \dfrac{15 + j20}{(15)^2 + (20)^2}$

$= 0.03845 - j0.0577 + 0.0240 + j0.0320 = 0.06245 - j0.0257$ mho.

$\dot{Z} = \dfrac{1}{0.06245 - j0.0257} = \dfrac{0.06245 + j0.0257}{(0.06245)^2 + (0.0257)^2} = 13.68 + j5.64$ ohms

$|Z| = \sqrt{(13.68)^2 + (5.64)^2} = 14.80$ ohms. *Ans.*

(*b*) $\dot{I} = \dfrac{\dot{E}}{\dot{Z}} = \dfrac{120}{13.68 + j5.64} \cdot \dfrac{13.68 - j5.64}{13.68 - j5.64}$

$= \dfrac{1{,}644 - j677}{187.6 + 31.2} = \dfrac{1{,}644 - j677}{218.8}$

$= 7.52 - j3.09$ amp.

$|I| = \sqrt{(7\ 52)^2 + (3.09)^2} = 8.14$ amp.

(c) $\cos\theta = \dfrac{13.68}{14.80} = 0.924 = \text{P.F.}$

(d) $\theta = 22.3°$.

The equivalent impedance may also be found by (92).

$$\begin{aligned} Z &= \frac{(8 + j12)(15 - j20)}{(8 + j12) + (15 - j20)} \\ &= \frac{120 - j160 + j180 + 240}{23 - j8} = \frac{360 + j20}{23 - j8} \\ &= \frac{360 + j20}{23 - j8} \cdot \frac{23 + j8}{23 + j8} = \frac{8{,}120}{593} + j\frac{3{,}340}{593} \\ &= 13.69 + j5.64 \text{ ohms.} \quad \textit{Ans.} \end{aligned}$$

With two impedances in parallel the use of (92) has the advantage that decimal quantities having several ciphers immediately following the decimal point are avoided. This condition occurs with impedances whose values are in the hundreds of ohms or greater. With more than two impedances in parallel, however, the method given by (91) is to be preferred.

60. Series-parallel Circuit.—The solution of series-parallel circuits is accomplished in the same manner as for direct-current series-parallel circuits (see Vol. I, Chap. II), except that complex impedances rather than simple resistances are involved. The impedance of the parallel circuit is first found by the methods of Sec. 58 or 59. This impedance, in complex, is then added to the impedance (in complex) in series with the parallel part of the circuit to find the total impedance of the circuit. The problem is then solved by the method of Sec. 55.

Example.—Figure 76(a) shows a series-parallel circuit consisting of two impedances $R_1 + jX_1 = 6 + j3$ and $R_2 - jX_2 = 5 - j8$ ohms in parallel and in

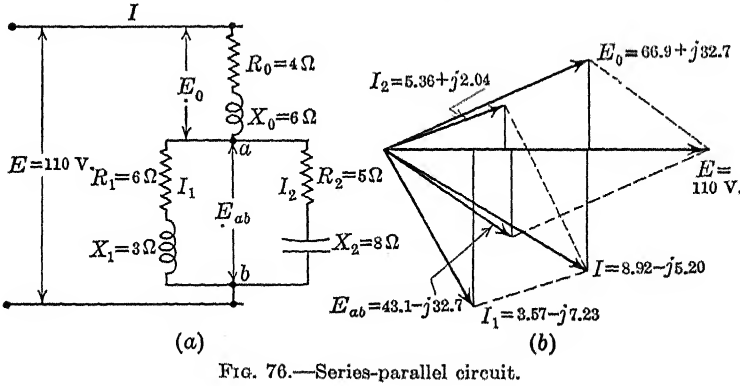


FIG. 76.—Series-parallel circuit.

series with the impedance $R_0 + jX_0 = 4 + j6$ ohms. The entire circuit is connected across 110-volt 50-cycle mains. Determine (a) impedance of parallel

circuit; (*b*) impedance of entire circuit; (*c*) total current I; (*d*) voltage E_{ab} across parallel circuit; (*e*)current in each impedance; (*f*) power in each circuit element.

(*a*)
$$\frac{1}{Z'} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{6+j3} + \frac{1}{5-j8}$$
$$= \frac{6-j3}{(6)^2+(3)^2} + \frac{5+j8}{(5)^2+(8)^2} = 0.1333 - j0.0667 + 0.0562 + j0.0899$$
$$= 0.1895 + j0.0232 \text{ mho.}$$
$$Z' = \frac{1}{0.1895+j0.0232} = \frac{0.1895 - j0.0232}{(0.1895)^2+(0.0232)^2} = 5.20 - j0.637 \text{ ohm.} \quad \textit{Ans.}$$
$$|Z'| = \sqrt{(5.20)^2 + (0.637)^2} = 5.23 \text{ ohms.} \quad \textit{Ans.}$$

(*b*)
$$Z = (5.20 - j0.637) + (4 + j6) = (5.20 + 4) + j(-0.637 + 6)$$
$$= 9.20 + j5.36 \text{ ohms.} \quad \textit{Ans.}$$
$$|Z| = \sqrt{(9.20)^2 + (5.36)^2} = 10.65 \text{ ohms.} \quad \textit{Ans.}$$

(*c*) The line-voltage vector is taken along the axis of reals.
$$I = \frac{110}{9.20+j5.36} = \frac{110}{9.20+j5.36}\cdot\frac{9.20-j5.36}{9.20-j5.36}$$
$$= \frac{110\cdot 9.20}{(9.20)^2+(5.36)^2} - j\frac{110\cdot 5.36}{(9.20)^2+(5.36)^2} = 8.92 - j5.20 \text{ amp.} \quad \textit{Ans.}$$
$$|I| = \sqrt{(8.92)^2 + (5.20)^2} = 10.33 \text{ amp.} \quad \textit{Ans.}$$

Also,
$$|I| = \frac{110}{|Z|} = 110/10.65 = 10.33 \text{ amp (\textit{check}).}$$

(*d*)
$$E_{ab} = IZ' = (8.92 - j5.20)(5.20 - j0.637) = 43.1 - j32.7 \text{ volts.} \quad \textit{Ans.}$$
$$|E_{ab}| = \sqrt{(43.1)^2 + (32.7)^2} = 54.1 \text{ volts.} \quad \textit{Ans.}$$

Also, the voltage across $R_0 + jX_0$

$$E_0 = (8.92 - j5.20)(4 + j6) = 66.9 + j32.7 \text{ volts.} \quad \textit{Ans.}$$
$$E_0 + E_{ab} = 110 + j0 \text{ (\textit{check}).}$$

(*e*)
$$I_1 = \frac{E_{ab}}{Z_1} = E_{ab}\cdot\frac{1}{Z_1}.$$

From (*a*),

$$\frac{1}{Z_1} = 0.1333 - j0.0667 \text{ mho.}$$
$$I_1 = (43.1 - j32.7)(0.1333 - j0.0667) = 3.57 - j7.23 \text{ amp.} \quad \textit{Ans.}$$
$$|I_1| = \sqrt{(3.57)^2 + (7.23)^2} = 8.06 \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = \frac{E_{ab}}{Z_2} = (43.1 - j32.7)(0.0562 + j0.0899) = 5.36 + j2.04 \text{ amp.} \quad \textit{Ans.}$$

since, from (*a*), $1/Z_2 = 0.0562 + j0.0899$,

$$|I_2| = \sqrt{(5.36)^2 + (2.04)^2} = 5.73 \text{ amp.} \quad \textit{Ans.}$$
$$I_1 + I_2 = I \text{ (\textit{check}).}$$

(*f*)
$$P_0 = (66.9\cdot 8.92) - (32.7\cdot 5.20) \quad \text{(see Sec. 56)}$$
$$= 597 - 170 = 427 \text{ watts.} \quad \textit{Ans.}$$
$$P_1 = (43.1\cdot 3.57) + (32.7\cdot 7.23) = 153.7 + 236 = 389.7 \text{ watts.} \quad \textit{Ans.}$$
$$P_2 = (43.1\cdot 5.36) - (32.7\cdot 2.04) = 231 - 66.7 = 164.3 \text{ watts.} \quad \textit{Ans.}$$

Also,
$$P_0 = I_0^2R_0;\ P_1 = I_1^2R_1;\ P_2 = I_2^2R_2 \text{ (\textit{check}).}$$

The total power

$$P = 427 + 389.7 + 164.3 = 981.0 \text{ watts}$$
$$= (110 \cdot 8.92) = (10.33)^2 \cdot 9.20 \ (\textit{check}).$$

The vector diagram for this circuit is shown in Fig. 76(*b*). To avoid confusion, the individual resistance and reactance drops I_1R_1, I_1X_1, I_2R_2, I_2X_2 are omitted.

61. Solution of Series-parallel Circuits with Polar Vectors.—The method of equivalent impedance given in Sec. 59 is equally applicable to quantities expressed in polar form. For example, let it be required to solve the series-parallel circuit of Sec. 60 by means of polar operators.

(*a*) $$Z_1 = \sqrt{(6)^2 + (3)^2}/\tan^{-1} 3/6 = 6.71/26.6^\circ.$$
$$Z_2 = \sqrt{(5)^2 + (8)^2}\backslash\tan^{-1} 8/5 = 9.42\backslash 58.0^\circ.$$

Using Eq. (92), (p. 85),

$$Z' = \frac{6.71/26.6^\circ \cdot 9.42\backslash 58.0^\circ}{(6 + 5) + j(3 - 8)} = \frac{63.2\backslash 31.4^\circ}{12.08\backslash 24.4^\circ}$$
$$= 5.23\backslash 7.0^\circ. \quad \textit{Ans.}$$
$$= 5.23(\cos 7.0^\circ - j \sin 7.0^\circ) = 5.20 - j0.637 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) $$Z = (4 + 5.20) + j(6.0 - 0.637) = 9.20 + j5.36 \text{ ohms}$$
$$= 10.65/30.2^\circ \text{ ohms.} \quad \textit{Ans.}$$

(*c*) $$I = \frac{110/0^\circ}{10.65/30.2^\circ} = 10.33\backslash 30.2^\circ$$
$$= 10.33\ (\cos 30.2^\circ - j \sin 30.2^\circ) = 8.92 - j5.20 \text{ amp.} \quad \textit{Ans.}$$

(*d*) $$E_{ab} = IZ' = 10.33\backslash 30.2^\circ \cdot 5.23\backslash 7.0^\circ$$
$$= 54.0\backslash 37.2^\circ \text{ volts.} \quad \textit{Ans.}$$

(*e*) $$I_1 = \frac{54.0\backslash 37.2^\circ}{6.71/26.6^\circ} = 8.06\backslash 63.8^\circ \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = \frac{54.0\backslash 37.2^\circ}{9.42\backslash 58.0^\circ} = 5.73/20.8^\circ \text{ amp.} \quad \textit{Ans.}$$

(*f*) $$P_0 = (10.33)^2 4 = 427 \text{ watts.} \quad \textit{Ans.}$$
$$P_1 = 54.0 \cdot 8.06 \cos (63.8^\circ - 37.2^\circ) = 389 \text{ watts.} \quad \textit{Ans.}$$
$$P_2 = 54.0 \cdot 5.73 \cos (37.2^\circ + 20.8^\circ) = 164 \text{ watts.} \quad \textit{Ans.}$$

Also,
$$P_1 = I_1^2R_1; P_2 = I_2^2R_2.$$

62. Admittance, Conductance, Susceptance.—*Admittance, conductance,* and *susceptance* are parameters similar to impedance, resistance, and reactance. Admittance is the reciprocal of impedance, and thus the two are related in the same manner as conductance and resistance are related in the d-c circuit. As is shown in the preceding sections, a-c circuits and networks can be solved by means of impedances expressed as complex quantities. However, the solutions of such problems often are facilitated by the use of admittance.

In (87) (p. 80), the current $\dot{I}$ in an inductive circuit,

$$\dot{I} = \dot{E}\left[\frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}\right] \qquad \text{amp,} \qquad [(87)]$$

and in (88) for the capacitive circuit,

$$\dot{I} = \dot{E}\left[\frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}\right] \qquad \text{amp.} \qquad [(88)]$$

The quantity within each of the brackets is called the *admittance* of the circuit and is denoted by $\dot{Y}$. Since current is equal to the product of voltage and admittance, admittance is in the nature of d-c conductance and is expressed in reciprocal ohms or mhos (℧).

Hence

$$\dot{I} = \dot{E}\dot{Y}. \qquad (93)$$

Since

$$\dot{I} = \frac{\dot{E}}{\dot{Z}},$$

$$\dot{Y} = \frac{1}{\dot{Z}} \qquad (94)$$

and

$$\dot{Z} = \frac{1}{\dot{Y}}. \qquad (95)$$

It is also apparent that $|Y| = 1/|Z|$ and $|Z| = 1/|Y|$. Omitting subscripts in (87) and (88) the quantity

$$\frac{R}{R^2 + X^2} = G \qquad \text{mhos} \qquad (96)$$

is the *conductance* of the circuit. Note that, when $X = 0, G = 1/R$, as with d-c circuits. Also,

$$\frac{X}{R^2 + X^2} = B \qquad \text{mhos} \qquad (97)$$

is the *susceptance* of the circuit. Note that, when $R = 0$, $B = 1/X$. It follows from (87), (88), (96), (97) that

$$\dot{Y} = G \pm jB \qquad \text{mhos.} \qquad (98)$$

The *negative* sign is used for the *inductive* circuit and the *positive* sign for the *capacitive* circuit. Note that, with an *inductive* circuit, the susceptance is *negative*, whereas the *reactance* is *positive;* with a capacitive circuit, the *susceptance* is *positive*, and the *reactance* is *negative*.

From (87), (88), (98),

$$\dot{I} = \dot{E}\dot{Y} = \dot{E}(G \pm jB) \qquad \text{amp.} \qquad (99)$$

In Fig. 77(a) is shown the vector diagram for the inductive circuit in which jB is negative and tle current lags; in Fig. 77(b) is shown the vector diagram for the capacitive circuit in which jB is positive and the current leads. In both diagrams the voltage is taken along the axis of reals and $\dot{E} = E + j0$. From Fig. 77(a) and (b) it follows

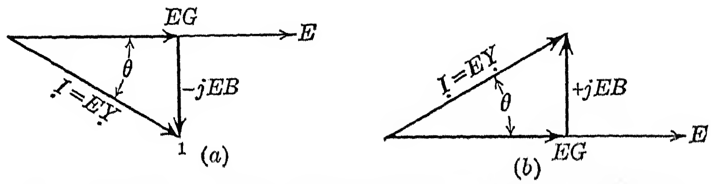


FIG. 77.—Complex-vector diagram with voltage along axis of reals.

that with an inductive circuit the susceptance must be negative in order that the current may lag and with a capacitive circuit the susceptance must be positive in order that the current may lead.

From Fig. 77(a) and (b),

$$\tan\theta = \frac{B}{G}, \qquad (100)$$

$$\cos\theta = \frac{G}{Y}. \qquad (101)$$

Note that EG is the energy current (Sec. 36, p. 62). Hence, power

$$P = E \cdot EG = E^2G \qquad \text{watts.} \qquad (102)$$

The quadrature current is EB and the reactive power

$$Q = E \cdot EB = E^2B \qquad \text{vars.} \qquad (103)$$

For unity power factor,

$$\Sigma B = B_1 + B_2 + B_3 + \cdots = 0. \qquad (104)$$

Since $\dot{Z} = 1/\dot{Y}$,

$$\dot{Z} = \frac{1}{G+jB} = \frac{1}{G+jB} \cdot \frac{G-jB}{G-jB}$$
$$= \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2} = R - jX \quad \text{ohms.}$$

Hence,

$$R = \frac{G}{G^2+B^2} \qquad \text{ohms,} \qquad (105)$$

$$X = \frac{B}{G^2+B^2} \qquad \text{ohms.} \qquad (106)$$

63. Parallel Circuit Using Admittances.—The parallel circuit, Fig. 75 (p. 84), may be solved by means of admittances, the voltage being taken along the axis of reals.

$$G_1 = \frac{R_1}{R_1^2 + X_L^2}; \qquad -B_1 = \frac{-X_L}{R_1^2 + X_L^2} \qquad \text{mhos.}$$
$$\dot{Y}_1 = G_1 - jB_1 \qquad \text{mhos.}$$
$$G_2 = \frac{R_2}{R_2^2 + X_C^2}; \qquad B_2 = \frac{X_C}{R_2^2 + X_C^2} \qquad \text{mhos.}$$
$$\dot{Y}_2 = G_2 + jB_2 \qquad \text{mhos.}$$

The total circuit admittance

$$\dot{Y} = \dot{Y}_1 + \dot{Y}_2 = (G_1 + G_2) + j(-B_1 + B_2) = G \pm jB \quad \text{mhos.}$$
$$\dot{I} = \dot{E}\dot{Y} = \dot{E}(G \pm jB) \qquad \text{amp.}$$
$$P = E \cdot EG = E^2G \qquad \text{watts.}$$

Power factor

$$\text{P.F.} = \cos\theta = \frac{G}{|Y|}.$$

64. Series-parallel Circuit Using Admittances.—The series-parallel circuit also may be solved by the use of admittances, the method being not unlike that described in Sec. 60 (p. 86). The admittance of the parallel-circuit element is first found. The impedance, the reciprocal of the admittance, is then obtained and is added to the series impedance, giving the total impedance of the circuit. This treatment of the parallel circuit is similar to that used in the d-c circuit when the conductance of each parallel element is first found and then these are added to give the total conductance of the parallel circuit. The resistance is the reciprocal of this conductance. The procedure is illustrated by solving the example of Sec. 60, Fig. 76.

Example.—In Fig. 76 determine (*a*) admittance of each parallel branch; (*b*) admittance of parallel circuit; (*c*) impedance of parallel circuit; (*d*) impedance of entire circuit; (*e*) admittance of entire circuit; (*f*) current; (*g*) voltage E_{ab} across parallel circuit; (*h*) current in each parallel branch.

(*a*) $\dot{Y}_1 = G_1 - jB_1 = \dfrac{6}{36 + 9} - j\dfrac{3}{36 + 9} = 0.1333 - j0.0667$ mho. *Ans.*

$\dot{Y}_2 = G_2 + jB_2 = \dfrac{5}{25 + 64} + j\dfrac{8}{25 + 64} = 0.0562 + j0.0899$ mho. *Ans.*

(*b*) $\dot{Y}' = \dot{Y}_1 + \dot{Y}_2 = G' + jB' = 0.1895 + j0.0232$ mho. *Ans.*

(*c*) $$\dot{Z}' = \frac{1}{\dot{Y}'} = \frac{0.1895}{(0.1895)^2 + (0.0232)^2} - j\frac{0.0232}{(0.1895)^2 + (0.0232)^2}$$

[see Eq. (95) p. 89]

$$= \frac{0.1895}{0.0364} - j\frac{0.0232}{0.0364} = 5.20 - j0.637 \text{ ohm.} \quad \textit{Ans.}$$

(d) $Z = (5.20 - j0.637) + (4 + j6) = 9.20 + j5.36$ ohms. *Ans.*

(e) $$Y = \frac{1}{Z} = \frac{1}{9.20 + j5.36} \cdot \frac{9.20 - j5.36}{9.20 - j5.36}$$

$$= \frac{9.20}{84.7 + 28.7} - j\frac{5.36}{84.7 + 28.7} = 0.0811 - j0.0473 \text{ mho.} \quad \textit{Ans.}$$

(f) $$I = EY = (110 + j0)(0.0811 - j0.0473)$$
$$= 8.92 - j5.20 \text{ amp.} \quad \textit{Ans.}$$

(g) $$E_{ab} = IZ' = (8.92 - j5.20)(5.20 - j0.637)$$
$$= 43.1 - j32.7 \text{ volts.} \quad \textit{Ans.}$$

(h) $$I_1 = E_{ab}Y_1 = (43.1 - j32.7)(0.1333 - j0.0667)$$
$$= 3.57 - j7.23 \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = E_{ab}Y_2 = (43.1 - j32.7)(0.0562 + j0.0899)$$
$$= 5.36 + j2.04 \text{ amp.} \quad \textit{Ans.}$$

These results may be compared with those in Sec. 60. Other quantities such as P_0, P_1, P_2, P, and power factor are found in the same manner as in Sec. 60.

The foregoing methods illustrate the fact that alternating-current networks in the steady state may be solved by means of complex quantities, expressed either in terms of real and imaginary components or in polar (or exponential) form. Problems are solved exactly as are similar direct-current problems, complex impedances being substituted for resistances.

Example.—Consider the example of Sec. 58, Fig. 75, in which $R_1 = 8$ ohms, $X_L = 12$ ohms, $R_2 = 15$ ohms, $X_C = 20$ ohms.

Determine (a) admittance of each branch; (b) admittance of entire circuit; (c) impedance of entire circuit; (d) current in each branch; (e) total current; (f) power in each branch; (g) total power; (h) power factor of entire circuit.

(a) $$G_1 = \frac{8}{64 + 144} = \frac{8}{208} = 0.0384 \text{ mho.}$$

$$B_1 = -\frac{12}{64 + 144} = \frac{-12}{208} = -0.0577 \text{ mho.}$$

$$Y_1 = 0.0384 - j0.0577 \text{ mho.} \quad \textit{Ans.}$$

$$G_2 = \frac{15}{225 + 400} = \frac{15}{625} = 0.0240 \text{ mho.}$$

$$B_2 = \frac{20}{225 + 400} = \frac{20}{625} = 0.0320 \text{ mho.}$$

$$Y_2 = 0.0240 + j0.0320 \text{ mho.} \quad \textit{Ans.}$$

(b) $$Y = G + jB = (0.0384 + 0.0240) + j(-0.0577 + 0.0320)$$
$$= 0.0624 - j0.0257 \text{ mho.} \quad \textit{Ans.}$$

$$|Y| = \sqrt{(0.0624)^2 + (0.0257)^2} = 0.0676 \text{ mho.} \quad \textit{Ans.}$$

(c) $$R = \frac{0.0624}{(0.0624)^2 + (0.0257)^2} = 13.68 \text{ ohms.} \qquad [(105)]$$

$$X = \frac{0.0257}{(0.0624)^2 + (0.0257)^2} = 5.64 \text{ ohms.} \qquad [(106)]$$

$$Z = 13.68 + j5.64 \text{ ohms.} \quad \textit{Ans.}$$

$$|Z| = \sqrt{(13.68)^2 + (5.64)^2} = 14.80 \text{ ohms.} \quad \textit{Ans.}$$

$$14.80 = \frac{1}{0.0676} \ (\textit{check}).$$

(*d*) $\dot{I}_1 = 120(0.0384 - j0.0577) = 4.61 - j6.92$ amp. *Ans.*

$|I_1| = \sqrt{(4.61)^2 + (6.92)^2} = 8.32$ amp. *Ans.*

$\dot{I}_2 = 120(0.0240 + j0.0320) = 2.88 + j3.84$ amp. *Ans.*

$|I_2| = \sqrt{(2.88)^2 + (3.84)^2} = 4.80$ amp. *Ans.*

(*e*) $\dot{I} = \dot{I}_1 + \dot{I}_2 = (4.61 + 2.88) + j(-6.92 + 3.84)$
$= 7.49 - j3.08$ amp. *Ans.*

$|I| = \sqrt{(7.49)^2 + (3.08)^2} = 8.10$ amp. *Ans.*

Also,

$\dot{I} = \dot{E}\dot{Y} = 120(0.0624 - j0.0257)$
$= 7.49 - j3.08$ amp (*check*).

(*f*) $P_1 = E^2G_1 = (120)^2 0.0384 = 553$ watts. *Ans.*

$P_2 = E^2G_2 = (120)^2 0.0240 = 346$ watts. *Ans.*

Also,

$P_1 = \dot{I}_1^2 R_1; P_2 = \dot{I}_2^2 R_2.$

(*g*) $P = 553 + 346 = 899$ watts. *Ans.*

(*h*) $\cos\theta = \dfrac{G}{Y} = \dfrac{0.0624}{0.0676} = 0.924.$ *Ans.*

Also,

$$\cos\theta = \frac{P}{EI} = \frac{899}{120 \cdot 8.10} = 0.924 \ (check).$$

(Compare these results with those obtained in Sec. 58.)

CHAPTER IV

ALTERNATING-CURRENT INSTRUMENTS AND MEASUREMENTS

With direct-current ammeters and voltmeters the magnetic field in which the moving coil operates is unidirectional and constant in magnitude, so that a very strong field can be produced by a permanent magnet. Hence, for a given torque, the current in the moving coil can be small, and the instrument consumes but little power. However, with alternating currents the magnetic field must be alternating and must be produced by the current being measured. Also, except where great accuracy is not desired, iron cannot be used in the magnetic circuit. Hence, alternating-current instruments require much more power to operate than do direct-current instruments. Also, there are the effects of inductance, induced currents in metal adjacent to the coils, and there are frequency errors that are not present with direct-current instruments. Hence the design of alternating-current instruments is more involved than the design of direct-current instruments.

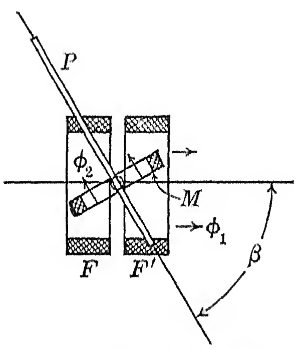


FIG. 78.—Principle of electrodynamometer.

65. Electrodynamometer Principle.—Some alternating-current instruments operate on the electrodynamometer principle, Fig. 78.

Two fixed coils FF' are in series and so connected that their magnetic fields act in conjunction. These coils may be considered as two parts of a single coil opened in the middle to allow the spindle of the moving coil to pass through.

M is a movable coil mounted on a vertical spindle. There is a hardened steel pivot at each end of the spindle, which turns in jeweled bearings. Two spiral springs similar to those used with direct-current instruments (Vol. I, Chap. V) oppose the turning of coil M and at the same time conduct current to the coil. As the springs can conduct but a very small current, the movable coil is wound with fine wire.

Assume that at some instant the direction of the magnetic field ϕ_1, due to the fixed coils, is from left to right. At the same instant, the current in coil M produces a field ϕ_2 whose direction is along the

axis of M. Coils tend to align themselves so that the number of magnetic linkages in the system is a maximum. The moving coil M, therefore, tends to turn in a clockwise direction so that its field will act in conjunction with ϕ_1. The turning of M is opposed by the control springs.

The torque developed is proportional to ϕ_1, ϕ_2, and $\sin \beta$, where β is the angle between the axis of coil M and the axis of coils FF'. As ϕ_1 and ϕ_2 are proportional to the currents in the coils FF' and M, the torque is proportional to the product of the two currents and $\sin \beta$.

66. Electrodynamometer Voltmeter.—Some alternating-current voltmeters operate on the electrodynamometer principle. The fixed coils FF', Fig. 79, are wound with fine wire and are connected in series with the moving coil M. A high resistance R is connected in series with the dynamometer to limit the current when the instrument is connected across the line. The current in the dynamometer, therefore, is proportional to the line voltage. The current causes coil M to turn, and the pointer attached to it moves over a scale graduated in volts. The scale is not divided uniformly, as is that of the direct-current voltmeter, for the deflections are very nearly proportional to the *square* of the voltage. The divisions at the lower part of the scale are crowded so that poor precision is obtained. The divisions at the middle and upper portions of the scale, however, are usually such that they may be read with precision.

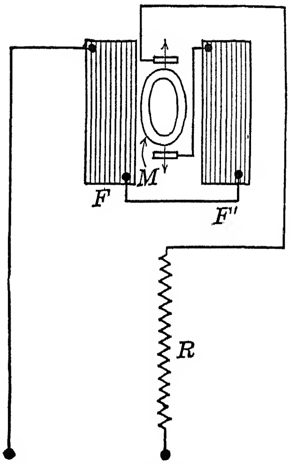


FIG. 79.—Diagram of dynamometer voltmeter.

This dynamometer type of voltmeter takes about five times as much current as a direct-current voltmeter of the same rating and consumes an appreciable amount of power. As the moving coil operates in a comparatively weak field, this type of instrument is very susceptible to stray fields. Unless the instrument is shielded, wires carrying currents, inductive apparatus, and even iron alone, if brought too near, may cause large errors in the indications of this type of voltmeter.

This instrument may be used for direct current as well as for alternating current. Reversed direct-current readings should be taken in order to eliminate the effect of the earth's field and of any stray fields. As the deflections depend on the *square* of the voltage, the instrument reads rms values.

67. Inclined-coil Voltmeters.—The inclined-coil type of voltmeter operates on the electrodynamometer principle. It differs from the type described in Sec. 66 only in the geometrical relations of its fixed and moving coils. The axis of the fixed coil, Fig. 80, is set at a considerable angle with the vertical. The axis of the moving coil makes a considerable angle with the spindle. This moving coil is connected in series with the fixed coil, the current being conducted to the moving coil through light springs. A resistance to limit the current is connected in series with the instrument.

When the pointer is at the zero position, there is a considerable angle between the axes of the fixed and moving coils. When current flows through the instrument, the moving coil tends to take such a

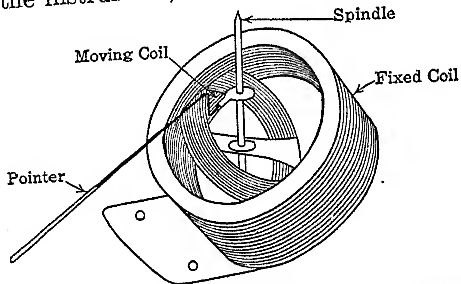


FIG. 80.—General Electric inclined-coil instrument.

position that its axis coincides with the axis of the fixed coil, so that their magnetic fields act in conjunction. In turning, the moving coil is opposed by flat spiral springs. The scale is calibrated in volts.

As this instrument is of the electrodynamometer type, it is adapted to both direct and alternating currents.

68. Dynamometer Ammeters.—Owing to the difficulty of conducting even moderately large currents into the moving coil, dynamometer ammeters of the portable type and of the switchboard type are not common. It is not a simple matter to use a shunt, since the division of current between the moving coil and the shunt depends on the respective impedances and the impedances depend on the frequency. Hence, in its simplest form, the instrument would be accurate at one frequency only and with irregular wave shapes would be in error since such waves contain currents of higher frequencies.

With the dynamometer-type ammeter, usually the entire current flows in the fixed coil, and only the movable coil is connected across the shunt. This reduces the voltage drop in the shunt. However, for most purposes the iron-vane type of instrument described in Secs. 74 and 75 is so much simpler and less expensive that the shunted type is little used.

69. Wattmeter.—Alternating-current power is equal to the product of the rms current and the rms voltage only when the power factor is unity. The ammeter and voltmeter method, therefore, as used with direct currents, seldom can be used to measure alternating-current power. Consequently, a *wattmeter* is necessary for measuring alternating-current power.

The wattmeter, Fig. 81, operates on the electrodynamometer principle. M is a moving coil wound with fine wire and is practically identical with the moving coil of the dynamometer voltmeter, Fig. 79. It is connected across the line in series with a high resistance R. The current is led into this coil through springs. The two fixed coils FF' are wound with a few turns of heavy wire, capable of carrying the load current. As there is no iron in the magnetic circuit, the field due to the current coils FF' is proportional to the load current at every instant. The current in the moving coil M is proportional to the voltage at every instant. For any given position of the moving coil, therefore, the torque is proportional at every instant to the product of the current and voltage or to the instantaneous power of the circuit. If the power factor is other than unity, there is negative torque for part of the cycle. That is, during the periods when there are negative loops in the power curve, Fig. 24 (p. 28), the current in the fixed coil and the current in the moving coil are in such directions as to produce negative torque. The torque varies from instant to instant and the torque-time curve is a double-frequency sine wave similar to the power wave, Fig. 24 (p. 28). Because of its relatively large moment of inertia, the moving-coil system assumes a deflection proportional to the average torque or *average power*. The torque is also a function of the angle between the fixed- and moving-coil axes, but this factor is taken into account by the scale calibration.

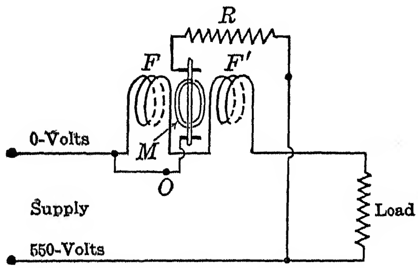


FIG. 81.—Connections for wattmeter.

Where intermediate accuracy is satisfactory, the recent improvements in magnetic materials have made it possible to employ laminated iron in the magnetic circuit of wattmeters, including a fixed cylindrical core within the moving coil. Such wattmeters, called *electrodynamic wattmeters*, can be made more compact than the noniron type, and the instrument losses are materially reduced.

When correctly adjusted, a wattmeter reads the product

$$E_w \cdot I_w \cdot \cos \alpha,$$

where E_w is the voltage across the potential circuit, I_w the current in the current coil, and α the angle between E_w and I_w.

It should be noted in Fig. 81 that the voltage terminal marked O is connected directly to one end of the moving coil. This terminal always should be connected directly to that side of the line to which the current coil is connected. The fixed and moving coils are then at the same potential. If the moving coil is connected to the other

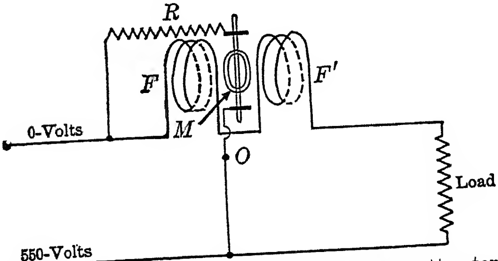


FIG. 82.—Incorrect method for connecting wattmeter.

side of the line, the potential difference between fixed and moving coils is equal to the full-line potential, Fig. 82. In this diagram, the fixed coils are considered as being at zero or ground potential. The moving coil is then at the potential of the other side of the line, or 550 volts, and this is the difference of potential that exists between fixed and moving coils. This is dangerous from the insulation standpoint, and electrostatic forces existing between the fixed and moving coils may cause an error in the instrument reading. (The wattmeter is described briefly in Vol. I, Chap. V.)

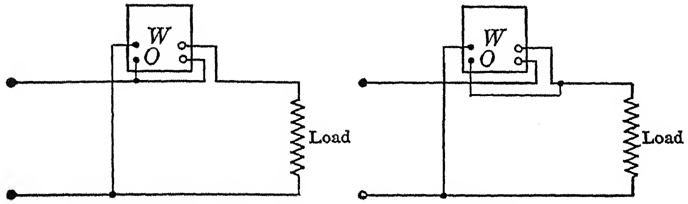
70. Wattmeter Connections.—In Fig. 83(a), wattmeter W is shown measuring the power taken by a load. In order to measure this power correctly, the wattmeter current coil should carry the *load* current, and the wattmeter voltage coil, in series with its resistance, should be connected directly across the *load*.

The current in the wattmeter current coil is the same as the load current; the wattmeter potential circuit is not connected directly across the load, however, but is measuring a potential in excess of the load potential by the amount of the impedance drop in the wattmeter current coil. The wattmeter reads too high, therefore, by the amount of power consumed in its own current coil. Under these conditions, the true power

$$P = P' - I^2R_c, \tag{107}$$

where P' is the power indicated by the wattmeter, I is the current in the wattmeter current coil, and R_c is the resistance of this coil. This loss ordinarily is of the magnitude of 1 to 3 watts at the rated current of the instrument and often may be neglected.

If the wattmeter be connected as in Fig. 83(*b*), the wattmeter potential circuit is connected directly across the load, but the wattmeter current coil carries the potential-coil current in addition to the load current. In fact, the wattmeter potential circuit may be



(*a*) (*b*)

FIG. 83.—Methods for connecting wattmeter.

considered as a small load in parallel with the actual load whose power is to be measured. The power consumed by this potential circuit must be deducted, therefore, from the wattmeter reading. The true power taken by the load is

$$P = P' - \frac{E^2}{R_p}, \tag{108}$$

where P' is the wattmeter reading, E the load voltage, and R_p the resistance of the wattmeter potential-coil circuit.

An idea of the magnitude of this correction may be obtained from the following example.

Example.—A wattmeter indicates 157 watts when connected as shown in Fig. 83(*b*). The line voltage is 120 volts, and the resistance of the wattmeter potential circuit is 2,000 ohms. What power is taken by the load?

$$P = 157 - \frac{120^2}{2,000} = 157 - 7.2 = 149.8 \text{ watts.}$$

It will be noted that a considerable percentage error would result in this case if the wattmeter loss were neglected.

When correction for instrument loss is necessary, the connection of Fig. 83(*b*) is preferable since the voltage, hence the potential-circuit loss, is usually constant. Also, the resistance of the potential circuit usually is given with the instrument.

Compensated wattmeters have a small auxiliary coil, of the same number of turns as the fixed coils and interwound with them, connected in series with the potential circuit, opposing the mmf of the

fixed coils. This auxiliary coil produces a small countertorque that compensates for the power loss in the potential circuit.

71. Wattmeter Ratings.—The current and potential circuits of a wattmeter must each have a rating corresponding to the current and voltage of the circuit to which the wattmeter is connected. A wattmeter is rated in amperes and volts rather than in watts, because the indicated watts show neither the amperes in the current coil nor the voltage across the potential circuit.

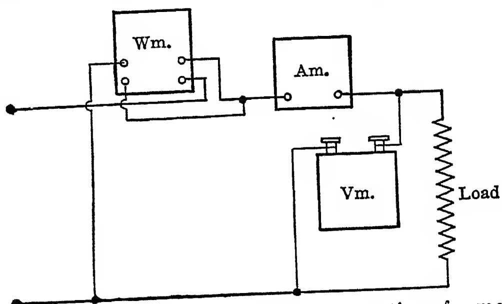


FIG. 84.—Wattmeter, ammeter, and voltmeter connections for measuring power.

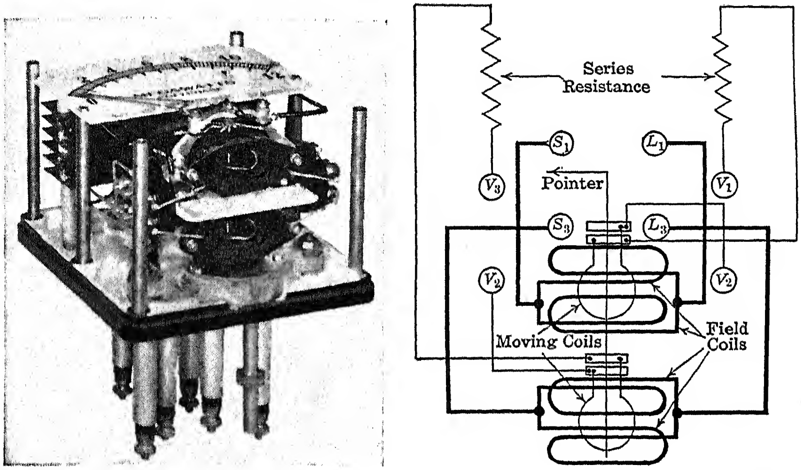
If the current in an ammeter or the voltage across a voltmeter exceeds the rating of the instrument, the pointer goes off scale and so warns the user. A wattmeter may be overloaded considerably and yet the load power factor be so low that the needle is well on the scale. For this reason, a voltmeter and an ammeter should be used ordinarily in conjunction with a wattmeter, so as to determine if either voltage or current exceeds the wattmeter rating.

Corrections for the power taken by ammeters and voltmeters often are necessary. For example, in Fig. 84, the I^2R-loss of the ammeter and the E^2/R-loss of the voltmeter must be deducted from the wattmeter reading, in addition to the wattmeter potential-circuit loss. The ammeter reads too high by the current taken by the voltmeter. This voltmeter current must be subtracted *vectorially* from the ammeter reading in order to obtain the true load current.

72. Polyphase Wattmeter.—Ordinarily, it requires two or more wattmeters to measure the total power of a two- or a three-phase circuit (Chap. V). If the load fluctuates, it is difficult to obtain accurate simultaneous readings of two wattmeters. At power factors less than 0.5, in a three-phase circuit, one of the wattmeters reverses its reading (Sec. 96, p. 139). This necessitates reversing the connections of one of the wattmeters, which is often inconvenient. If both wattmeters be combined in one, that is, if both moving coils be mounted on the same spindle, the turning moments for the two ele-

ments add or subtract automatically, and the total power is read on a single scale.

Figure 85(*a*) shows the construction of a Weston switchboard-type polyphase wattmeter in which the two elements are clearly shown, and Fig. 85(*b*) gives the interior connections. Figure 86 shows one method for connecting a General Electric polyphase wattmeter in



(*a*) Internal mechanism. (*b*) Internal connection, front view.
FIG. 85.—Weston switchboard-type polyphase wattmeter.

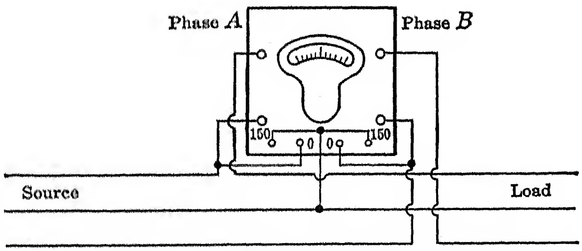


FIG. 86.—Connections for General Electric polyphase wattmeter on three-phase circuit.

a 3-wire 3-phase circuit. Note the symmetry of the connections. The two outer lines from the source connect to the two front current binding posts. Each of the two potential binding posts o, o connects to the source side of its current line. With this connection the wattmeter measures only the power consumed in its current coils.

Although it is often more convenient to use a polyphase wattmeter, two single instruments are better adapted to precision work since it is a simple matter to apply individual scale corrections. Each element of a polyphase wattmeter must be carefully shielded so that there is no mutually inductive action of one element on the other.

73. Wattmeter Calibration.—A dynamometer wattmeter ordinarily is calibrated with direct current, the connections for calibration being shown in Fig. 87. The voltage across the potential circuit is measured with a standard direct-current voltmeter. The current is measured accurately by means of a potentiometer, although a standardized direct-current ammeter is often sufficiently accurate.

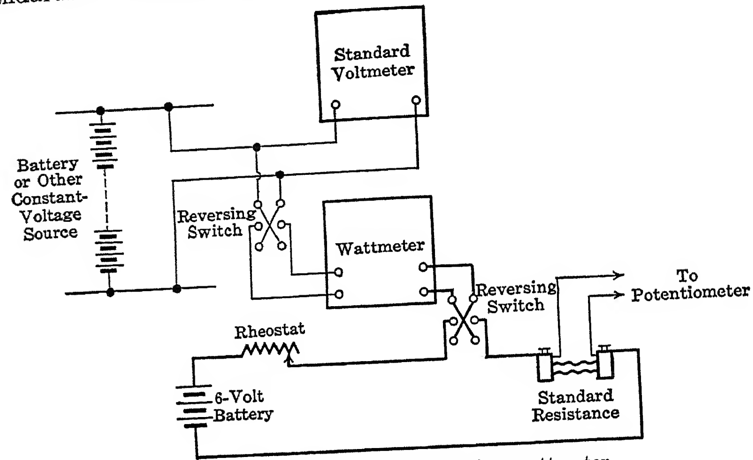


FIG. 87.—Connections for calibrating wattmeter.

Both current and potential are reversed at each reading to eliminate the effect of the earth's field or of any stray field. [One position of the switches must give the connection of Fig. 82, so that the calibration voltage should not be high.] The true power in watts is given by the product of current and voltage, since direct current is used.

IRON-VANE INSTRUMENTS

74. Voltmeters.—In Vol. I (Chap. V), it is pointed out that d-c instruments depending on the solenoid action of an iron plunger are not satisfactory as ammeters. By the use of light iron vanes, jeweled bearings, etc., satisfactory types of commercial alternating-current instruments, based on the principle of magnetized iron, have been developed.

One such type, manufactured by the Weston Electrical Instrument Company, is shown in Fig. 88.

A small strip of soft iron M, bent into cylindrical form, is mounted axially on a spindle, which is free to turn. Another similar strip F, more or less wedge-shaped and with a larger radius than M, is fixed within a cylindrical coil. The cylindrical coil is wound with fine wire and is in series with a high resistance. When connected across the

line, the current through the instrument is proportional to the circuit voltage. When current flows through this exciting coil, both iron vanes become magnetized. The upper edges of the two strips will always have the same magnetic polarity; and the lower edges will always have the same magnetic polarity, but when the upper edges are *n*-poles, as shown, the lower edges are *s*-poles. There will always be a repulsion, therefore, between the two upper edges and also between the two lower edges of the iron strips. This repulsion tends to rotate the spindle against the action of two springs. A pointer mounted on the spindle moves over a graduated scale and indicates the voltage.

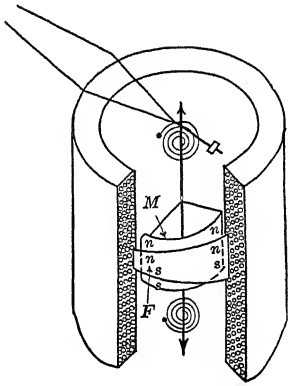


Fig. 88.—Weston iron-vane type instrument.

This type of instrument can be used for direct current with a precision of 1 or 2 per cent. Its obvious advantages are its simplicity, its low cost, and the fact that there is no current conducted to the moving element. When carefully calibrated, a precision of 0.5 per cent and better can be obtained with alternating current. This type of instrument cannot be calibrated with a high degree of precision with direct current on account of the effect of hysteresis on the vanes. It should be calibrated by comparison with an alternating-current standard. Air damping is obtained by the use of a light aluminum vane moving in a restricted space.

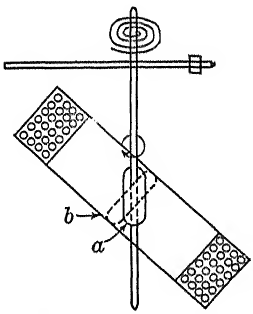


Fig. 89.—Inclined coil, iron-vane type of instrument.

The iron-vane principle has been applied to the inclined-coil type of instrument. A small iron vane, mounted obliquely on the spindle, Fig. 89, replaces the inclined moving coil of Fig. 80 (p. 96). When the pointer is at zero, this vane lies at an angle to the coil axis, as at *a*. When current flows in the coil, the vane attempts to take such a position that the direction of its axis shall coincide with that of the magnetic field, which acts along the coil axis. This position is shown at *b*. The vane, in seeking this position, turns the spindle that carries the pointer. The turning moment is opposed by springs. Iron laminations that surround the coil shield the instrument from stray magnetic fields. Magnetic damping produced by a light aluminum vane moving between the poles of permanent magnets is employed.

75. Ammeters.—Owing to the difficulty of conducting any except the smallest currents into the moving system of dynamometer instruments, iron-vane ammeters are practically the only type used for commercial instruments. The Weston iron-vane ammeter operates on the same principle as the iron-vane voltmeter (Sec. 74). The magnetizing coil in the ammeter is wound with a few turns of larger size wire rather than with the large number of turns of fine wire used in the voltmeter.

The General Electric Company's inclined-coil ammeter is of the same construction as the voltmeter, except that the coil is wound with larger size wire rather than with fine wire, Fig. 89.

76. Thermocouple Instruments.—Alternating currents are also measured by means of the thermocouple principle. It is well known that if the junction of two wires of unlike metals (such as iron and a copper-nickel alloy) be heated and the free, or "cold," ends are connected to a millivoltmeter, an emf results (Seebeck effect). In thermocouple instruments the heater is a wire of resistor alloy connected between two metal blocks, Fig. 90(a), and through which the current to be measured flows. The heater wire has practically zero temperature coefficient of resistance, and the instrument scale can be calibrated in terms of the current in the heater circuit. The thermal emf is proportional to the difference of temperature between the hot junction and the points at which the thermoleads are connected to the copper leads. Temperature changes, due to ambient conditions, which affect the cold ends to a different degree from the hot junction, introduce error. To avoid such errors, it is common practice to terminate the resistor wire or heater in rather massive metal blocks and then bring the points at which the thermo-leads connect with the copper leads into good thermal contact with the blocks. Usually this is accomplished by connecting the leads to thin copper plates, Fig. 90(a), and separating the plates from the blocks by thin mica. This is called *cold-junction compensation.*

The millivoltmeters used with thermocouples must be necessarily much more sensitive than those used with shunts, for the output voltage of the thermocouple may be only 15 mv and the internal resistance 5 ohms. Accordingly, thermocouple instruments are delicate and should be handled carefully. (With ordinary shunts the full-scale voltage drop is 50 mv.)

Since the heater temperature increases as the square of the current (I^2R), if the usual d-c millivoltmeter with uniform scale were used the scale would follow a square law, which is undesirable for many applications. To correct for this, the air gap is increased so that the

flux density is diminished as the coil moves toward the upscale position, Fig. 90(*b*). This diminishes the sensitivity as the pointer moves upscale, and thus a nearly linear scale is obtained.

Since the operation of the instrument is based on the heating effect of the current (I^2R), its indications are proportional to rms values. By using a small heater wire and a high resistance in series, the instrument can be used as a voltmeter. This type of instrument is used commonly for radio-frequency measurements; and since its indications are based on the heating effect of the current, it can be calibrated with direct current.

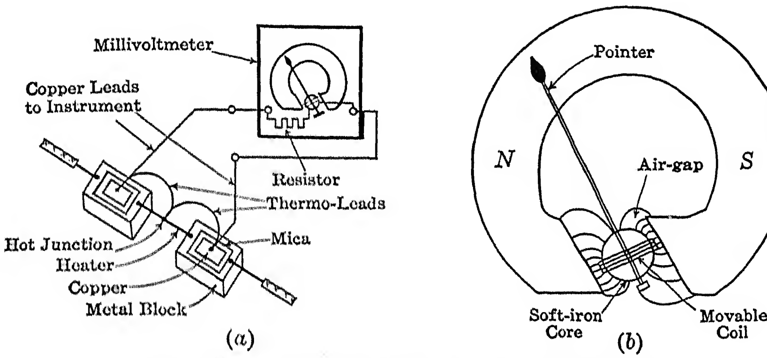
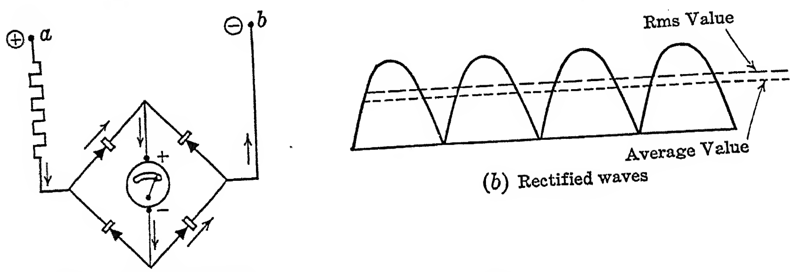


FIG. 90.—Thermocouple instrument. (*General Electric Co.*)

77. Rectifier-type Instruments.—Another method of using a d-c instrument of the permanent-magnet type for measuring alternating current is to employ a rectifier. The indications of the d-c instrument will be proportional to the *average* value of the rectified wave, Fig. 91(*b*) (see Sec. 7, p. 13). With a sine wave the ratio of rms value to average value, or the form factor, is 1.11 so that for a sine wave under these conditions the instrument indications have a definite rms value. The copper-oxide or selenium type of rectifier is employed ordinarily, and the bridge connection, Fig. 91(*a*), is used since it gives full-wave rectification, Fig. 91(*b*) (see Sec. 336, p. 555). In Fig. 91(*a*), the arrows show the direction of current when terminal *a* is positive. It is clear that when terminal *b* is positive the directive action of the rectifiers still causes the current to enter the positive terminal of the instrument.

In using the instrument, it must be remembered that except for sine waves the form factor is usually other than 1.11 so that the instrument may be considerably in error with nonsinusoidal waves. Also, some aging of rectifiers occurs in service, and the instrument is affected by temperatures of 50°C or greater. Hence. with such instruments

the stated accuracy is given usually as ± 5 per cent of full scale. However, because of their sensitivity and uniform scale, they are widely used. The circuit to the instrument never should be opened when the rectifier is connected to the line, since this causes full voltage to be impressed across the rectifier plates, which may damage them. The instrument may be used with a shunt to measure current or in series with a high resistance to measure voltage. Rectifier instruments are widely used for radio-frequency measurements.



(a) Bridge rectifier circuit (b) Rectified waves

FIG. 91.—Rectifier-type a-c instrument.

78. Alternating-current Watt-hour Meter.—The direct-current watt-hour meter can be used with alternating current, as the reversal of line voltage reverses both its armature and field current simultaneously and the direction of the torque remains unchanged. At low power factors, however, considerable error may be introduced by the inductance of the armature circuit. This causes the armature current to lag the line voltage by a small angle; and although this has negligible effect at or near unity power factor, the error at low power factor is quite pronounced. This error may be compensated by shunting the current coils of the meter with a low noninductive resistance.

The induction watt-hour meter is so much cheaper and so superior to the direct-current type that there is little necessity for using the direct-current type on alternating-current circuits.

A rear view of a typical induction meter is shown in Fig. 92. P is a potential coil that is highly inductive and is placed on one lug of the laminated magnetic circuit, this lug being over the aluminum disk D. CC are two series or current coils placed on two projecting lugs beneath the disk. These coils are so wound that, if one tends to send flux upward, the other tends to send it downward. A small auxiliary or compensating winding cw is placed on the potential lug, and its ends are connected to the resistance R. In order that the meter may register correctly, the potential-coil flux must lag the line voltage by

90°. As it is impossible to make the resistance of the potential coil zero, its current will lag by an angle less than 90°. At low power factors, this introduces considerable error in the meter registration. By properly adjusting the resistance R, however, the potential-coil *flux* may be brought into the 90° relation, and the meter will register substantially correctly at all power factors. To adjust the compensation, the meter is made correct at unity power factor, and then the power factor is dropped to some low value, as 0.5. If the registration

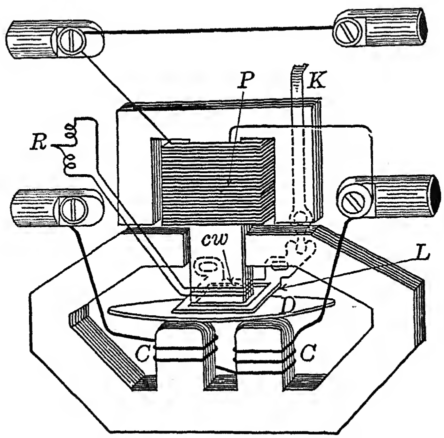


FIG. 92.—Diagram of induction watt-hour meter.

is now in error, this is due to improper compensation. The meter is again made to register correctly by changing the resistance R, the two small wires of this resistance being either twisted or untwisted and then soldered. If the meter underregisters when the load current lags, the resistance R should be *decreased;* if the meter overregisters with lagging current, the resistance R should be *increased.* The reverse is true with leading current.

L is a small metallic stamping placed under the potential lug and can be moved laterally by means of the lever K. Its function is to provide the small torque necessary to compensate the friction of the meter. The operation of this adjustment is as follows: Figure 93 shows the stamping under the lug, set off center. When the flux begins to increase downwardly through the lug, a current is induced in the short-circuited stamping. This current, by Lenz's law, opposes the flux entering the stamping, so that during this period the flux is crowded to the left-hand side of the lug, as shown. When the flux begins to decrease, the current in the short-circuited stamping tends to oppose the decrease in the flux. This retards the time phase of the

flux in the right-hand side of the lug with respect to that in the left-hand side of the lug. The result is a sweeping of the flux from left to right across the lug. This sliding flux cuts the disk and causes eddy currents to be induced in it. These currents, reacting with the flux, produce a torque tending to drive the disk in the direction in which the stamping is displaced from its position of symmetry. This is the "shaded-pole" principle, which is also used to start small single-phase induction motors (see Sec. 212, p. 379).

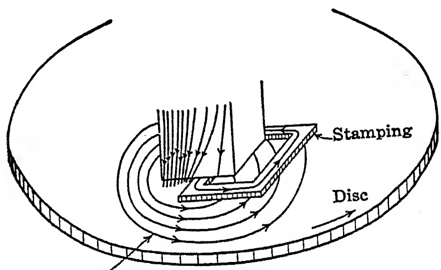


FIG. 93.—Shaded-pole principle of light-load adjustment.

The meter operates on the induction-motor principle (p. 305) although the field "glides" linearly along the air gap rather than having a circular direction about a cylindrical armature. The production of the driving torque at unity power factor is illustrated in Fig. 94. The line voltage E, the instantaneous values of which are shown in (a), is impressed across the potential circuit of the watt-hour meter. If the meter is properly lagged, the potential flux ϕ_p lags E by 90°. The current wave I is in phase with the voltage wave E; also, the flux ϕ_i, due to the current coil, is in phase with I. In (b) is shown the magnetic polarities of the meter poles for the various times indicated in (a). At 1, the current is zero so that no flux is produced by the current coils. The potential-coil flux is a negative maximum so that the potential pole is S. The two current lugs, therefore, must be N-poles. At 2, the potential-coil flux is zero, but the current is a maximum. Therefore, the lower poles will be S and N as shown, and the potential lug will have an N on one side and an S on the other. At 3, the upper lug is N, and the two lower ones S. Times 4 and 5 also are shown, 5 corresponding to 1.

In (1), the entire upper lug is an S-pole. In (2), this S-pole has diminished in magnitude and has moved toward the right-hand side of the lug, and an N-pole appears on the left-hand side of this lug. In (3), an N-pole occupies the entire upper lug; in (4), this has diminished and moved toward the right side of the lug.

A similar cycle takes place on the two lower lugs. In (1), both lugs are N-poles, making one large N-pole. In (2), this large N-pole has diminished and moved toward the right, being followed by an S-pole appearing on the left. In (3), the N-pole has disappeared on the lower lug, both lugs becoming S-poles, etc. By following the

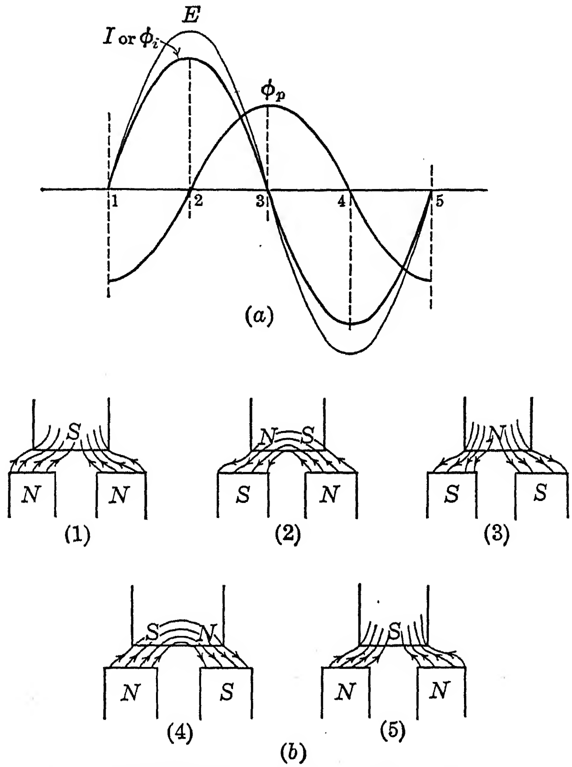


Fig. 94.—Gliding field in air gap of induction watt-hour meter.

cycle, it will be observed that an N-pole moves from left to right on both the upper and the lower lugs. Similarly, an S-pole does likewise, following the N-pole. The field, therefore, "glides" laterally through the gap. In so doing, it cuts the disk and induces eddy currents therein. These eddy currents react with the gliding field, and by Lenz's law the disk tends to follow the field (Sec. 182, p. 307).

If the power factor be zero, ϕ_i, Fig. 94(a), either will be in time phase with ϕ_p if the current lags or will be 180° out of phase with ϕ_p if the current leads. In either case, if instantaneous values of flux be taken, Fig. 94(b), it will be found that there is no lateral displacement of the field in the gap but merely a sinusoidal pulsation of flux in the

gap. Under these conditions, the torque acting on the disk is zero. Just as in the d-c meter, the retarding torque is produced by the disk's cutting a field of constant strength produced by *permanent* magnets. This causes a *retarding* torque that is proportional to the angular velocity of the disk. Both the *driving* torque (motor action) and the *retarding* torque (generator action) are produced on the same disk.

79. Calibration of the Induction Watt-hour Meter.—The induction watt-hour meter is calibrated in much the same manner as the

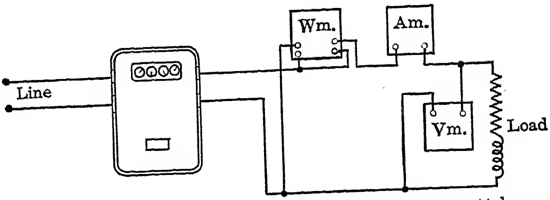


FIG. 95.—Connections for testing alternating-current watt-hour meter.

d-c watt-hour meter. A standard indicating wattmeter is used to measure the average power over a stated interval, and the revolutions of the disk of the watt-hour meter are counted with the aid of a stop watch. The average meter watts are calculated by means of the equation

$$W = \frac{K \cdot N \cdot 3{,}600}{t}, \tag{109}$$

where K is the meter constant, N the revolutions of the disk, and t the time in seconds.

The connections for making the test are shown in Fig. 95.[1] As a rule, an ammeter and a voltmeter are used with such a test in order to determine the power factor. Instrument losses should be carefully investigated and corrections made if necessary.

After the meter is adjusted at full load and unity power factor by means of the retarding magnets, it is adjusted at light load by means of the light-load adjustment. The power factor is lowered. Any error occurring now must be due to improper lagging. The registration then is made correct by adjusting the resistance R, Fig. 92, which is in series with the lagging coil. If the meter registers low with lagging current, the resistance R should be decreased; if it registers high, the

[1] Most laboratories are provided with phase shifters for changing the phase angle between voltage and current so that any desired power factor may be obtained. (See F. A. LAWS, "Electrical Measurements.") Where several meters are calibrated simultaneously, as by manufacturers and utilities, one method is to compare the angular velocities of the disks of the meters under test with the angular velocity of the disk of a rotating standard. Stroboscopic methods also are employed.

resistance R should be increased. With leading current these operations should be reversed.

The induction watt-hour meter has advantages over the d-c meter. As there is no coil-wound armature in addition to the disk, the rotating element of the induction meter is much lighter than that of the d-c meter. It has, moreover, no commutator or delicate brushes, which increase friction and are frequent sources of trouble with the d-c meter.

The induction meter also is made in the polyphase type. Two single-phase elements act on a common spindle. There are two sets of damping magnets.[1]

80. Frequency Indicators.—Some types of frequency indicators are based on the effect of frequency upon the current in electric cir-

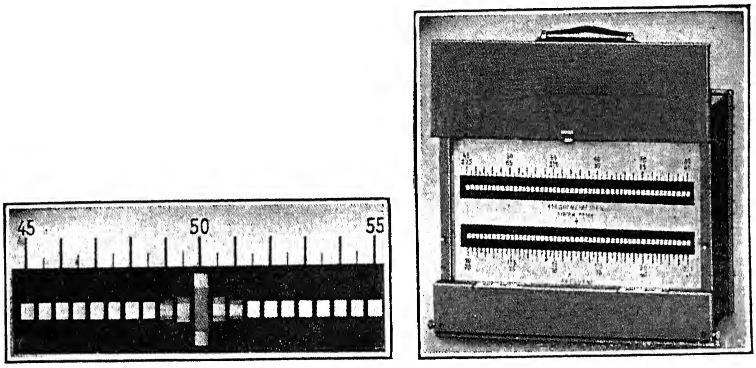


FIG. 96.—Frahm vibrating-reed frequency meter.

cuits. The moving element is actuated by the joint effect of the currents in two shunt circuits, one containing inductance and the other either resistance or capacitance. A change of frequency produces opposite effects on phase and magnitude of the currents in the two circuits and causes the moving element to deflect. Hence the instrument scale may be calibrated in terms of frequency.

In another type, current is supplied to the moving element through three circuits having different resonant frequencies, such as 72, 58, and 36 cycles for a 60-cycle instrument. As the frequency changes, the current in the moving element changes in both phase and magnitude, which causes the element to change its position, and the scale can be graduated in cycles.[1]

A simpler but less precise type of frequency indicator is based on the principle of mechanical resonance. A number of steel reeds, each having a white index on its end, are clamped between two metal strips.

[1] For a more detailed analysis, see F. A. LAWS, "Electrical Measurements," or "Standard Handbook," Sec. 3.

The mechanical frequency of vibration of each reed is different. Behind this bank of reeds there is an electromagnet, the coil of which is excited by the circuit whose frequency it is desired to measure. The reed whose natural frequency is the same as the frequency of the circuit will vibrate with the greatest amplitude, Fig. 96. With the exception of one or two reeds whose natural frequency is near this value, none of the others will be affected. The frequency is determined, therefore, by noting the scale reading opposite the reed that vibrates with the greatest amplitude, Fig. 96. Were the reeds unpolarized, they would be attracted equally well by either a north or a south pole. An adjacent permanent magnet keeps the reeds polarized, so that the reed of a particular mechanical frequency will respond to the

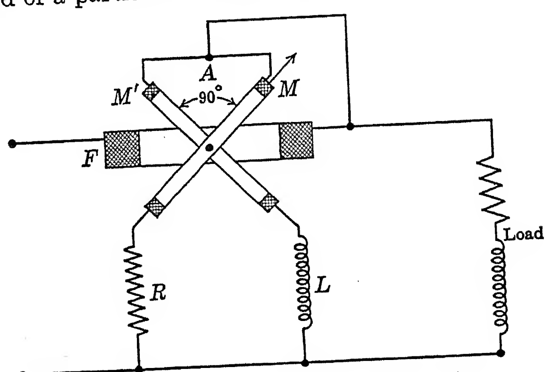


FIG. 97.—Principle of Tuma phase meter.

same electrical frequency. The reeds are usually arranged so that there is a reed for every half-cycle.

81. Power-factor Indicators.—Power-factor indicators and synchroscopes are based on the principle of the Tuma phase meter. In Fig. 97, F is a fixed coil carrying the circuit current. MM' are two flat coils wound with fine wire; they are fastened together rigidly and mounted on a spindle free to rotate. There is no mechanical control whatever of this moving element. The angle between the coils is 90°, or nearly so. The windings of the two coils MM' are connected together at the common point A, and A is connected to the same side of the circuit as F. A noninductive resistance R is connected between M and the other side of the line. A high inductance L is connected between M' and the side of the line opposite A. Assume for the moment that the currents in M and M' differ by 90° in time phase. Also assume that the power factor of the load is unity. Under the assumed conditions, the current in coil M' lags the line voltage by 90°, hence lags the flux due to coil F by 90°, and therefore exerts no

torque. The current in coil M is in time phase with the line voltage and hence with the flux due to coil F; coil M, therefore, will move into the plane of coil F, as there is no restraining torque. Hence, at unity power factor, the entire moving element takes such a position that the coil M is in the plane of coil F.

If the power factor of the load is zero, the current and the voltage differ in phase by 90°. Hence, the current in coil M and the flux due to coil F have a time-phase difference of 90°, and coil M exerts no turning moment. The current in coil M', however, is now in time phase with the flux due to coil F, and, therefore, coil M' will move

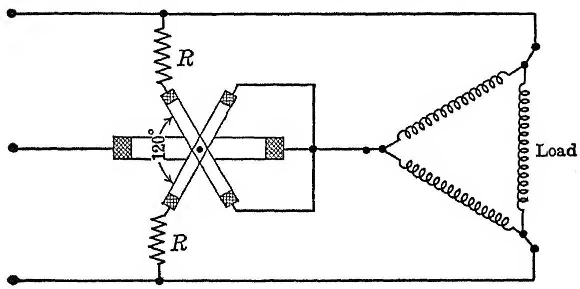


FIG. 98.—Three-phase power-factor indicator.

into the plane of coil F. The moving system then will have a position 90° from its position at unity power factor. That is, when the current changes its time phase by 90°, the moving element of the indicator changes its space position by 90°. The direction in which the element turns depends on whether the current lags or leads the voltage. For intermediate power factors, it can be shown that the angle of the moving system corresponds to the circuit power-factor angle. If the scale is calibrated in degrees, the pointer can be made to indicate the power-factor *angle* of the circuit. To make the indicator read power factor, it is necessary merely to make the scale divisions proportional to the cosine of the power-factor angle. In practice, the current is led into the moving system through strips of annealed silver foil, which exert no appreciable control on the moving system.

As it is impossible to obtain either a pure resistance or a pure inductance, the currents in coils M and M' will not differ by exactly 90° in time phase. It can be shown that, if the space angle between coils M and M' be made equal to the angle of phase difference of their currents, the instrument indicates correctly.

If the angle between the two coils MM' be made 120°, Fig. 98, the instrument can be made to indicate 3-phase power factor, *if the system is balanced.* A noninductive resistance R now is connected in series with each of the moving coils. The fixed coil is connected in one line

of the 3-phase system, and the common terminal of the two moving coils connects to this same line. The other terminal of each of the moving coils connects to one of the other two lines of the 3-phase system, Fig. 98. This is the scheme of connections for the power-factor indicator of the General Electric Company, so often seen on switchboards. The instrument indicates the 3-phase power factor if the system is very nearly balanced. If the system is unbalanced, the reading has little if any significance.

82. Synchroscope.—Before connecting an alternator to the bus bars and in parallel with other alternators, it is necessary not only that its

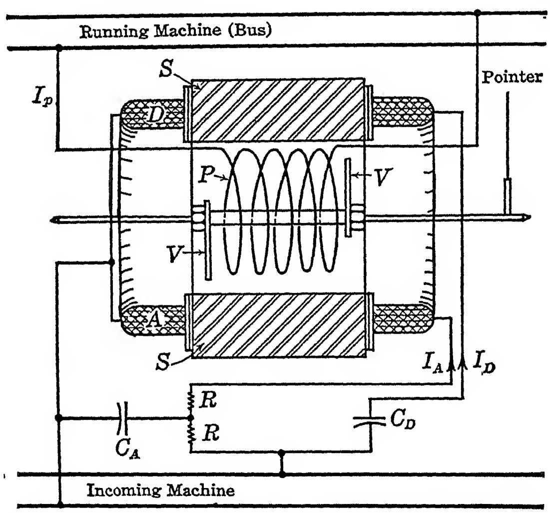


FIG. 99.—Cross section of General Electric synchroscope.

voltage be the same as that of the bus bars but that it be in phase opposition as well. This corresponds to having d-c generators of the same polarity before connecting them in parallel.

A synchroscope is an instrument for indicating when machines are in the proper phase relation for connecting in parallel and at the same time for showing whether the incoming machine is running fast or slow. This type of instrument is based on the principle of the power-factor indicator (Sec. 81).

A diagram of the General Electric type of synchroscope is shown in Fig. 99. A and D show the cross section of the 2-phase windings of a small cylindrical stator similar to an induction-motor stator. A cross section of the iron core is shown crosshatched as S. There are two distributed windings, displaced 90° from each other (see Fig. 265, p. 308). One of these windings is energized by current I_A and the other by I_D, which are displaced 90° in time-phase by the resistance-ca-

pacitance phase-splitting network R, R, C_A, C_D. Thus a true rotating field is produced within the stator. The moving element consists of two light nickel-iron vanes V, V, connected by a sleeve of the same material and mounted on a horizontal spindle with no mechanical control. The vanes point in opposite directions. The moving system is excited from the running machine by a coil P concentric with the sleeve. Thus, when the end of one vane is an N-pole, the end of the other is an S-pole.

Assume that the field produced by the winding energized by I_A is parallel to the paper and that energized by I_D is perpendicular to the paper. Also assume that the current I_P in coil P is in phase with the voltage of the running machine and that I_A is in phase with the voltage of the incoming machine. When the emf of the incoming machine is in phase with that of the running machine, the field due to I_A will be in time-phase with I_P and the vanes V, V will assume a position parallel to the paper as shown and the pointer can be adjusted to the position over the index, Fig. 100. The field due to I_D will have no effect under these conditions since the flux is displaced 90° in time-phase from the current I_P. On the other hand, if the phase of the emf of the incoming machine differs by 90° in time-phase from that of the running machine, current I_D will be in phase with I_P and the vanes V, V, and pointer will assume a position 90° to that shown in Figs. 99 and 100. Thus the pointer indicates the phase angle between the incoming and running machines. If there is a difference between the frequencies of the incoming and running machines, the pointer rotates at a speed which is equal to this difference, the direction of rotation showing whether the incoming machine is "fast" or "slow." The generator switch usually is thrown when the pointer is rotating slowly in the "fast" direction and is approaching the index. In Fig. 100 is shown the exterior view of the assembled synchroscope.

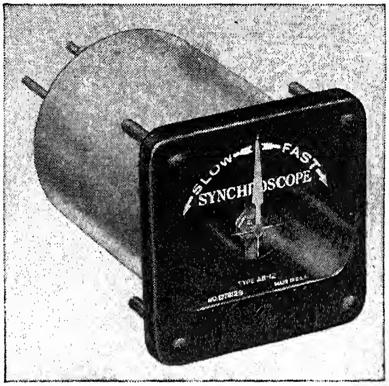


FIG. 100.—Exterior view, General Electric synchroscope.

83. Electromagnetic Oscilloscope and Oscillograph.[1]—It is often desired to investigate transient conditions in electric circuits, such,

[1] The term "oscillograph" is used when a photographic or other record of a varying electrical quantity is made. The term "oscilloscope" is used when the instrument only makes visible the varying electrical quantity.

for example, as the current and voltage relations during the blowing of a fuse or during the short circuit of an alternator or in oscillations produced by switching, etc. Further, it is desirable to have apparatus that will show the current and voltage waves in alternating-current circuits during steady conditions. The oscillograph is an instrument that is capable of meeting these requirements.

Its principle is quite simple, being that of a D'Arsonval galvanometer (Vol. I, Chap. V), Fig. 101. A small phosphor-bronze or silver-alloy strip, or filament, is stretched over two clefts CC, around a small pulley P and back again. The spring S acting on the pulley keeps the two lengths of the strip in tension. This strip is placed between the poles of a strong permanent magnet or an electromagnet. When a

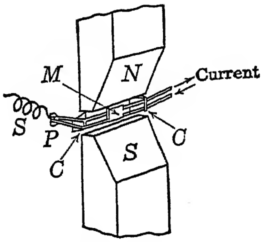


FIG. 101.—Vibrating element of oscillograph.

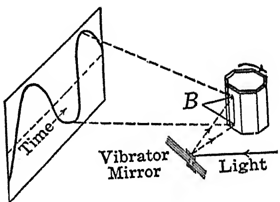


FIG. 102.—Method of drawing out vibrating beam into wave.

current flows through the filament, one length of the filament moves outward and the other inward. A very small mirror M is cemented across the two lengths of the filament and is given a rocking motion by the movement of the filament. If a beam of light be reflected from the mirror, it will be drawn out into a straight line by the mirror vibration. If the beam of light be made to strike a rotating mirror, in the manner shown in Fig. 102, the rotation of the mirror introduces a time element and the wave is drawn out so that its characteristics are shown.

The instrument is merely a galvanometer having a single turn and a very light moving element whose moment of inertia is very small. Also, the filament is under considerable tension so that its natural frequency of vibration is high, being from 3,000 to 10,000 cycles per sec. These characteristics are necessary in order that the filament may respond accurately to the comparatively high frequency variations which it is called upon to follow. The moving element is usually immersed in oil so that its movement is properly damped and the filament is kept cool.

Figure 103 shows the general arrangement of a typical oscilloscope or

oscillograph. The light from the filament of an automobile headlight first passes through two spherical focusing lenses and then strikes the two total-reflecting prisms. These prisms deflect the beams at right angles and direct them through the slits to the vibrator mirrors. The slits are adjustable and serve to reduce the section of the beam so that a fine line is obtained when the wave is traced or photographed, Fig. 104. The mirrors reflect the light back to the rotating mirror, which in turn reflects it, drawn out as a wave, through cylindrical lenses to the viewing screen and also to the camera if a photographic record is desired. The cylindrical lenses further concentrate the beam, but in one plane only. The rotating mirror is driven usually by a small syn-

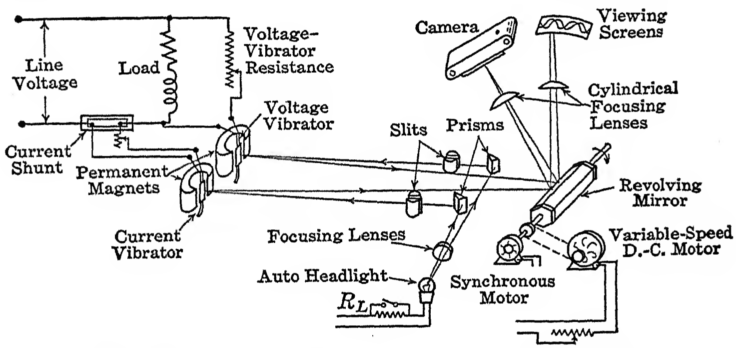


FIG. 103.—Optical system and connections of oscillograph.

chronous motor operated from the same circuit to which the vibrators are connected, so that the waves on the viewing screen remain stationary in position. A standard roll-film camera casing may be used for photographing, Fig. 103. It is necessary to provide a suitable attachment to hold it and a properly timed shutter for giving but a single exposure.

Another method of obtaining a photographic record is to wind the film about a cylindrical drum within a light-tight casing provided with a narrow transverse opening and a shutter. The casing and cylinder are located so that the light comes directly from the mirrors to the film without striking the rotating mirror. Were the film drum stationary, only a straight line would appear on the film, due to the deflections of the vibrators. When the drum is rotated, however, a time axis is provided by the motion of the film.

In some designs a resistance R_L in series with the automobile headlight is momentarily short-circuited by the shutter mechanism when a photograph is being taken, thus momentarily giving an intense beam.

In viewing and in photographing, speeds other than the fixed speed provided by the synchronous motor frequently are desired. To obtain

such flexibility of speed a variable-speed direct-current motor also is provided for driving the film drum and the mirror.

The oscillograph vibrators are connected into the circuit in the same manner as d-c ammeter and voltmeter coils are connected, Fig. 103. As the current vibrator can carry but a small current—about 0.1 amp—it is connected in parallel with a noninductive shunt that is in series with the line. The voltage vibrator is connected across the

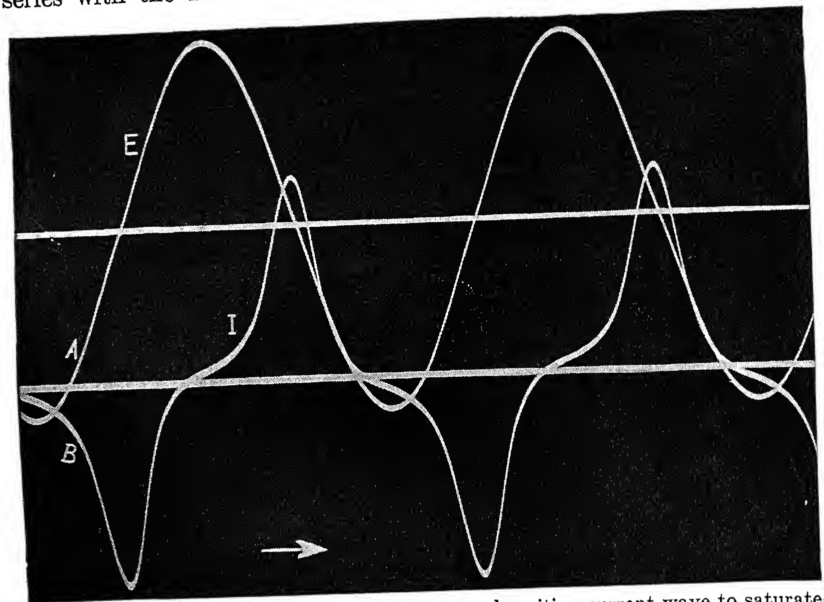


FIG. 104.[1]—Oscillogram showing voltage wave, and exciting-current wave to saturated transformer core. (*Courtesy of C. T. Weller, General Electric Co.*)

line in series with a high noninductive resistance. The current vibrator then will vibrate with an amplitude proportional to the circuit current and in phase with it. The current through the voltage vibrator will be proportional to the circuit voltage and in phase with it.

Figure 104 shows an oscillogram of a sinusoidal voltage wave E, applied to a saturated transformer core, and the resulting exciting current I. Note that the saturated core "peaks" the current wave, introducing harmonics (see p. 67 and footnote, p. 253).

84. Cathode-ray Oscilloscope.—Although the electromagnetic-type oscilloscope, or oscillograph, can respond accurately to frequencies as high as 5,000 cycles per sec, even the small moment of inertia of the vibrating system is too great for accurate response to high frequencies,

[1] From "Deviation Factor vs. Output of Sine-wave Generators" by C. T. Weller, *Gen. Elec. Rev.*, March, 1946, pp. 60–65.

particularly those in higher audio range and in the radio range. Also, the magnetic oscillograph has far too much inertia to follow ultra-high-speed transients such as lightning and lightning-generator discharges. In the cathode-ray oscilloscope, however, the deflected element is an electron beam, whose inertia is sensibly zero, so that the oscilloscope can respond accurately to transients that occur even in a fraction of a microsecond (millionth of a second).

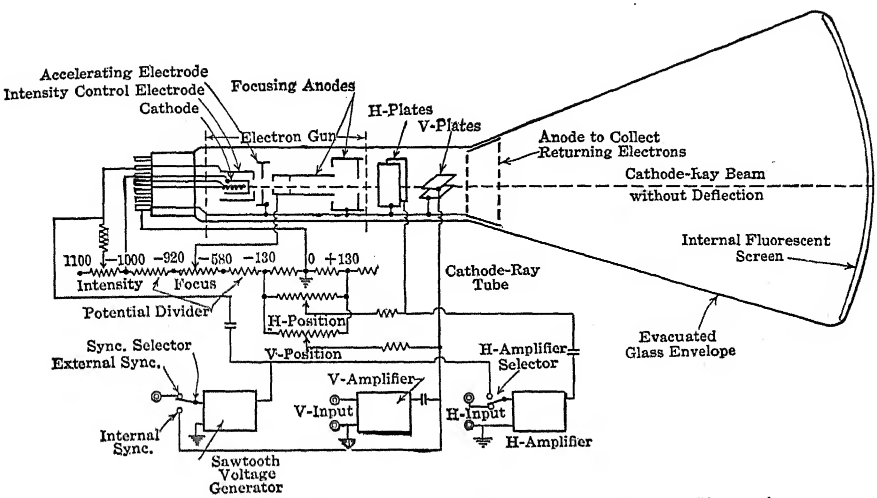


FIG. 105.—Basic components of representative cathode-ray oscillograph.

A block-and-circuit diagram of the oscilloscope is shown in Fig. 105. The left-hand end of the tube contains the "electron gun," one element of which consists of an indirectly heated tungsten cathode. The heated oxide coating on the end of the cathode sleeve emits electrons. These are accelerated and drawn into a thin beam by the electric field produced by the intensity-control electrode and the focusing electrodes. The beam converges and has a minimum cross section in the vicinity of the intensity-control electrode or grid. The beam then diverges until it passes through the focusing anodes. The electric field produced by these anodes causes the beam to converge so that it reaches the fluorescent screen "focused" in a small spot. The intensity-control electrode usually is operated at a potential of about 100 volts below or negative to that of the cathode. By varying this bias by means of the intensity control, the beam current and brightness of the spot on the screen may be regulated and even shut off entirely. By varying the potential of the first focusing anode with respect to that of the accelerating electrode, by means of the focus control, the spot may be properly focused on the screen. The

different potentials that are applied to the foregoing electrodes are obtained from a potential divider supplied with direct current from a rectifier and filter, not shown.

After the electron beam leaves the electron gun, it passes between two horizontal and between two vertical electrostatic-deflection plates designated as the H and the V plates. Also the symbols X and Y are used by some manufacturers to designate these plates and their associated control circuits and amplifiers. By impressing potential on these plates the beam may be deflected both horizontally and vertically. Since the electron beam consists of negative charges in motion, it deflects toward the positive plate. Usually, the wave whose amplitude is to be observed is impressed on the vertical deflecting plates, which cause the beam to be deflected in a vertical plane in proportion to the amplitude of the wave. The time axis is produced by a saw-tooth voltage generator, or "sweep circuit," which applies a practically uniformly increasing potential to the horizontal deflecting plates and so causes the beam to sweep across the tube, usually from left to right. The beam is quickly "snapped" back from the right position to the left. The return may be "trace-blanked" by applying a suitable potential to the intensity-control electrode at the instant of the return. The sweep may be internally synchronized with the a-c input to the vertical deflection plates or synchronized with any desired external signal by means of the synchronizing selector switch.

The beam may be positioned in both the horizontal and the vertical direction by means of the horizontal and vertical positioning controls, which adjust the magnitude of the d-c polarizing potentials applied to the horizontal and vertical plates. The "deflection factor" of the cathode-ray tube is about 20 to 200 d-c volts per in. Where the signal voltage is too low to give sufficient deflection of the beam on the screen, it may be amplified by means of wide-range amplifiers built into the oscilloscope as conveniently available auxiliary devices.

The d-c polarizing voltages necessary for the cathode-ray-tube electrodes and the plate voltages for the amplifiers and saw-tooth-voltage generator are supplied by electronic rectifiers also incorporated within the oscilloscope.

The screen material, which is coated on the inside wall at the right-hand end of the tube, fluoresces, usually green, when struck by the electron beam. The waves may be photographed, if desired. In order to prevent accumulation of charge on the inner wall of the tube, which would cause the beam to deflect erratically, a conducting coat is applied to the inner wall, and the coat is connected to the common ground.

Since the electron beam consists of electric charges in motion, and hence constitutes a current, it deflects in a magnetic field in accordance with Fleming's left-hand rule (Vol. I, Chap. XIII). Hence the beam may be focused or made to deflect in response to current waves by causing the current to flow in coils placed outside the tube and producing a magnetic field within the tube. Magnetic focusing and deflection are used primarily in television applications and other services requiring larger tubes with higher accelerating potentials.

If the saw-tooth-voltage generator, or sweep circuit, is disconnected from the horizontal-deflecting plates and two separate external a-c signals are impressed simultaneously on the vertical and horizontal deflection plates, the resulting figure on the screen, known as a Lissajous figure, may be analyzed to determine the frequency ratio of the two signals, their phase angle, and the ratio of their amplitudes. A "translating" device that is capable of converting the inherent voltage scales of the oscilloscope into current or flux or any other desired scale makes it possible to use the cathode-ray oscilloscope for such applications as the instantaneous determination of vacuum-tube characteristics, examination of hysteresis loops, and many other applications. The cathode-ray oscilloscope and also oscillograph have become important and versatile instruments for research and for observing the characteristics of both electrical and mechanical equipment.

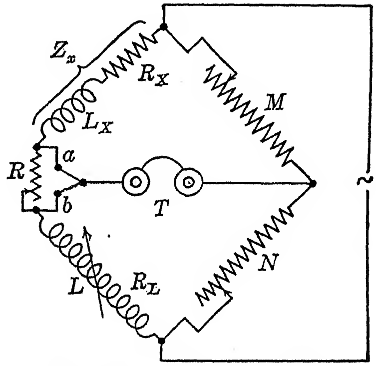


FIG. 106.—Impedance bridge.

85. Impedance Bridge.—Impedances may be measured with a bridge in the same manner that resistances are measured by direct current with the Wheatstone bridge (Vol. I, Chap. V). The usual connections are shown in Fig. 106. The unknown impedance Z_x, whose resistance is R_x and inductance L_x, forms one arm of the bridge. Two of the arms M and N are noninductive resistances. One arm, such as M, should be variable over a wide range. N may be adjustable to decimal values such as 1, 10, 100, etc., ohms. The fourth arm L of the bridge consists of a variable inductance standard, or variometer, L, whose resistance is R_L. The variable resistance R may be connected in either bridge arm L or Z_x by moving the detector contact to either a or b. If the frequency is in the sensitive audio range, from 200 to 2,500 cycles, headphones T may be used as a detector. If low fre-

quencies are used, such as from 20 to perhaps 200 cycles, a tuned vibration galvanometer may be used as a detector. If the impedances and resistances remain constant, the bridge balance will be independent of frequency.

If the bridge is in balance and the detector contact is at a,

$$\frac{L_x}{L} = \frac{M}{N} \quad \text{and} \quad \frac{R_x}{R + R_L} = \frac{M}{N}; \tag{110}$$

if the detector contact is at b,

$$\frac{L_x}{L} = \frac{M}{N} \text{ as before,} \quad \text{and} \quad \frac{R_x + R}{R_L} = \frac{M}{N}. \tag{111}$$

These equations show that the inductance balance is independent of any resistances in the Z_x, L-arms of the bridge. With the values of M and N that are necessary to balance the inductances, it may be impossible at the same time to balance the resistances. Hence, it is necessary to be able to connect R in either arm and adjust it for a balance.

It is not necessary that L be variable. A balance may be obtained with L a fixed standard by adjusting M, N, and R. The impedance Z_x may be a capacitive impedance $1/\omega C_x$, R_x where ω is 2π times the frequency and C_x is the unknown capacitance. A capacitance C in the arm L is then necessary for a balance (see Vol. I, Chap. V). When the bridge is in balance,

$$\frac{C_x}{C} = \frac{N}{M}. \tag{112}$$

Obviously, the positions of the alternating-current supply and the detector may be interchanged. There are many modifications of this bridge.[1]

[1] See F. A. Laws, "Electrical Measurements."

CHAPTER V

POLYPHASE SYSTEMS

86. Reasons for Use of Polyphase Systems.—In many applications of alternating current, there are objections to the use of single-phase power.

In a single-phase circuit, the power delivered is pulsating. Even when the current and voltage are in phase, the power is zero twice in each cycle, Fig. 22 (p. 24). When the power factor is less than unity, not only is the power zero four times in each cycle, but it is *negative* twice in each cycle. This means that the circuit returns power to the source for a part of the time. This is analogous to a single-cylinder gasoline engine in which the flywheel returns energy to the cylinder during the compression part of the cycle. Over the complete cycle, both the single-phase circuit and the flywheel receive an excess of energy over that which they return to the source. The pulsating nature of the power in single-phase circuits is objectionable for many applications.

A polyphase circuit is somewhat like a multicylinder gasoline engine. With the engine, the power delivered to the flywheel is practically steady, as one or more cylinders are firing when the others are compressing. This same condition exists in polyphase electrical systems. Although the power of any one phase may be negative at times, the *total power* is constant if the loads are balanced. This makes polyphase systems highly desirable, particularly for power loads.

The rating of a given motor, or generator, increases with the number of phases—an important consideration. Below are the approximate power ratings of a given machine for different numbers of phases, the single-phase rating being assumed as 100.

Single-phase	100
2-phase	140
3-phase	148
6-phase	148
Direct-current	154

The same machine operating 3-phase or 6-phase has about 50 per cent greater rating than when operating single-phase. A machine has the same rating whether connected 3-phase or 6-phase, because the same windings are used in the same manner for each. [The fore-

going table does not apply to synchronous converters. The ratio of polyphase to single-phase rating in converters is much greater than that shown in the table (see p. 437).]

Single-phase synchronous and induction motors, without auxiliary means, have no starting torque, whereas the starting torque of such motors when operating polyphase is substantial.

In single-phase synchronous machines, a pulsating armature reaction induces eddy currents in the field structure, causing heating. This effect is negligible in polyphase machines with balanced loads.

A minor consideration in favor of 3-phase power transmission is the fact that, with a fixed voltage between conductors, the 3-phase system requires but three-fourths the weight of copper of a single-phase system, other conditions such as distance, power loss, etc., being fixed.

87. Double-subscript Notation.—The solution of problems involving circuits and systems containing a number of currents and voltages is simplified and less susceptible to error if the current and voltage vectors are designated by some systematic notation, of which the following is one type:

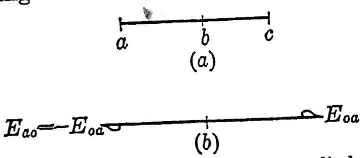


FIG. 107.—Subscript notation applied to voltage and current vectors.

The system is based on a simple geometrical proposition involving the relations among the segments into which a line may be divided. For example, consider the line ac, Fig. 107(a). The distance

$$ac = ab + bc. \tag{I}$$

Note that the first and last letters on the left-hand side of the equation are the same as the first and last letters on the right-hand side. Also, the last letter of the first term of the right-hand side is the same as the first letter of the second term of the right-hand side. These relations among the letters may be applied, for example, in determining the length of the segment ab. Applying the relations cited for (I), $ab = ac + cb$. Transposing (I) algebraically, $ab = ac - bc$. It therefore follows that $cb = -bc$, or reversing the order of the letters reverses the sign of the quantity that they represent. The foregoing relations among letters denoting points on a line are applicable also to the addition and subtraction of d-c voltages and currents and also of alternating vector quantities, the letters, however, under these conditions being used as subscripts.

If a voltage in a circuit acts in such a direction as to cause a current to flow from a to b, the positive direction of voltage is from a to b, and the voltage may be represented by E_{ab}, the order of the subscripts

denoting that the voltage is acting from a to b. For example, if a d-c emf be impressed across a simple resistance ab, the end a of the resistance being positive, the direction of current will be from a to b. Hence this emf is denoted by E_{ab} and the resulting current by I_{ab}.

At first sight it might appear that with alternating current a definite direction cannot be assumed, since both voltage and current reverse in sign during every cycle. As a matter of fact, the direction of a voltage or of a current by itself is not important, but rather the *phase relations* among voltages and currents. For example, if a horizontal vector E_{oa} to the right, Fig. 107(b), is a given voltage vector, the voltage vector E_{ao} to the left represents a voltage equal in magnitude but in

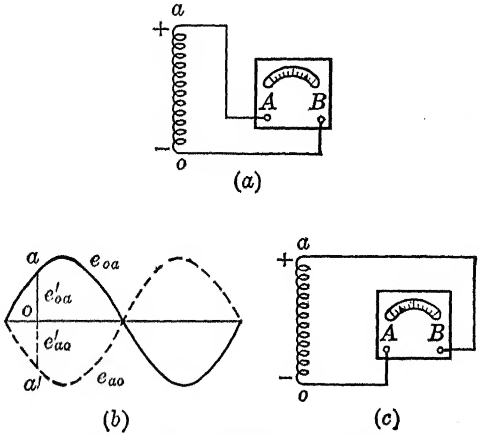


Fig. 108.—Subscript reversal.

opposition to E_{oa}. That is, $E_{ao} = -E_{oa}$. This is illustrated as follows:

In Fig. 108(a) is shown a coil oa of an alternator armature in which a sinusoidal emf is being induced. This emf is represented in (b) by the sine curve e_{oa}. At the instant o the instantaneous emf e'_{oa} induced in the coil is given by the ordinate oa, the terminal a being positive and the terminal o negative. Assume that a zero-center d-c voltmeter is capable of measuring the instantaneous emf e'_{oa}. When the voltmeter is connected with its right-hand binding post B to the terminal o and its left-hand binding post A to the terminal a as shown in (a), the voltmeter reads positive, the pointer deflecting to the right of center. That is, the emf e'_{oa} is positive. If the value of the instantaneous emf e'_{ao} is desired, the binding post A of the voltmeter must be transferred from the terminal a to the terminal o of the coil and likewise the binding post B must be transferred from the terminal o to the terminal a, as shown in (c). Obviously, the voltmeter pointer

now reverses its deflection, moving to the left of center, the magnitude of the deflection remaining unchanged. This shows that e'_{ao} is opposite and equal to e'_{oa}. Now assume that the voltmeter is replaced by an oscilloscope, the binding posts of the oscilloscope also being designated as A and B. When binding post B is connected to terminal o and binding post A to terminal a, as in (a), the sine curve e_{oa}, Fig. 108(b), is shown on the oscilloscope screen. When the binding post B is connected to terminal a and the binding post A to terminal o, the sine curve e_{ao}, shown dotted, 180° out of phase with e_{oa}, is obtained. Since the instantaneous values of e_{oa} and e_{ao}, Fig.

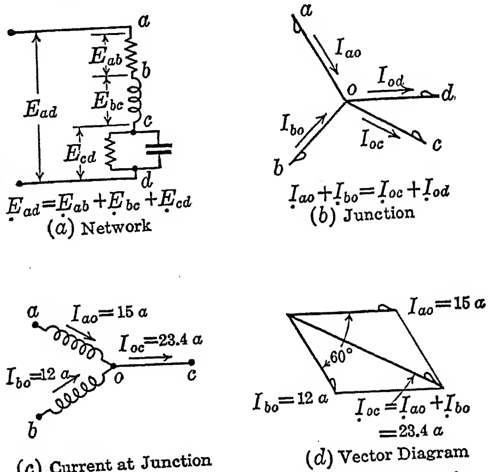


FIG. 109.—Examples of double-subscript notation.

107(b), are equal and opposite, their rms values, E_{oa} and E_{ao} are equal and opposite. That is, $E_{ao} = -E_{oa}$.

Consider the series-parallel circuit, Fig. 109(a). The total impressed voltage is the vector sum of the component voltages, that is, $\dot{E}_{ad} = \dot{E}_{ab} + \dot{E}_{bc} + \dot{E}_{cd}$. With the exception of the first term on each side of the equation, it is to be noted that when several voltages in series are being added the first letter of each subscript is the same as the last letter of the preceding subscript; also, the first and last subscript letters on one side of the equation are the same as the first and last subscript letters on the other side of the equation. This relation is similar to the addition of sectors of a line, Fig. 107(a).

When the notation is applied to currents, the principle is that of Kirchhoff's first law. Consider the junction o, Fig. 109(b), at which the four wires ao, bo, co, do meet. The current from a toward the junction o is I_{ao}, and that from b to o is I_{bo}; the two currents flowing away from the junction are I_{oc} and I_{od}. By Kirchhoff's first law, using

vectors, the total current flowing toward a junction must be equal to the total current flowing away from the junction. Hence

$$\dot{I}_{ao} + \dot{I}_{bo} = \dot{I}_{oc} + \dot{I}_{od}.$$

Reversing the order of a subscript reverses the algebraic sign of a quantity. Reversing the subscripts on the right-hand side of the equation and transferring the quantities on the right-hand side to the left-hand side of the equation give

$$\dot{I}_{ao} + \dot{I}_{bo} + \dot{I}_{co} + \dot{I}_{do} = 0. \tag{113}$$

Multiplying through by -1 and reversing the subscripts give

$$\dot{I}_{oa} + \dot{I}_{ob} + \dot{I}_{oc} + \dot{I}_{od} = 0. \tag{114}$$

Hence, if the currents at any junction are all placed on the same side of the equation, either all the first letters of the subscripts or all the last letters of the subscripts are the same.

To illustrate the use of the notation, consider Fig. 109(*c*), which shows two transformer secondaries, *ao* and *bo*, connected at *o*, to which the wire *oc* to the external circuit is also connected. A current I_{ao} equal to 15 amp rms flows from *a* to *o*, a current I_{bo} of 12 amp rms flows from *b* to *o*, and I_{ao} leads I_{bo} by 60°, as shown by the vector diagram in (*d*). It is required to determine the current I_{oc} in the wire *oc*. By Kirchhoff's first law, using vectors, $\dot{I}_{oc} = \dot{I}_{ao} + \dot{I}_{bo}$. The vector diagram is shown in (*d*). By trigonometry, I_{oc} is found to be 23.4 amp rms.

It is to be noted that by using this system of notation the likelihood of error is minimized.

Further details involving the use of this system of notation are given in its application to polyphase currents in the following sections.

88. Generation of Three-phase Emfs.—The 3-phase system is the most used of the polyphase systems. This is due to the fact that the 3-phase system has the least number of wires of any symmetrical polyphase system,[1] the line voltages are equal, and with a neutral conductor two different values of voltage are available.

The generation of 3-phase emfs by simple coils rotating in a bipolar magnetic field is shown in Fig. 110(*a*). Three simple coils a_1a, b_1b, c_1c, fastened rigidly together 120° apart, rotate in a counterclockwise direction. The shaded sides of the three coils are 120° apart, and the terminals *a*, *b*, *c* from these sides may be said to be *corresponding* terminals, the significance of which will be shown later. Likewise,

[1] There are only three wires in the 2-phase 3-wire system shown in Fig. 132 (p. 148), but this is an unsymmetrical system.

terminals a_1, b_1, c_1, also 120° apart, are corresponding terminals. Figure 110(*d*) shows similar coils a_1a, b_1b, c_1c placed on a cylindrical laminated iron core rotating counterclockwise in a bipolar field. In (*d*) the turns of the coils can be seen more clearly than in (*a*), and the direction of the instantaneous induced emfs is shown by arrows.

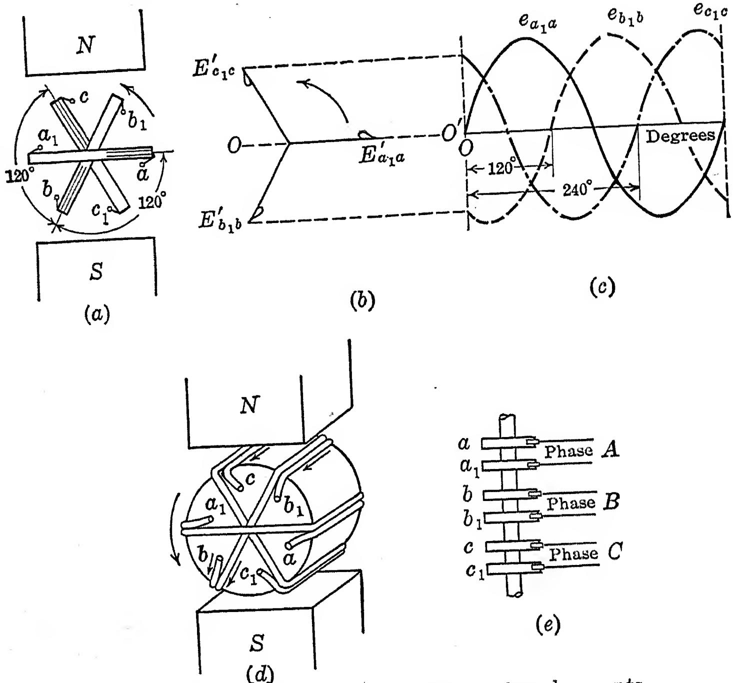


FIG. 110.—Generation of three-phase emfs and currents.

The current can be conducted from each of the three coils to the external circuit by means of a pair of slip rings, aa_1, bb_1, cc_1, Fig. 110(*e*), the terminals a and a_1 of coil a_1a being connected to rings a and a_1, etc.

At the instant shown in (*a*) and (*d*), the emf induced in coil a_1a is zero and is increasing in a positive direction; the emf induced in coil b_1b is approaching its maximum negative value, terminal b being *negative;* the emf induced in coil c_1c with terminal c positive has passed its maximum positive value, and is diminishing, Fig. 110(*c*), *O*-deg. These three emfs also can be considered as generated by the three rotating vectors E'_{a_1a}, E'_{b_1b}, E'_{c_1c}, Fig. 110(*b*), the three vectors being parallel to coils a_1a, b_1b, c_1c, in (*a*) and (*d*). Also, the right-hand end of vector E'_{a_1a} corresponds in position to terminal a, the lower end of vector E'_{b_1b} corresponds in position to terminal b, etc. (see Fig. 15, p. 18).

In (c) are shown the three emf waves e_{a_1a}, e_{b_1b}, e_{c_1c}, induced in the coils a_1a, b_1b, c_1c. It will be noted that at the instant under consideration the emf in coil a_1a is zero and increasing positively; that in coil b_1b is negative and approaching its negative maximum value; that in coil c_1c is positive and decreasing in value, the three values of emf thus corresponding to the positions of the coils in (a) and (d). The emf e_{b_1b} lags emf e_{a_1a} by 120°, and emf e_{c_1c} lags e_{a_1a} by 240°, corresponding to the angles between the ends a and b and a and c of the coils.

Figure 110(c) shows that, for any particular instant of time, the algebraic sum of the three emfs is zero. When one emf is zero, each of the other two has 86.6 per cent of its maximum value and these two have opposite signs. When any one emf is at its maximum, each of the other two has the opposite sign to this maximum and is 50 per cent of its maximum value.

The equations of the three emf waves are

$$e_{a_1a} = \sqrt{2}E \sin \omega t, \tag{115a}$$
$$e_{b_1b} = \sqrt{2}E \sin (\omega t - 120°), \tag{115b}$$
$$e_{c_1c} = \sqrt{2}E \sin (\omega t - 240°), \tag{115c}$$

where E is the rms value of each emf.

Each of the coils of Fig. 110(a) and (d) can be connected through its two slip rings to a single-phase circuit. This gives six slip rings and three independent single-phase circuits, such as phase A, phase B, phase C, Fig. 110(e). With the type of alternator having a rotating field and stationary armature, the usual type, the six slip rings would not be necessary, but six leads would be taken directly from the armature.

In practice, however, a 3-phase alternator seldom supplies three independent circuits by the use of six wires, but the phases are combined to give 3- or 4-wire 3-phase systems.

It is to be noted in Fig. 110(a), (b), (c), (d) that, when side a of coil a_1a is under the center of the N-pole, the emf of terminal a is positive maximum, and under these conditions e_{a_1a} is positive, Fig. 110(c). [This corresponds to 90° in Fig. 110(c)]. At a later time, corresponding to 120 electrical degrees after coil side a is under the center of the N-pole, the side b of coil b_1b comes under the center of the N-pole, the emf of terminal b is positive maximum, and under these conditions e_{b_1b} is positive and lagging e_{a_1a} by 120°; likewise, at a still later time corresponding to 240° after coil side a is under the center of the N-pole, the emf of terminal c becomes positive, and the emf e_{c_1c} lags e_{a_1a} by 240°. Hence, the maximum values of these three emfs may be represented by the three vectors E'_{a_1a}, E'_{b_1b}, E'_{c_1c}, Fig. 110(b),

which differ in phase by 120°. In Fig. 111 the three vectors E_{oa}, E_{ob}, E_{oc}, differing in phase by 120°, represent to scale the rms values. (The terminals a_1, b_1, c_1 are connected together to form terminal o.)

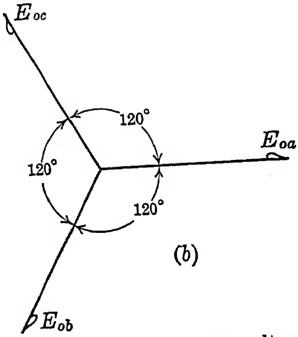


FIG. 111.—Three-phase voltage vector diagram.

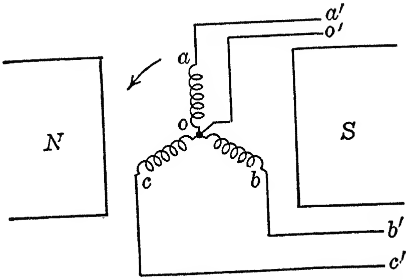


FIG. 112.—Y-connection of generator coils.

89. Y-connection.—The three coils of Fig. 110(a) and (d) are shown in simple diagrammatic form in Fig. 112. The three *corresponding* terminals a_1, b_1, c_1 are now connected at the common junction o. This is the Y-connection. Ordinarily, only three wires, aa', bb', cc', lead to the external circuit, although a neutral wire oo' is sometimes carried along, giving a 3-phase 4-wire system.

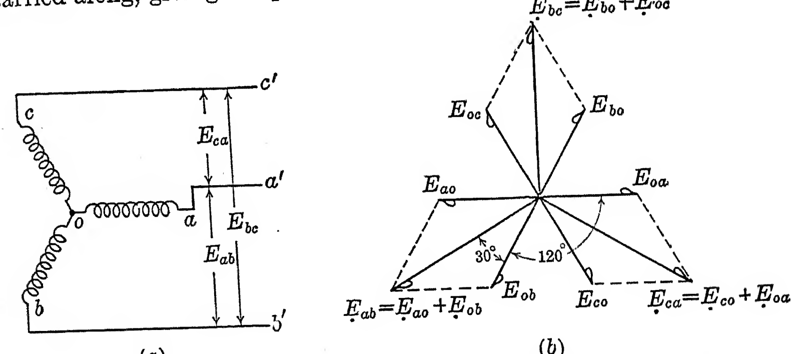


FIG. 113.—Y-connection and corresponding voltage vector diagram.

Figure 113(a) shows the three coils of Fig. 112 connected in Y to give a 3-phase system of which the three line wires are aa', bb', cc'. In Fig. 113(b) are shown the voltage vectors E_{oa}, E_{ob}, E_{oc}, corresponding to the emfs in the three coils oa, ob, and oc. These three emfs are the *phase* or *Y-voltages*. Let it be required to find the three line voltages E_{ab}, E_{bc}, E_{ca}. The line voltage $\dot{E}_{ab} = \dot{E}_{ao} + \dot{E}_{ob}$ (Sec. 87). E_{ao} is not on the original diagram but is obtained by reversing E_{oa}. E_{ao} is then added vectorially to E_{ob}, giving E_{ab}.

From geometry, E_{ab} lags the phase voltage E_{ob} by 30° and the coil voltage E_{oa} by 150°. Also, E_{ab} is numerically equal to

$$\sqrt{3}E_{ob} = 1.732E_{ob}.$$

In a similar manner, $\dot{E}_{bc} = \dot{E}_{bo} + \dot{E}_{oc}$, and $\dot{E}_{ca} = \dot{E}_{co} + \dot{E}_{oa}$. These three line voltages are shown in Fig. 114a.

In a balanced Y-system, the three line voltages are equal and differ in phase by 120°. *Each line voltage differs in phase by* 30° *from one of its phase voltages. The three line voltages are each equal in magnitude to* $\sqrt{3}$, *or* 1.732, *times the phase voltage.*

It is evident from Fig. 113(a) that the three phase, or coil currents I_{oa}, I_{ob}, I_{oc} are equal to the three line currents $I_{aa'}$, $I_{bb'}$, $I_{cc'}$, as coil and line are in series.

Therefore, in a Y-system the line and phase currents are equal.

As the three coils meet at a common junction o, by Kirchhoff's first law the sum of the three currents must be zero, provided that there is no neutral current. That is, $\dot{I}_{oa} + \dot{I}_{ob} + \dot{I}_{oc} = 0$. This is true whether or not the currents are balanced.

90. Currents in Y-system.—At the right, Fig. 114(a), is shown a Y-connected load consisting of three equal resistors $a'o'$, $b'o'$, $c'o'$. This load is supplied by the Y-connected energy source at the left, which is similar to that shown in Figs. 112 and 113(a). A neutral conductor $o'o$, of negligible resistance, connects the neutral of the load with that of the source. The three conductors aa', bb', cc' connecting the source and the load have negligible resistance. Inasmuch as the three loads are balanced, the current in the neutral is zero, as will be shown later.

Although it is pointed out in Sec. 87 that the order of subscripts does not purpose to show the direction of alternating-current flow, the order of the subscripts is frequently used to show the direction of *energy* flow. Thus in the source at the left, Fig. 114(a), the emfs E_{oa}, E_{ob}, E_{oc}, as well as the corresponding currents I_{oa}, I_{ob}, I_{oc}, indicate that the energy flows out of the source and since it flows into the load the terminal voltages to neutral at the load are given by $V_{a'o'}$, $V_{b'o'}$, $V_{c'o'}$. Upon applying Kirchhoff's second law to circuit $oaa'o'a$ and remembering that $V_{a'o'}$ is a voltage *drop* $(= -I_{a'o'}Z_{a'o'})$

$$+E_{oa} - V_{a'o'} = 0$$

or $E_{oa} = V_{a'o'}$. This is shown in Fig. 114(b), where $V_{a'o'}$ is in phase with E_{oa} in Fig. 113(b). Similarly, $V_{b'o'}$ is in phase with E_{ob} and $V_{c'o'}$ with E_{oc}. The three terminal voltages at the load, $V_{a'b'}$, $V_{b'c'}$, $V_{c'a'}$, are found by adding vectorially the resistor voltages. Thus,

$V_{a'b'} = V_{a'o'} + V_{o'b'}$. If $V_{a'b'}$, $V_{b'c'}$, $V_{c'a'}$ are reversed and then are compared with Fig. 113(*b*), $V_{b'a'} = E_{ab}$; $V_{c'b'} = E_{bc}$; $V_{a'c'} = E_{ca}$. The opposite order of subscripts is due to the fact that one system is a source of induced emf and the other is a load. It is customary to use E in dealing with an induced emf and V in dealing with a load or a terminal voltage.

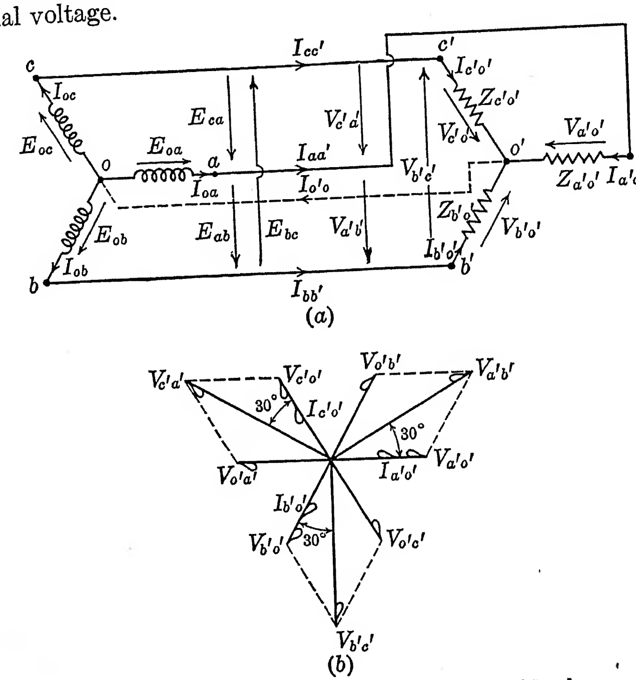


FIG. 114.—Y-connected power source and load.

A study of Fig. 114(*a*) shows that $\dot{I}_{oa} = \dot{I}_{aa'} = \dot{I}_{a'o'}$; $\dot{I}_{ob} = \dot{I}_{bb'} = \dot{I}_{b'o'}$; $\dot{I}_{oc} = \dot{I}_{cc'} = \dot{I}_{c'o'}$ again showing that in a Y-system the coil, or phase, current is also the line current. The currents $I_{a'o'}$, $I_{b'o'}$, $I_{c'o'}$, for unity power factor, are shown vectorially in Fig. 114(*b*). Since the three currents $I_{a'o'}$, $I_{b'o'}$, $I_{c'o'}$ are equal and differ in phase by 120°, their vector sum is zero and hence the neutral current $I_{o'o}$ is zero, since $\dot{I}_{o'o} = \dot{I}_{a'o'} + \dot{I}_{b'o'} + \dot{I}_{c'o'} = 0$.

91. Power in Y-system.—Figure 115 shows the three currents I_{oa}, I_{ob}, I_{oc}, of coils oa, o[illegible] oc, F[illegible] 113(*a*) [also see Fig. 113(*b*)]. Unity power factor is assumed [illegible] ree currents, therefore, are in phase with their coil voltages. [illegible] ed system is assumed, and the three currents, therefore, are e[illegible] nagnitude.

As is shown in Sec. [illegible] coil current I_{oa} and the line current $I_{aa'}$ are the same current [illegible] line current $I_{aa'}$, therefore, is 30° out

of phase with the line voltage E_{ca}, when the power factor is unity. This relation holds for each phase.

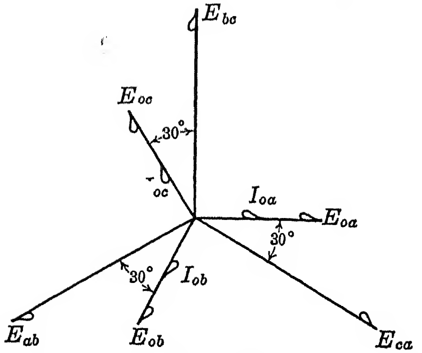


FIG. 115.—Relation of line to coil voltage and current in a Y-system, unity power factor.

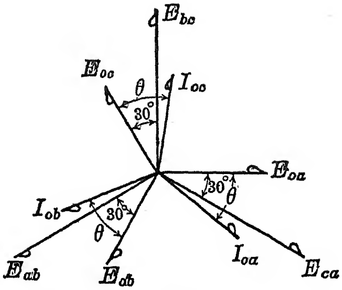


FIG. 116.—Relation of line to coil voltage and current in a Y-system. Power factor = cos θ.

The power delivered at unity power factor by coil *oa*, which is equal to that delivered by each of the other two coils, is

$$P' = E_{oa}I_{oa} \qquad \text{watts,}$$

and the total power delivered by the generator is three times P', or

$$P = 3E_{coil}I_{coil} \qquad \text{watts.}$$

As the power delivered to the line is the same as that delivered by the generator, upon substituting $E_{line}/\sqrt{3}$ for the value of E_{coil},

$$P = \frac{3}{\sqrt{3}} E_{line}I_{coil} = \sqrt{3}\, E_{line}I_{line} \qquad \text{watts,} \quad (116)$$

the coil current and the line current being the same.

In a balanced 3-phase system, the line power at unity power factor is equal to $\sqrt{3}$ times the product of line voltage and line current.

Figure 116 shows the same 3-phase system when the power factor differs from unity. Each coil current lags its respective coil voltage by the angle θ.

The total coil power is now three times that in the individual coil, or

$$P = 3E_{coil}I_{coil} \cos \theta_{coil} \qquad \text{watts.}$$

The system power is

$$P = \sqrt{3}\, E_{line}I_{line} \cos \theta_{coil} \qquad \text{watts,} \quad (117)$$

and the system kilowatts are

$$\frac{\sqrt{3}}{1,000} E_{line}I_{line} \cos \theta_{coil} \qquad \text{watts.} \quad (118)$$

Therefore, in a balanced 3-phase system, the system power factor is the cosine of the angle between **coil** *current and* **coil** *voltage.*

The angles between line currents and line voltages are not power-factor angles, for they involve the factors $\theta - 30°$, Fig. 116, and $\theta + 30°$, θ being the coil power-factor angle.

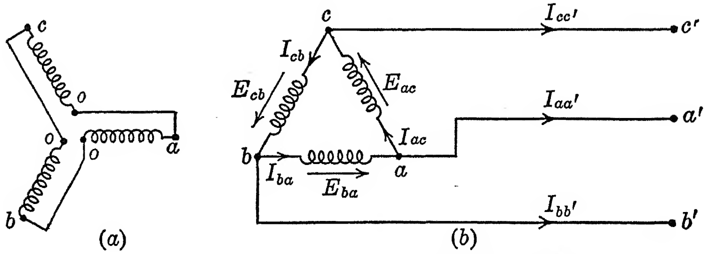


FIG. 117.—Delta-connection of alternator coils.

The system power factor, which is the coil power factor, is

$$\text{P.F.} = \frac{P}{\sqrt{3}\,E_{line}I_{line}} = \frac{P}{\sqrt{3}\,EI}, \tag{119}$$

where P is the total system power in watts and E and I are E_{line} and I_{line}.

It follows that the system volt-amperes,

$$Va = \sqrt{3}\,EI, \tag{120}$$

and the kva,

$$Kva = \frac{\sqrt{3}\,EI}{1{,}000}. \tag{121}$$

If the system is unbalanced, that is, if the currents or voltages are not equal or do not differ in phase by 120°, the question arises as to just what the system power factor is under these conditions. Where such unbalancing is not very great, (119) is used, line currents and voltages being averaged. The system power factor has practically no significance when the unbalancing is considerable.

Example.—A 3-phase alternator has three armature coils each rated at 1,330 volts and 150 amp. What is the voltage, kva, and current rating of this alternator if the three coils are connected in Y?

$$E_{line} = \sqrt{3} \cdot 1{,}330 = 2{,}300 \text{ volts. } \textit{Ans.}$$
$$\text{Kva rating} = \sqrt{3} \cdot 2{,}300 \cdot 150 = 600{,}000 \text{ va} = 600 \text{ kva. } \textit{Ans.}$$
$$\text{Current rating} = 150 \text{ amp. } \textit{Ans.}$$

92. Delta Connection.—The three coils of Fig. 113 may be connected as shown in Fig. 117(*a*), the diagram being simplified in Fig. 117(*b*). The end of each coil, which, in Fig. 113, was connected to the

neutral o, is now connected to the outer end of the next coil. As points o and a are now connected directly together ($E_{co} = E_{ca}$, etc.), the o's are now superfluous and are dropped.

Figure 118(a) shows vectorially the three voltages E_{ba}, E_{cb}, E_{ac}, acting from b to a, c to b, a to c, respectively.

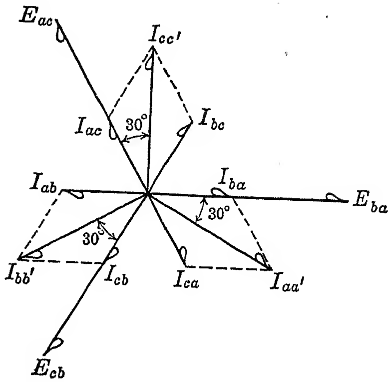


FIG. 118.—Relation of line voltage and current to coil values in delta-connected generator, unity power factor.

At first sight, Fig. 117 looks like a short circuit, the three coils, each containing a source of emf, being in series and short-circuited. The actual conditions existing in this closed circuit may be shown by the use of the subscript notation. Assume that the coil bc is broken at c', Fig. 119(a). The emf

$$\dot{E}_{bc'} = \dot{E}_{ba} + \dot{E}_{ac'}.$$

The vector sum of these two emfs, shown in Fig. 119(b), lies along voltage $E_{bc'}$ and is equal to it. The emf $E_{c'c} = 0$, therefore, and points c and c' can be connected without any resulting current. This is the same condition that exists when two d-c generators having equal emfs are connected in parallel. No current flows between the two if the proper polarity is observed.

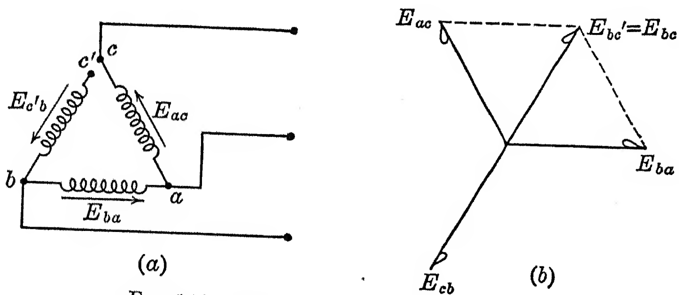


FIG. 119.—Sum of three delta emfs is zero.

Since the three emfs E_{ba}, E_{ac}, E_{cb} ($= E_{oa}$, E_{oc}, E_{ob}), Fig. 117(a), are in series, it follows, from a study of Fig. 110(c) (p. 128) that their sum at every instant is zero.

The three coil currents I_{ba}, I_{ac}, I_{cb}, of Fig. 117 are shown vectorially in Fig. 118, in phase with their respective voltages (power-factor unity), a balanced system again being assumed. The line current

$$\dot{I}_{aa'} = \dot{I}_{ba} + \dot{I}_{ca}.$$

This addition is made vectorially in Fig. 118, giving $I_{aa'}$, differing in phase from E_{ba} by 30°. It will be noted that in magnitude $I_{aa'}$ is $\sqrt{3}$ times the coil current. Line currents $I_{bb'}$ and $I_{cc'}$ may be found in a similar manner, Fig. 118. In the delta system, therefore, there is a phase difference of 30° between line current and line voltage at unity power factor, just as in the Y-system.

It is obvious that line voltage is equal to coil voltage in a delta system. Moreover, the sum of the three *voltages* acting around the delta must be zero, by Kirchhoff's second law.

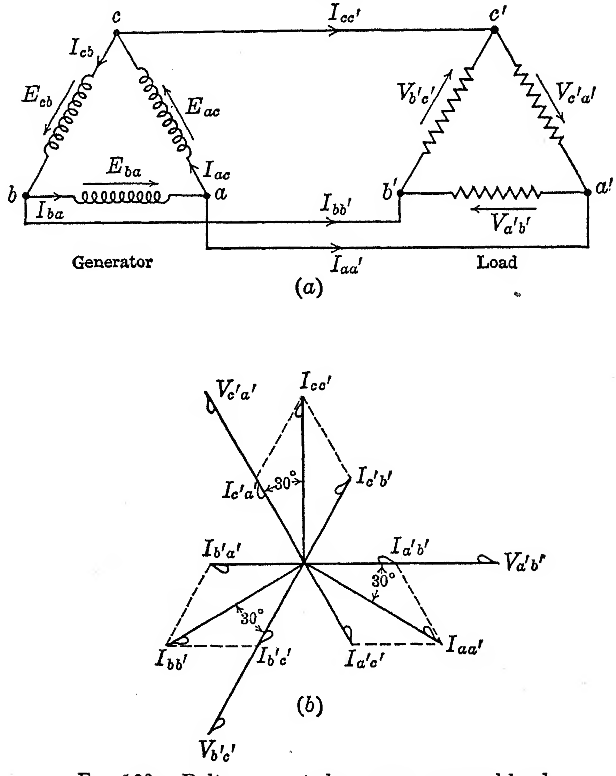
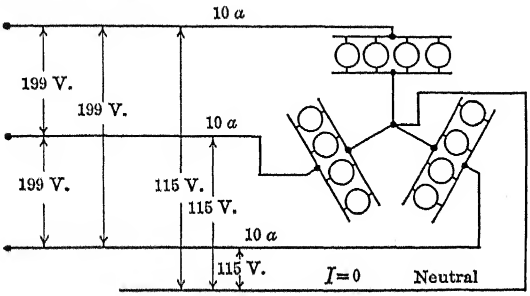


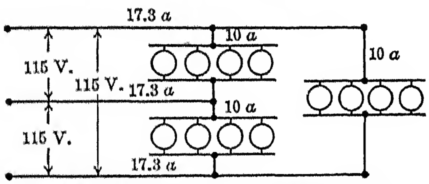
FIG. 120.—Delta-connected power source and load.

93. Load Currents in Delta System.—In Fig. 120(*a*) is shown a delta-connected load receiving energy from a delta-connected generator, the resistance of the connecting leads being negligible. At the load the positive direction of current is a' to b', b' to c', c' to a'. Upon applying Kirchhoff's second law to circuit $baa'b'b$ and remembering that $V_{a'b'}$ is a voltage *drop*, $E_{ba} - V_{a'b'} = 0$ or $V_{a'b'} = E_{ba}$. Similarly, $V_{b'c'} = E_{cb}$, $V_{c'a'} = E_{ac}$. The three voltage drops $V_{a'b'}$, $V_{b'c'}$, $V_{c'a'}$ are shown in the vector diagram, Fig. 120(*b*).

By applying Kirchhoff's first law to junction a', line current $I_{aa'} = I_{a'b'} + I_{a'c'}$. Similarly, $I_{bb'} = I_{b'c'} + I_{b'a'}$; $I_{cc'} = I_{c'a'} + I_{c'b'}$. These relations are shown vectorially in Fig. 120(*b*), unity power factor being assumed. Again note that at unity power factor there is a phase difference of 30° between line voltage and line current, as between $V_{a'b'}$ and $I_{aa'}$ [also compare Fig. 120(*b*) with Fig. 118].



115-Volt Lamp Banks Connected in Y



115-Volt Lamp Banks Connected in Delta

FIG. 121.—Lamp loads in Y and in delta.

To recapitulate, *in a delta system the line voltage is equal to the phase voltage, and by Kirchhoff's second law the sum of the three voltages around the delta is zero. In a balanced delta system the line current is* $\sqrt{3}$ *times the phase current.*

Although Fig. 114(*a*) shows a Y-load with a Y-connected source and Fig. 120(*a*) shows a delta load with a delta-connected source, the load may be connected in either Y or delta, irrespective of the connection of the source.

Figure 121 shows three lamp loads, each requiring 10 amp, at 115 volts. They are connected first in Y and then in delta. In order to supply the proper voltage in each case, there are 199 volts across lines in the Y-system and 115 volts in the delta system. There are 10 amp per line in the Y-system and 17.3 amp per line in the delta system. The power supplied is the same in each system.

94. Power in Delta System.—The total power in a delta system is

$$P = 3E_{coil}I_{coil} \cos \theta_{coil}. \qquad \text{(I)}$$

This power is equal to that in the line, as there is no intervening

loss. Also, the line current

$$I_{line} = \sqrt{3}\, I_{coil},$$

and

$$E_{line} = E_{coil}.$$

Hence, substituting in (I),

$$P = \sqrt{3}\, E_{line} I_{line} \cos \theta_{coil}.$$

This equation is the same as Eq. (117) (p. 133) for the Y-system. This should be so, for the relations in a 3-phase line are the same whether the power originates in a delta- or in a Y-connected generator.

The power factor of the delta system is the same as that for a Y-system.

$$\text{P.F.} = \frac{P}{\sqrt{3}\, EI} = \cos \theta_{coil}, \tag{122}$$

where P is the total power of the system in watts and E and I are the line voltage and line current.

The denominator $\sqrt{3}\, EI$ [(122)] gives the *volt-amperes* of the three-phase system. The *kva* of the 3-phase system is given by

$$\frac{\sqrt{3}\, EI}{1{,}000} \text{ [see (121), p. 134].}$$

METHODS OF MEASURING POWER IN THREE-PHASE SYSTEM

95. Three-wattmeter Method.—Let (1), (2), (3), Fig. 122(a), be the three coils of either a Y-connected alternator or a Y-connected load.

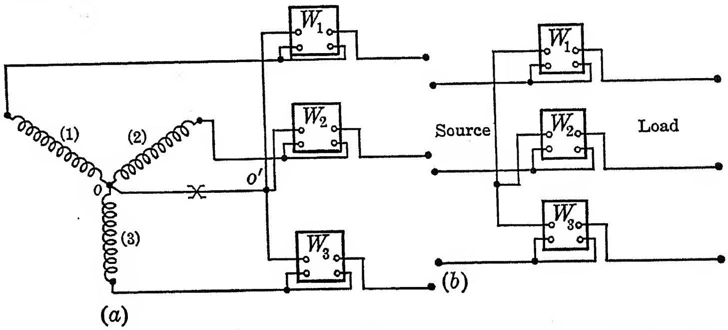


Fig. 122.—The three-wattmeter method of measuring three-phase power.

If the neutral of the Y is accessible, it is possible to measure the power of each coil, or phase, by connecting the current coil of a wattmeter in series with the phase and the wattmeter potential coil across the phase, Fig. 122(a). W_1, W_2, W_3, therefore, measure the power in loads 1, 2, 3, regardless of power factor, degree of balance, etc.

The total power

$$P = W_1 + W_2 + W_3 \qquad \text{watts.} \qquad (123)$$

If the loads are balanced,

$$W_1 = W_2 = W_3 \qquad \text{watts.}$$

If the potential circuits of the three wattmeters have equal resistances, these three potential circuits constitute a balanced Y-load, having a neutral O'. As coils 1, 2, 3 and these three wattmeter potential circuits are both symmetrical systems, O' must be at the same potential as O. No current flows, therefore, between O and O', and the line can be cut at X without changing existing conditions. Figure 122(b) shows the three-wattmeter connection for a 3-phase system. It can be shown that the total power is the sum of the wattmeter readings even though the wattmeter potential circuits have different resistances.[1] Under these conditions, however, the wattmeters may not all have the same reading, even with balanced loads.

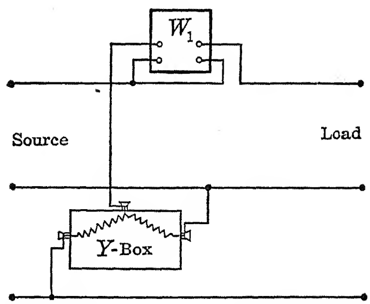


FIG. 123.—Y-box for measuring three-phase power.

The three-wattmeter method is well adapted to measuring power in a system where the power factor is continually changing, as in obtaining the phase characteristics of a synchronous motor. If the three instruments have equal potential-circuit resistances, they read alike, regardless of power factor, if the loads are balanced. The three-wattmeter method is necessary in a 3-phase 4-wire system, as a system of n wires ordinarily requires at least $n - 1$ wattmeters in order to measure the power correctly.

The Y-box.—The use of the Y-box is based on the principle that each of the three wattmeters of Fig. 122 reads the same, if the loads are balanced. Under these conditions, the total power $P = 3W_1$. If two resistances, each equal to the resistance of the potential coil of W_1, be used in conjunction with this potential coil, the wattmeters W_2 and W_3 are not necessary. As a rule, these two equal resistances are mounted in the same box and are connected as shown in Fig. 123. Accurate results can be obtained with this method only when the loads are balanced.

96. Two-wattmeter Method.—The power in a 3-phase, 3-wire system can be measured by two wattmeters connected as shown in Fig. 124. The current coils of the two instruments are connected in two

[1] LAWS, F. A. "Electrical Measurements," 2d ed. p. 341 *et seq.*

of the lines, and the potential coil of each instrument is connected from its own current coil to the line in which there is no current coil. Under these conditions, the total power passing through the system is

$$P = W_1 \pm W_2 \qquad \text{watts.} \qquad (124)$$

regardless of power factor, balance, etc. The choice of the plus or the minus sign will be explained later.

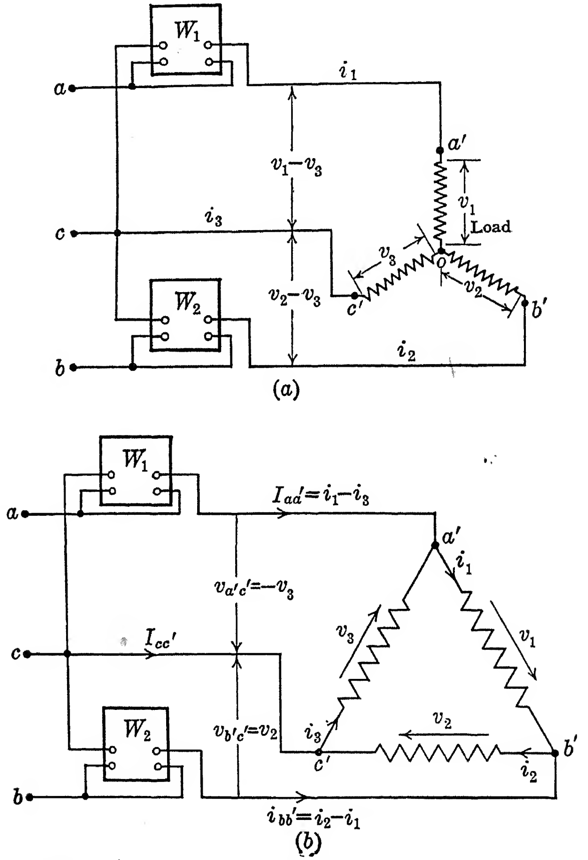


FIG. 124.—Two-wattmeter method for measuring 3-phase power.

One method of proving that these instruments give the correct power is as follows: In Fig. 124(a) let v_1, v_2, v_3, and i_1, i_2, i_3 be the voltages and currents of the three loads at any particular *instant.* These being *instantaneous* values, the power at the instant under consideration is equal to the sum of their products, regardless of power factor. That is, the instantaneous power

$$p = v_1 i_1 + v_2 i_2 + v_3 i_3. \qquad \text{watts.} \qquad \text{(I)}$$

At junction o, by Kirchhoff's first law,

$$i_1 + i_2 + i_3 = 0,$$
$$i_3 = -(i_1 + i_2). \qquad \text{(II)}$$

Substituting (II) in (I)

$$\begin{aligned} p &= v_1 i_1 + v_2 i_2 - v_3(i_1 + i_2) \\ &= (v_1 - v_3)i_1 + (v_2 - v_3)i_2 \qquad \text{watts.} \qquad \text{(III)} \end{aligned}$$

As the line voltages in a Y-system are the *differences* of the proper coil voltages (Sec. 89 p. 130), that is, one coil voltage is reversed when added to find the line voltage [also see Fig. 125(b)], the instantaneous values of power measured by the wattmeters are

$$\begin{aligned} w_1 &= (v_1 - v_3)i_1 \qquad \text{watts,} \\ w_2 &= (v_2 - v_3)i_2 \qquad \text{watts.} \end{aligned}$$

Hence, at every instant the power $w_1 + w_2$ measured by the two wattmeters is equal to the total instantaneous power p of the system.

A similar proof for a delta-connected load is as follows: In Fig. 124(b), let the instantaneous values of the line, or delta, voltages be v_1, v_2, v_3, and let the delta currents be i_1, i_2, i_3. Again, the instantaneous power is

$$p = v_1 i_1 + v_2 i_2 + v_3 i_3 \qquad \text{watts.} \qquad \text{(IV)}$$

By Kirchhoff's second law, around the delta,

$$v_1 + v_2 + v_3 = 0,$$
$$v_1 = -(v_2 + v_3). \qquad \text{(V)}$$

Substituting (V) in (IV),

$$\begin{aligned} p &= -(v_2 + v_3)i_1 + v_2 i_2 + v_3 i_3 \\ &= -v_3(i_1 - i_3) + v_2(i_2 - i_1) \qquad \text{watts.} \qquad \text{(VI)} \end{aligned}$$

The current to W_1 is $I_{aa'}$; and, at junction a', $I_{aa'} = (i_1 - i_3)$. Likewise, the current to W_2 is $I_{bb'}$; and, at junction b',

$$I_{bb'} = (i_2 - i_1).$$

The voltage to W_1 is $V_{a'c'} = -v_3$, and the voltage to W_2 is

$$V_{b'c'} = v_2.$$

[Note the direction of v_3 and v_2 arrows, Fig. 124(b)].

Hence the first term in (VI) gives the reading of W_1, and the second term gives the reading of W_2.

That is, the instantaneous values of power measured by the two wattmeters are

$$w_1 = -v_3(i_1 - i_3) \qquad \text{watts.}$$
$$w_2 = v_2(i_2 - i_1) \qquad \text{watts.}$$

Hence, at every instant the sum $w_1 + w_2$ gives the total instantaneous power p.

Y-connected Load. It is shown, Secs. 89, 91, and 92, that a phase difference of 30° exists between line voltage and line current at unity power factor. For power factors other than unity, this phase difference becomes $30° \pm \theta$, where θ is the power-factor angle of the coil.

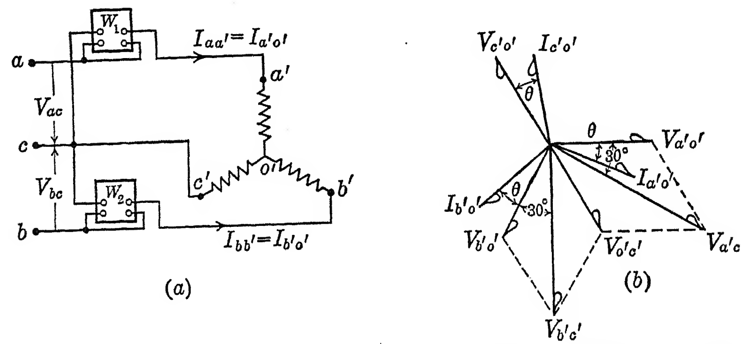


FIG. 125.—Two-wattmeter method and vector diagram for balanced Y-load.

Figure 125(a) shows two wattmeters, W_1 and W_2, measuring the power taken by a balanced 3-phase Y-connected load. The wattmeter W_1 is connected so that the current $I_{aa'} = I_{a'o}$ flows in its current coil, and the voltage $V_{ac} = V_{a'c'}$ (the voltage drop in the current coil being neglected) is across its potential circuit. The reading of W_1, therefore, is equal to the product of I_{ao}, $V_{a'c'}$, and the cosine of the angle between them. Figure 125(b) gives the vector diagram for the load. The three coil voltages $V_{a'o'}$, $V_{b'o'}$, $V_{c'o'}$ are equal in magnitude and differ in phase by 120°. The coil currents $I_{a'o'}$, $I_{b'o'}$, $I_{c'o'}$ are equal in magnitude and lag their coil voltages by the angle θ. The voltage $V_{a'c'}$ is found by reversing $V_{c'o'}$, giving $V_{o'c'}$, and then adding $V_{a'o'}$ and $V_{o'c'}$ vectorially ($V_{a'c'} = V_{a'o'} + V_{o'c'}$). The current $I_{a'o'}$ is given. The angle between $V_{a'c'}$ and $I_{a'o'}$ is $30° - \theta$.

Hence, the reading of W_1 is

$$W_1 = V_{a'c'}I_{a'o'} \cos (30° - \theta)$$
$$= V_{line}I_{line} \cos (30° - \theta) \qquad \text{watts.} \quad (125)$$

Likewise, the wattmeter W_2 reads the product of $V_{b'c'}$, $I_{b'o'}$, and the cosine of the angle between them. From the vector diagram, Fig. 125(b), $V_{b'c'}$ is found by adding vectorially $V_{b'o'}$ and $V_{o'c'}$

$$(V_{b'c'} = V_{b'o'} + V_{o'c'}).$$

The current $I_{b'o'}$ is given. The angle between $V_{b'c'}$ and $I_{b'o'}$ is $30° + \theta$. The reading of W_2 is, therefore,

$$W_2 = V_{b'c'}I_{b'o'} \cos (30° + \theta)$$
$$= V_{line}I_{line} \cos (30° + \theta) \qquad \text{watts.} \quad (126)$$

Summarizing,

$$W_1 = VI \cos (30° - \theta) \qquad \text{watts,} \quad (127)$$
$$W_2 = VI \cos (30° + \theta) \qquad \text{watts,} \quad (128)$$

where V and I are line voltage and line current, the system being balanced.

Delta-connected Load.—A similar proof for a delta-connected load is given as follows: In Fig. 126(a) is shown a balanced delta-connected

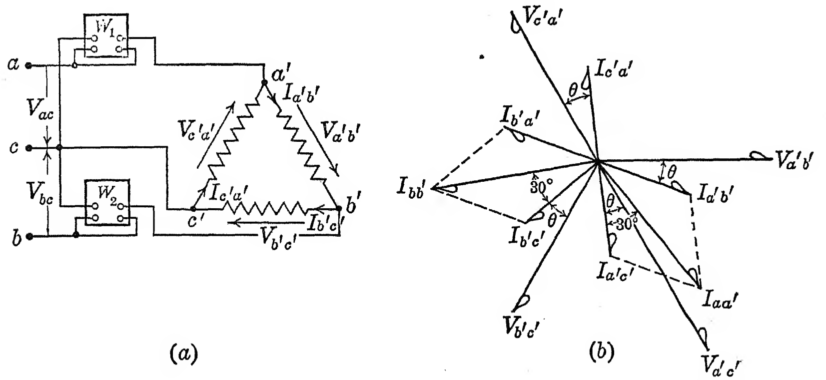


FIG. 126.—Two-wattmeter method and vector diagram for balanced delta-connected load.

load $a'b'$, $b'c'$, $c'a'$ similar to that in Fig. 124(b). The power factor is $\cos \theta$, the current lagging. Two wattmeters W_1 and W_2 measure the power and are connected in the same manner as in Fig. 125(a). The current coil of W_1 is in conductor aa' and that of W_2 in conductor bb'. The potential circuit of W_1 is connected across conductors ac and that of W_2 across conductors bc. The vector diagram is shown in (b). The three line voltages $V_{a'b'}$, $V_{b'c'}$, $V_{c'a'}$ are equal in magnitude and differ in phase by 120°; the three currents $I_{a'b'}$, $I_{b'c'}$, $I_{c'a'}$ are equal in magnitude and lag their line voltages by the angle θ. The current to W_1 is $I_{aa'}$, and the voltage is $V_{a'c'} = V_{ac}$, the voltage drop in the wattmeter current coil being neglected. The current

$$\dot{I}_{aa'} = \dot{I}_{a'b'} + \dot{I}_{a'c'}$$

is found by reversing $I_{c'a'}$ and adding. $V_{a'c'}$ is found by reversing $V_{c'a'}$. The angle between $V_{a'c'}$ and $I_{aa'}$ is $30° - \theta$. Hence the reading

of W_1 is

$$W_1 = V_{a'c'}I_{aa'} \cos (30° - \theta) \qquad \text{watts.} \qquad (125a)$$

Likewise, W_2 reads the product of $V_{b'c'} = V_{bc}$, the current $I_{bb'}$, and the cosine of the angle between them. $\dot{I}_{bb'} = \dot{I}_{b'c'} + \dot{I}_{b'a'}$ is found by reversing $I_{a'b'}$ and adding. The angle between $V_{b'c'}$ and $I_{bb'}$ is $30° + \theta$. Hence the reading of W_2 is

$$W_2 = V_{b'c'}I_{bb'} \cos (30° + \theta) \qquad \text{watts.} \qquad (126a)$$

Equations (125*a*) and (126*a*) are the same as Eqs. (125) and (126) so that (127) and (128) apply to both Y-connected and delta-connected loads.

W_1 and W_2 will have the same reading when $\theta = 0$ and $\theta = 180°$. Both conditions correspond to unity power factor. When θ equals 180°, however, the power has reversed. The two instruments also read the same at zero power factor ($\theta = 90°$), although this condition is seldom realized.

When θ is greater than 30°, Eq. (127) becomes

$$W_1 = VI \cos (\theta - 30°).$$

But $\cos (\theta - 30°)$ equals $\cos (30° - \theta)$ since $\cos A = \cos (-A)$ (p. 604). With leading current, $W_1 = VI \cos (30° + \theta)$, and

$$W_2 = VI \cos (30° - \theta).$$

When $\theta = 60°$, corresponding to a power factor of 0.5 lag, W_2, Eq. (128), reads zero, as $\cos (30° + 60°) = \cos 90° = 0$. In this case, the reading of W_1, Eq. (127), gives the total power. For angles greater than 60°, corresponding to power factors less than 0.5, $\cos (30° + \theta)$ is negative, W_2 reads negative, and the total power is

$$P = W_1 - W_2 \qquad \text{watts.} \qquad (129)$$

Discretion must be used, therefore, when two single instruments are employed, as the total power may be either the *sum* or the *difference* of the readings.

It may also be shown that

$$\tan \theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}, \qquad (130)$$

where θ is the coil power-factor angle. It is possible, therefore, to obtain the power factor in a balanced 3-phase system by means of the wattmeter readings alone.

Another convenient method for determining the power factor from the wattmeter readings for a balanced load is to divide the smaller

wattmeter reading by the larger,

$$\frac{W_2}{W_1} = \frac{\cos (30° + \theta)}{\cos (30° - \theta)}. \tag{131}$$

The power factor corresponding to this ratio is obtained by substituting different values of θ and solving for $\cos \theta$. The power factors corresponding to different values of W_2/W_1 are plotted as ordinates, Fig. 127. For example, when $\theta = 30°$, $\cos 30° = 0.866$, and

$$\frac{W_2}{W_1} = \frac{\cos 60°}{\cos 0°} = 0.500.$$

By the use of Fig. 127, the power factor is read directly, the ratio W_2/W_1 being known. It is seen that, when $W_2/W_1 = 1.0$, the power

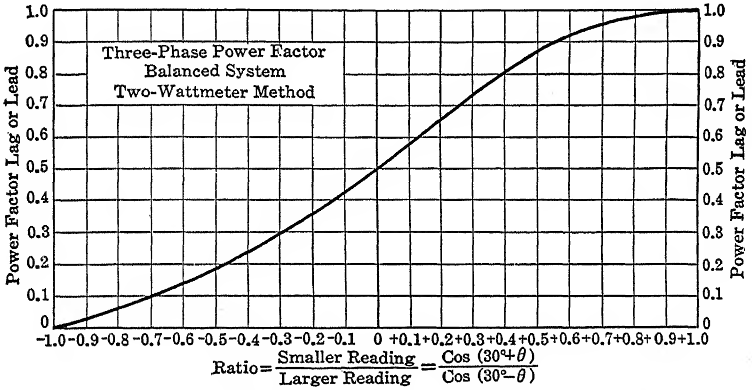


FIG. 127.—Power-factor diagram, two-wattmeter method.

factor is 1.0; when $W_2/W_1 = 0$, the power factor is 0.5; when W_2/W_1 is negative, that is, when it becomes necessary to reverse W_2, the power factor is less than 0.5.

Example.—In a test of a 3-phase induction motor, two wattmeters are used to measure the input. Their readings are 1,900 and 800 watts. Both instruments are known to be reading positive. What is the power factor of the motor at this load?

Using (130),

$$\tan \theta = \sqrt{3}\,\frac{1,900 - 800}{1,900 + 800} = \sqrt{3}\,\frac{1,100}{2,700} = 0.705,$$

$$\theta = 35.3°,$$

$$\cos \theta = \cos 35.3° = 0.816. \quad \textit{Ans.}$$

Also $W_2/W_1 = 800/1,900 = 0.421$, which may be used in Fig. 127 to verify the foregoing result.

If a polyphase wattmeter is used, Fig. 85 (p. 101), the adding or subtracting is done automatically, as both elements of the instrument

act on the same spindle. The polyphase instrument, therefore, if properly connected, reads the total power.

The two-wattmeter method cannot be used to measure power in a 3-phase 4-wire system unless the current in the neutral wire is zero. When the current in the neutral wire of Fig. 128 is zero, the power is correctly indicated by $W_1 \pm W_2$. Now apply load $B'O$ between line B and the neutral. The current to this load will complete its circuit from wire B through the neutral without going through the current coil of either wattmeter. As neither wattmeter, therefore, indicates this additional load, the two wattmeters are not sufficient to measure

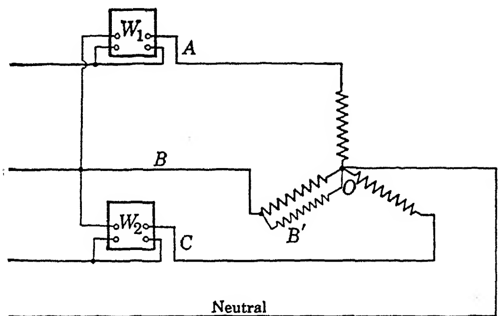


FIG. 128.—Two-wattmeter method not applicable generally to 4-wire system.

the power in such a 4-wire system under all conditions of load (also, see Fig. 122, p. 138, and Sec. 98, p. 149).

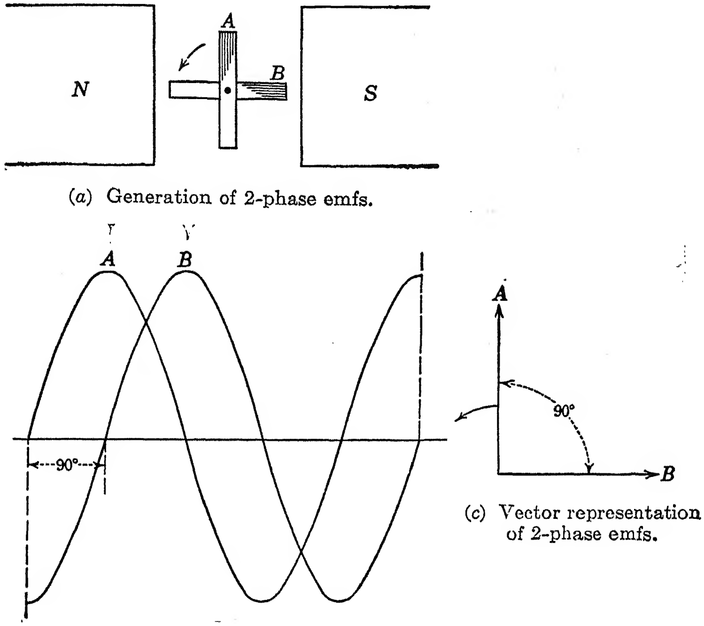
TWO-PHASE SYSTEMS

97. Two-phase and Four-phase (**Sometimes Called Quarter-phase**) **Systems.**—Although 3-phase systems for the most part have superseded other systems, there are still some 2-phase and 4-phase systems in operation. The 2-phase system is rarely used for transmission but is used for distribution, and in some instances it is specially advantageous to use 2-phase machines.

Two-phase emfs are induced in the elementary generator, Fig. 129(a), by two coils A and B, 90° apart. Figure 129(b) shows the emf waves induced by these coils. The emf of A leads that of B by 90°. When one emf is a maximum, the other is zero. Figure 129(c) shows these 2-phase emfs vectorially (also, see Figs. 265 and 266, pp. 308 and 310).

The two phases may be carried along, insulated from each other, to supply two separate single-phase circuits, or they may supply a common load such as an induction motor shown diagrammatically in Fig. 130. The two phases are *entirely insulated* from each other in Fig.

130 and no single load can be supplied between the two phases. Only one value of voltage is obtainable, moreover, as the voltages of the two phases are equal.



(a) Generation of 2-phase emfs.

(b) Two-phase emf waves.

(c) Vector representation of 2-phase emfs.

Fig. 129.—Two-phase emfs.

If, however, the generator coils be connected at their neutral points, a 4-phase *star* system results. If a neutral conductor be carried along with the other four conductors, a 4-phase, or quarter-phase, 5-wire

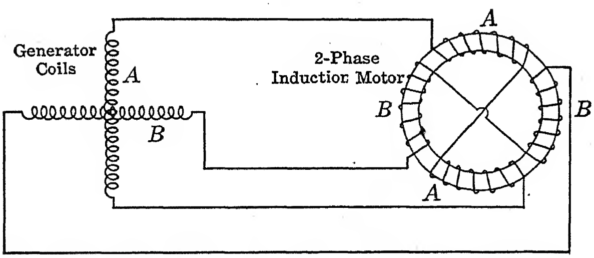


Fig. 130.—Two-phase circuit with phases isolated.

system results, Fig. 131(*a*). Three different voltages, moreover, are available. If the voltages between the outer wires of each phase be 200 volts, then 200, 100, and 141 volts are available, Fig. 131(*b*). This system is more readily unbalanced than the 3-phase system,

a fact that constitutes an objection to its use. Another objection is the greater number of wires.

If one end of the coil A be connected to one end of the coil B, a 3-wire 2-phase system results, Fig. 132. This gives two different values of voltage, 200 and 283 ($= 200\sqrt{2}$) volts. This system is

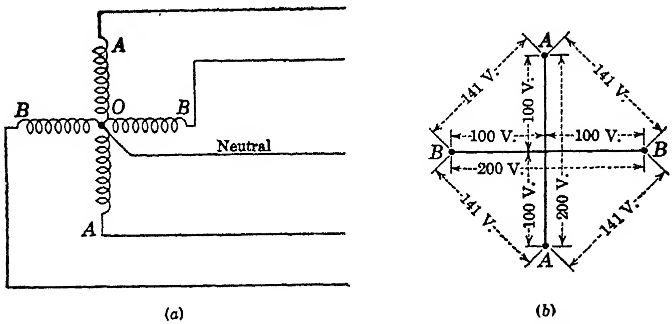


FIG. 131.—Two-phase interconnected system giving 4-phase, 5-wire star system.

little used because of the considerable amount of voltage unbalancing that results, even when moderate loads are applied. It should be noted that the common wire N carries a current $I\sqrt{2}$, where I is the current in each of the two outer wires. The wire N is not a true neutral conductor since its potential is not the center of gravity of the potentials of the system.

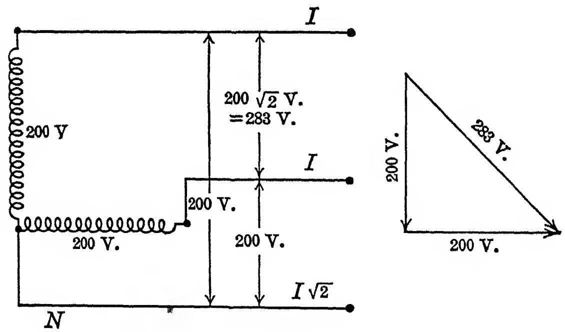


FIG. 132.—Two-phase, 3-wire system.

A 2-phase or 4-phase alternator may have a winding that consists of four coils. These coils may be connected in mesh, Fig. 133. This corresponds to the delta connection in a 3-phase system. As in the case of the delta, if these coils are properly connected, the winding is not short-circuited on itself.

The line voltage is equal to the coil voltage. The diametrical voltage is equal to $\sqrt{2}$ times the coil voltage. The line current is

equal to $\sqrt{2}$ times the coil current, because the line current is the resultant of two equal coil currents having 90° phase difference.

In Fig. 133, the coil voltage is 200 volts, and the diametrical voltage is $200\sqrt{2} = 283$ volts. The coil current is 100 amp, and the line

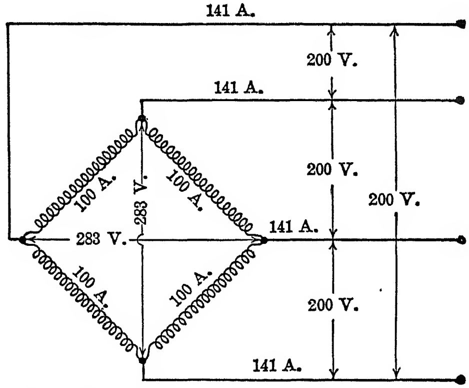


FIG. 133.—Mesh-connected, 2-phase winding.

current is 141 amp. The total kva rating of this system is

$$\frac{4 \cdot 200 \cdot 100}{1{,}000} = 80 \text{ kva.}$$

Considering the system from the point of view of the four line wires, it may be assumed to consist of two isolated systems similar to those in Fig. 130. The rating is computed as follows:

$$\text{Kva} = \frac{2(200\sqrt{2})\,(100\sqrt{2})}{1{,}000} = 80.$$

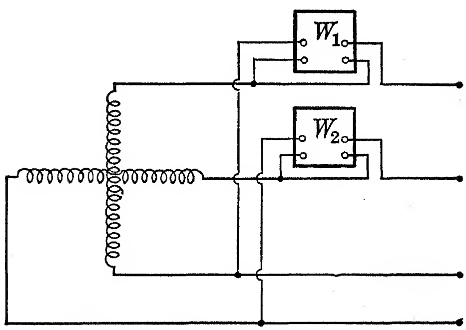


FIG. 134.—Measurement of power in isolated 2-phase 4-wire system.

98. Measurement of Power in Two-phase and Four-phase Systems.—In a 2-phase 4-wire system, consisting of two isolated single-phase systems, Fig. 134, the total power may be measured by two wattmeters, one in each of the single-phase systems, regardless of unbalance,

power factors, etc. If the system is interconnected to form a 4-phase system, Fig. 135, the loads must be balanced or this method is incorrect.

If the loads of a 4-wire interconnected system are not balanced, at least three wattmeters must be used, Fig. 135. The power is the algebraic sum of their readings (Fig. 122, p. 138). The power in a

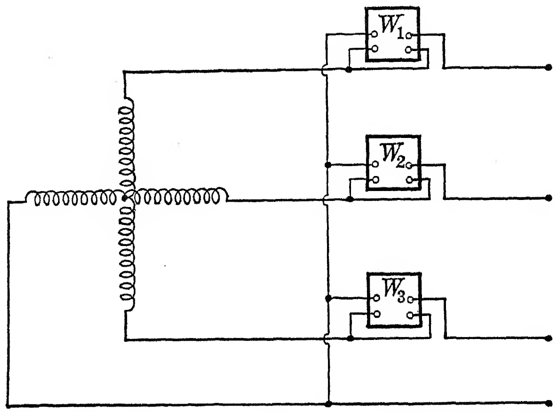


FIG. 135.—Measurement of power in 4-phase or 2-phase interconnected system (or any 4-wire system).

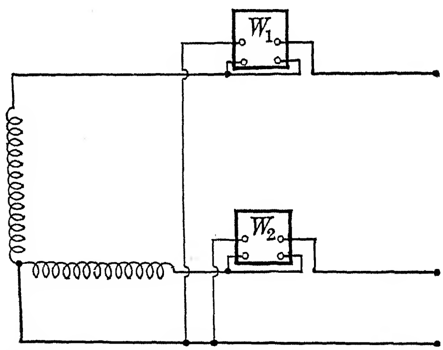


FIG. 136.—Measurement of power in 2-phase, 3-wire system (or any 3-wire system).

2-phase 3-wire system (or *any* 3-wire system) may be measured by two wattmeters connected as shown in Fig. 136. The wattmeter current coils may be connected, however, in either two of the three wires, (see Figs. 124(*a*), 125(*a*), 126(*a*), etc.).

99. Addition of Loads by the Kilovolt-ampere Method.—Frequently the polyphase loads on a line or feeder are specified by the kva and the power factor. However, they may be expressed also in kilowatts and power factor, or in amperes and power factor, if the voltage is constant and its value is known. Under the

last two conditions the loads are readily converted into kva and power factor. Since in a 3-phase system the kva are equal to $\sqrt{3}\,EI/1{,}000$, where E and I are line volts and line amperes (Sec. 91, p. 132, and Sec. 94, p. 137), and the voltage of the usual distribution system is substantially constant, the kva are equal to the product of a constant and the current. Hence the kva may be added like currents, the phase relations being taken into consideration. This is also true with other constant-potential polyphase systems, the constant differing from that of the 3-phase system. The method of determining the total kva on a line or feeder under these conditions is illustrated by the following example:

Example.—The following loads are connected along a 2,300-volt 3-phase line in which the voltage drop may be neglected: (1) 60 kva at 0.75 power-factor lag; (2) 80 kva at 0.9 power-factor lead; (3) 100 kw at 0.80 power-factor lag; and (4) 120 kva at unity power factor. Determine (*a*) total kilowatts; (*b*) total kva; (*c*) total amperes; (*d*) resultant power factor of system.

To compute the total kva and kilowatts, each load is resolved into kilowatts and kilovars (see Sec. 37, p. 64) as follows:

$$
\begin{array}{llll}
(1) & \cos\theta_1 = 0.75;\ \theta_1 = 41.4^\circ;\ \sin\theta_1 = 0.661; & \text{kw} = 60\cdot 0.75 = & 45; \\
 & \text{kvar} = 60\cdot 0.661 = \quad 39.7(-). & & \\
(2) & \cos\theta_2 = 0.90;\ \theta_2 = 25.8^\circ;\ \sin\theta_2 = 0.435; & \text{kw} = 80\cdot 0.9 = & 72; \\
 & \text{kvars} = 80\cdot 0.435 = \quad 34.8(+). & & \\
(3) & \cos\theta_3 = 0.80;\ \theta_3 = 36.9^\circ;\ \sin\theta_3 = 0.600; & \text{kw} = & 100; \\
 & \text{kvars} = 125\cdot 0.600 = \quad 75.0(-). & & \\
(4) & \cos\theta_4 = 1.0;\ \theta_4 = 0;\quad \sin\theta_4 = 0; & \text{kw} = & 120; \\
 & \text{kvars} = \quad 0. & & \\
\text{Total} & \text{kvars} = \quad 79.9(-). & \text{kw} = & 337.
\end{array}
$$

Lagging kilovars are considered (−) and leading kilovars (+).

(*a*) Kw = 337. *Ans.*

(*b*) Kva $= \sqrt{(337)^2 + (79.9)^2} = 346.$ *Ans.*

(*c*) $I = 346{,}000/(2{,}300\sqrt{3}) = 86.9$ amp. *Ans.*

(*d*) P.F. $= \text{kw/kva} = \dfrac{337}{346} = 0.974$, current lags. *Ans.*

The foregoing problem may be solved by the use of complex algebra. For example, the kva may be expressed as follows:

$$\text{kva} = \text{kw} \pm j(\text{kvars}).$$

100. Applications of Complex Algebra to Polyphase Circuits.—The solution of simple polyphase networks is generally not difficult if complex algebra is used. The methods of solving such problems are best illustrated by actual numerical examples.

Example 1.—Three impedances *a*, *b*, *c*, Fig. 137(*a*), having resistances 20, 15, 10 ohms and reactances of $+j10$, $-j15$, $+j20$ ohms, are connected in delta across the three conductors $A'A$, $B'B$, $C'C$ of a 110-volt 3-phase 60-cycle system. Impedance *a* is connected between conductors *A-B*, impedance *b* between conductors *B-C*, impedance *c* between conductors *C-A*. The sequence of phase rotation is *AB*, *BC*, *CA*. A wattmeter W_1 is connected with its current coil in conductor $A'A$ and its potential coil between conductors *A* and *C*; wattmeter W_2 is connected with its current coil in conductor $B'B$ and its potential coil between

conductors B and C. Determine (*a*) current in each impedance; (*b*) current in each line conductor; (*c*) reading of each wattmeter.

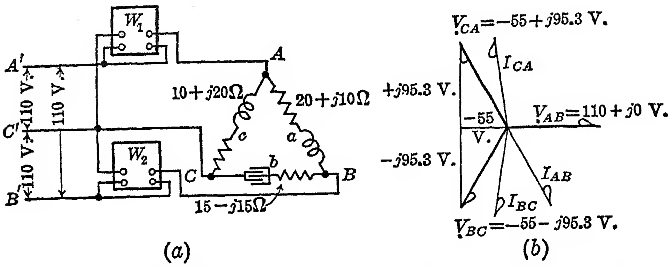


FIG. 137.—Unbalanced delta-connected loads and vector diagram.

(*a*) Assume that $\dot{V}_{AB}$ lies along the axis of reals, Fig. 137(*b*). Hence,

$$\dot{V}_{AB} = 110 + j0 = 110\underline{/0^\circ} \text{ volts,}$$
$$\dot{V}_{BC} = -55 - j95.3 = 110\overline{/120^\circ} \text{ volts,}$$
$$\dot{V}_{CA} = -55 + j95.3 = 110\underline{\backslash 120^\circ} \text{ volts.}$$

(*b*)

$$\dot{I}_a = \dot{I}_{AB} = \frac{110 + j0}{20 + j10} \cdot \frac{20 - j10}{20 - j10} = \frac{2{,}200}{500} - j\frac{1{,}100}{500} = 4.40 - j2.20 \text{ amp,}$$
$$|I_a| = \sqrt{(4.40)^2 + (2.20)^2} = 4.92 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_b = \dot{I}_{BC} = \frac{-55 - j95.3}{15 - j15} \cdot \frac{15 + j15}{15 + j15} = \frac{605 - j2{,}255}{450} = 1.345 - j5.01 \text{ amp,}$$
$$|I_b| = \sqrt{(1.345)^2 + (5.01)^2} = 5.18 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_c = \dot{I}_{CA} = \frac{-55 + j95.3}{10 + j20} \cdot \frac{10 - j20}{10 - j20} = \frac{1{,}356 + j2{,}053}{500} = 2.71 + j4.11 \text{ amp,}$$
$$|I_c| = \sqrt{(2.71)^2 + (4.11)^2} = 4.92 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_{A'A} = \dot{I}_{AB} + \dot{I}_{AC} = (4.40 - j2.20) + (-2.71 - j4.11) = 1.69 - j6.31 \text{ amp,}$$
$$|I_{A'A}| = \sqrt{(1.69)^2 + (6.31)^2} = 6.54 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_{B'B} = \dot{I}_{BC} + \dot{I}_{BA} = (1.345 - j5.01) + (-4.40 + j2.20) = -3.055 - j2.81 \text{ amp,}$$
$$|I_{B'B}| = \sqrt{(3.055)^2 + (2.81)^2} = 4.15 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_{C'C} = \dot{I}_{CA} + \dot{I}_{CB} = (2.71 + j4.11) + (-1.345 + j5.01) = 1.365 + j9.12 \text{ amp,}$$
$$|I_{C'C}| = \sqrt{(1.365)^2 + (9.12)^2} = 9.23 \text{ amp. } \textit{Ans.}$$
$$\dot{I}_{A'A} + \dot{I}_{B'B} + \dot{I}_{C'C} = 0 \ (\textit{check}).$$

(*c*) The voltage across the potential circuit of W_1 is $\dot{V}_{AC}$, and the current is $\dot{I}_{A'A}$. $\dot{V}_{AC} = 55 - j95.3$ volts, and $\dot{I}_{A'A} = 1.69 - j6.31$ amp.

Hence, by Sec. 56 (p. 81),

$$W_1 = 55 \cdot 1.69 + 95.3 \cdot 6.31 = 694 \text{ watts. } \textit{Ans.}$$

The voltage across the potential circuit of W_2 is $\dot{V}_{BC}$, and the current is $\dot{I}_{B'B}$.

$$\dot{V}_{BC} = -55 - j95.3 \text{ volts} \quad \text{and} \quad \dot{I}_{B'B} = -3.055 - j2.81 \text{ amp.}$$

Hence,

$$W_2 = (-55) \cdot (-3.055) + (-95.3) \cdot (-2.81) = 436 \text{ watts. } \textit{Ans.}$$

The total power

$$P = W_1 + W_2 = 1{,}130 \text{ watts. } \textit{Ans.}$$

Since the wattmeters are connected to measure the power by the two-wattmeter method, 1,130 watts is the power taken by the entire system. This may be checked by determining the I^2R-loss in each impedance. That is,

$$P_a + P_b + P_c = (4.92)^2 20 + (5.18)^2 15 + (4.92)^2 10 = 1{,}130 \text{ watts } (\textit{check}).$$

The foregoing example may also be solved using the polar-vector method, although a transformation to rectangular vectors is necessary in order to find the line currents.

For example,

$$V_{AB} = 110\underline{/0^\circ}; \qquad V_{BC} = 110\overline{/120^\circ}; \qquad V_{CA} = 110\underline{\backslash 120^\circ}.$$

$$Z_{AB} = 22.37\underline{/26.6^\circ}; \qquad Z_{BC} = 21.2\overline{\backslash 45^\circ}; \qquad Z_{CA} = 22.37\underline{/63.40^\circ}.$$

$$I_{AB} = \frac{110\underline{/0^\circ}}{22.37\underline{/26.6^\circ}} = 4.92\overline{\backslash 26.6^\circ} = 4.40 - j2.20 \text{ amp, etc.}$$

Example 2.—Three loads having resistances 20, 40, 50 ohms and reactances $+j30$, $-j50$, $j0$ ohms are connected from conductors $A'A$, $B'B$, $C'C$ to the neutral

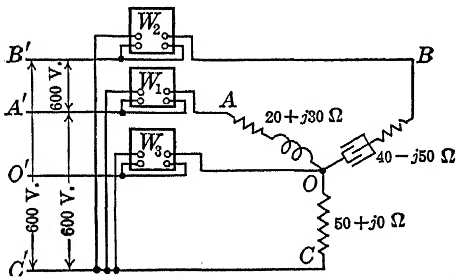


FIG. 138.—Unbalanced loads in 3-phase, 4-wire system.

$O'O$ of a balanced 600-volt 60-cycle 3-phase 4-wire system in which all system voltages are balanced, Fig. 138. Three wattmeters W_1, W_2, W_3 are connected with their current coils in conductors $A'A$, $B'B$, $O'O$ and their potential circuits between conductors A, B, O, and conductor C. The sequence of phase rotation is AO, BO, CO. Determine (*a*) current in each line conductor; (*b*) current in neutral; (*c*) reading of each wattmeter. (*d*) Show that the sum of the wattmeter readings is equal to the total power of the system, and compare with Fig. 122 (p. 138).

The voltage to neutral is $600/\sqrt{3} = 346.4$ volts.

$$V_{AO} = 346.4 + j0; \; V_{BO} = -173.2 - j300; \; V_{CO} = -173.2 + j300 \text{ volts.}$$

$$(a) \qquad I_{AO} = \frac{346.4}{20 + j30} = \frac{346.4 \cdot 20}{1{,}300} - j\frac{346.4 \cdot 30}{1{,}300} = 5.33 - j8.00 \text{ amp.,}$$

$$|I_{A'A}| = |I_{AO}| = \sqrt{(5.33)^2 + (8.00)^2} = 9.61 \text{ amp. } \textit{Ans.}$$

$$I_{BO} = \frac{-173.2 - j300}{40 - j50} = \frac{(-173.2 - j300)(40 + j50)}{4{,}100} = 1.968 - j5.04 \text{ amp,}$$

$$|I_{B'B}| = |I_{BO}| = \sqrt{(1.968)^2 + (5.04)^2} = 5.41 \text{ amp. } \textit{Ans.}$$

$$I_{CO} = \frac{-173.2 + j300}{50 + j0} = -3.464 + j6.0 \text{ amp,}$$

$$|I_{C'C}| = |I_{CO}| = \sqrt{(3.464)^2 + (6.0)^2} = 6.93 \text{ amp. } \textit{Ans.}$$

(b) $\dot{I}_{OO'} = \dot{I}_{AO} + \dot{I}_{BO} + \dot{I}_{CO}$
$= (5.33 - j8.00) + (1.968 - j5.04) + (-3.464 + j6.0) = 3.834 - j7.04$ amp,

$$|I_{OO'}| = \sqrt{(3.834)^2 + (7.04)^2} = 8.02 \text{ amp.} \quad \textit{Ans.}$$

(c) The current in wattmeter W_1 is $\dot{I}_{AO} = 5.33 - j8.00$, and its potential circuit is connected across $\dot{V}_{AC} = \dot{V}_{AO} + \dot{V}_{OC} = 519.6 - j300$. Hence, by Sec. 56 (p. 81),

$$W_1 = 519.6 \cdot 5.33 + 300 \cdot 8.00 = 5{,}170 \text{ watts.} \quad \textit{Ans.}$$

The current in W_2 is $\dot{I}_{BO} = 1.968 - j5.04$, and the potential across W_2 is $\dot{V}_{BC} = \dot{V}_{BO} + \dot{V}_{OC} = -j600$. Hence,

$$W_2 = 600 \cdot 5.04 = 3{,}024 \text{ watts.} \quad \textit{Ans.}$$

The current in W_3 is $\dot{I}_{O'O} = -3.834 + j7.04$, and the potential across W_3 is $\dot{V}_{OC} = 173.2 - j300$. Hence,

$$W_3 = 173.2 \cdot (-3.834) + (-300 \cdot 7.04) = -2{,}776 \text{ watts.} \quad \textit{Ans.}$$

The total power

$$P = W_1 + W_2 + W_3 = 5{,}170 + 3{,}024 - 2{,}776 = 5{,}418 \text{ watts.} \quad \textit{Ans.}$$

(d) The total power is also equal to the sum of the I^2R-losses in the impedances. That is,

$$P = I_{AO}^2 \cdot 20 + I_{BO}^2 \cdot 40 + I_{CO}^2 \cdot 50$$

$$= (9.61)^2 \cdot 20 + (5.41)^2 \cdot 40 + (6.93)^2 \cdot 50 = 5{,}418 \text{ watts } (\textit{check}). \quad \textit{Ans.}$$

Hence, the three wattmeters as connected in Fig. 138 measure the total power of the system, even though W_3 would normally read backward. The current connection of W_3 must be reversed, therefore, and its reading substracted from the sum of the other two readings.

In Fig. 122(*a*) (p. 138) the current coils of the three wattmeters are connected in the three line conductors, and the common potential connection is made to the neutral conductor of the system. Although this connection is the most usual one, the foregoing example illustrates the fact that the three current coils may be connected in *any* three of the system conductors and the common potential connection made to the fourth (also see Fig. 135, p. 150). Hence, to measure the power in any polyphase system of n conductors, the $n - 1$ wattmeters may have their current coils connected in *any* $n - 1$ of the conductors and the potential circuits to the remaining conductor.

101. Equivalent Delta Systems and Y-systems.—In Vol. I (Chap. III), it is shown that a delta connection of resistances may be replaced by an equivalent Y, and vice versa. By these means many more or less complicated passive networks may be reduced to more or less simple ones. With alternating current, similar transformations may

be made, the resistances being replaced by impedances expressed in complex. Thus, in Fig. 139 the delta system in (*a*) may be replaced by the Y-system in (*b*) as follows:

$$\dot{Z}_1 = \frac{\dot{Z}_{12}\dot{Z}_{31}}{\dot{Z}_n}, \quad (132) \qquad \dot{Z}_2 = \frac{\dot{Z}_{23}\dot{Z}_{12}}{\dot{Z}_n}, \quad (133) \qquad \dot{Z}_3 = \frac{\dot{Z}_{31}\dot{Z}_{23}}{\dot{Z}_n}, \quad (134)$$

where $\dot{Z}_n = \dot{Z}_{12} + \dot{Z}_{23} + \dot{Z}_{31}$.

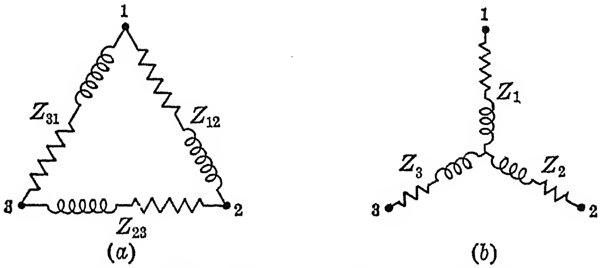


FIG. 139.—Equivalent delta and Y-systems.

Likewise, the Y-system may be replaced by the delta system,

$$\dot{Z}_{12} = \frac{\dot{Z}_0}{\dot{Z}_3}, \quad (135) \qquad \dot{Z}_{23} = \frac{\dot{Z}_0}{\dot{Z}_1}, \quad (136) \qquad \dot{Z}_{31} = \frac{\dot{Z}_0}{\dot{Z}_2}, \quad (137)$$

where $\dot{Z}_0 = \dot{Z}_1\dot{Z}_2 + \dot{Z}_2\dot{Z}_3 + \dot{Z}_3\dot{Z}_1$.

In the network, Fig. 140(*a*), where there are three unbanlaced Y-connected loads and no neutral conductor, the voltages to neutral

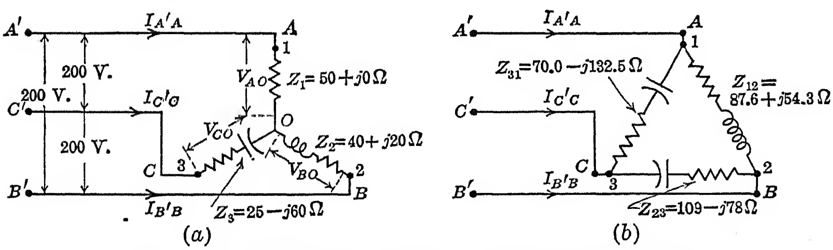


FIG. 140.—Y-system converted to equivalent delta system.

are unknown. Hence, if Kirchhoff's laws are used, three simultaneous equations involving complex quantities are necessary. However, by converting the Y-system in (*a*) into the equivalent delta system in (*b*), a direct solution is made possible.

Example.—In the Y-connected system, Fig. 140(*a*), determine (*a*) equivalent delta; (*b*) three line currents $I_{A'A}$, $I_{B'B}$, $I_{C'C}$; (*c*) three voltages to neutral, V_{AO}, V_{BO}, V_{CO}. The sequence of phase rotation is V_{AB}, V_{BC}, V_{CA}.

(a) Using (135), (136), (137),

$$Z_0 = (50 + j0)(40 + j20) + (40 + j20)(25 - j60) + (25 - j60)(50 + j0)$$
$$= 5{,}450 - j3{,}900 \text{ ohms},$$
$$Z_{12} = \frac{5{,}450 - j3{,}900}{25 - j60} \cdot \frac{25 + j60}{25 + j60} = 87.6 + j54.3 \text{ ohms.} \quad \textit{Ans.}$$
$$Z_{23} = \frac{5{,}450 - j3{,}900}{50 + j0} = 109.0 - j78.0 \text{ ohms.} \quad \textit{Ans.}$$
$$Z_{31} = \frac{5{,}450 - j3{,}900}{40 + j20} \cdot \frac{40 - j20}{40 - j20} = 70.0 - j132.5 \text{ ohms.} \quad \textit{Ans.}$$

The equivalent delta system is shown in Fig. 140(b).

(b) Let $V_{AB} = 200 + j0$ volts; $V_{BC} = -100 - j173.2$ volts;

$$V_{CA} = -100 + j173.2 \text{ volts.}$$

$$I_{AB} = \frac{200 + j0}{87.6 + j54.3} \cdot \frac{87.6 - j54.3}{87.6 - j54.3} = 1.649 - j1.022 \text{ amp,}$$
$$I_{BC} = \frac{-100 - j173.2}{109.0 - j78.0} \cdot \frac{109.0 + j78.0}{109.0 + j78.0} = 0.1453 - j1.485 \text{ amp,}$$
$$I_{CA} = \frac{-100 + j173.2}{70.0 - j132.5} \cdot \frac{70.0 + j132.5}{70.0 + j132.5} = -1.329 - j0.0501 \text{ amp,}$$
$$I_{A'A} = I_{AO} = I_{AB} + I_{AC} = (1.649 - j1.022) + (1.329 + j0.0501)$$
$$= 2.978 - j0.972 \text{ amp.} \quad \textit{Ans.}$$
$$I_{B'B} = I_{BO} = I_{BC} + I_{BA} = (0.1453 - j1.485) + (-1.649 + j1.022)$$
$$= -1.504 - j0.463 \text{ amp.} \quad \textit{Ans.}$$
$$I_{C'C} = I_{CO} = I_{CA} + I_{CB} = (-1.329 - j0.0501) + (-0.1453 + j1.485)$$
$$= -1.474 + j1.435 \text{ amp.} \quad \textit{Ans.}$$

$$I_{AA'} + I_{BB'} + I_{CC'} = 0 \ (\textit{check}).$$

(c) $V_{AO} = I_{AO}Z_1 = (2.978 - j0.972)(50 + j0) = 149.0 - j48.6$ volts. *Ans.*

$V_{BO} = I_{BO}Z_2 = (-1.504 - j0.463)(40 + j20) = -50.9 - j48.6$ volts. *Ans.*

$V_{CO} = I_{CO}Z_3 = (-1.474 + j1.435)(25 - j60) = 49.2 + j124.3$ volts. *Ans.*

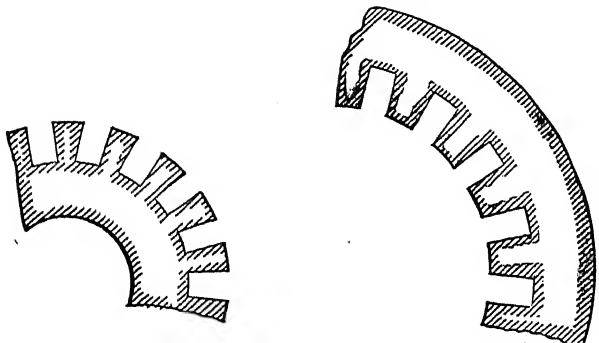
It will also be found as a further check that essentially

$$V_{AB} = V_{AO} + V_{OB} = 200 + j0 \text{ volts;}$$
$$V_{BC} = V_{BO} + V_{OC} = -100 - j173.2 \text{ volts;}$$
$$V_{CA} = V_{CO} + V_{OA} = -100 + j173.2 \text{ volts}$$

CHAPTER VI

THE ALTERNATOR

102. Rotating-field Type.—In commercial alternators, in general, the armature is stationary, and the field rotates. The generation of emf in an armature conductor depends only on *relative* motion of conductor and field flux so that either armature or field may be the rotating member. In direct-current machines, the commutator makes necessary either that the armature be the rotating member or that the brushes revolve with the field. As alternators have no commutator, it is not necessary that the armature be the rotating member. The stationary armature is illustrated in Figs. 153, 154, 155, etc.



(a) (b)

FIG. 141.—Effect of slot depth on the width of tooth necks in rotor and in stator.

This construction has two distinct advantages A rotating armature requires two or more slip rings for conducting the current from the armature to the external circuit. Such rings must be more or less exposed and are difficult to insulate, particularly for the higher voltages of 6,600 and 13,200 volts at which alternators are commonly operated. These rings are a frequent source of trouble, owing to arc-overs, short circuits, etc. A stationary armature requires no slip rings, and the armature leads can be continuously insulated conductors from the armature coils to the bus bars. It is more difficult to insulate the conductors in a rotating armature than in a stationary one, because of centrifugal force and the vibration resulting from rotation.

When the field is the rotating member, the field current must be conducted to the field winding through slip rings. As the field voltage

seldom exceeds 250 volts and the amount of power is small, no particular difficulty is encountered in the operation of such slip rings.

Usually, it is difficult to find sufficient space for the copper on the surface of an armature. This is particularly true with high-speed high-voltage machines having armatures of small diameter. Increased space for copper may be obtained by deepening the slots. If the armature be the rotating member, the deepening of the slots is limited by the contraction of the tooth necks, as shown in Fig. 141(*a*). No such difficulty is encountered if the armature be stationary, since the tooth necks increase in width with the deepening of the slots, Fig. 141(*b*). Since the armature usually operates at much higher voltage than the field, much more insulation is required. Space for this increased insulation is readily obtainable in the deep stator slots.

ALTERNATOR WINDINGS

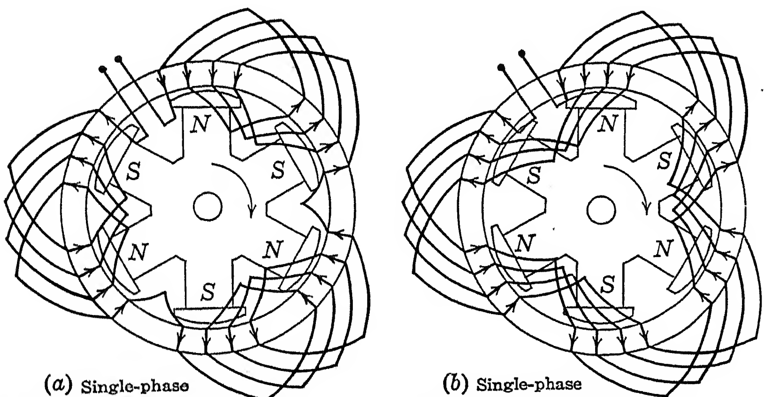
103. General Principles.—The usual direct-current armature generates alternating current; and if properly connected slip rings are provided, alternating current may be obtained. Hence, windings of the direct-current type, if provided with proper taps, are suitable for alternators and such windings are used (see Synchronous Converter, p. 426). However, the requirements of alternating-current systems make it advantageous in most cases to depart from the direct-current winding. For example, the usual direct-current winding is a *closed-coil* winding (see Vol. I, Chap. XI). Alternator windings may be either closed or open, the delta connection giving a closed-coil winding and the Y-connection an open-coil winding.

The general principles that govern direct-current windings hold for windings of alternators. The span of each coil must be approximately one pole pitch; that is, the two sides of any coil must lie under adjacent poles. The coils must be so connected that their emfs add. In addition, it is desirable that the winding be designed to give a sine wave, at least approximately. The cost of the winding should be low, so that it is important that the coils be formed and insulated before they are placed in the slots.

Alternator windings may be divided into two general classes, the barrel type, Figs. 143 to 147, in which diamond-shaped coils, usually formed, are used; and the spiral type, Figs. 150, 151. The barrel type may be half-coil, Fig. 143(*a*); whole coil, Fig. 143(*b*); single-layer, Figs. 142, 143(*a*); or two-layer, Figs. 143(*b*), (*c*) to 147. However, in the United States the two-layer lap winding is used almost exclusively. In Europe, diamond-shaped coils are not used so extensively, spiral windings being used to a considerable extent.

104. Single-phase Windings.—Single-phase windings are practically never used. As a matter of fact, single-phase power is generated only occasionally, such as for single-phase railroad supply. Even then, the generators are usually wound 3-phase, and single-phase power is taken from two terminals of a Y-connection. Since the rating of a given alternator wher operated single-phase is only approximately 60 per cent of its rating when operated polyphase, polyphase operation is almost universal.

Since polyphase windings are merely two or more single-phase windings symmetrically spaced on the armature, single-phase windings will be described first. If the principle of the single-phase winding is understood, little difficulty is experienced in understanding the polyphase winding.



(*a*) Single-phase partially distributed lap winding

(*b*) Single-phase partially distributed wave winding

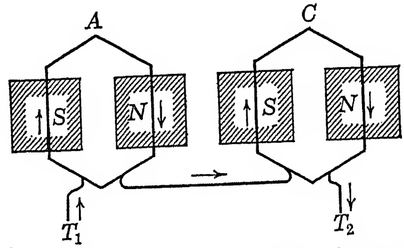
FIG. 142.—Single-phase lap and wave windings.

In d-c dynamos the wave winding gives a higher emf than the lap winding, if the number of armature conductors, poles, and other conditions are the same. In the alternator, the wave and lap windings give the same emf, if the number of armature conductors, poles, and other conditions are the same. This is illustrated in Fig. 142, where a single-layer 6-pole distributed lap winding is shown in (*a*) and a wave winding, otherwise similar, is shown in (*b*). An inspection of the two windings shows that each has the same number of series-connected conductors between terminals. Hence, other conditions being the same, equal emfs must be induced. Because the connections of the lap winding are somewhat simpler than those of the wave winding, the lap winding is used almost exclusively.

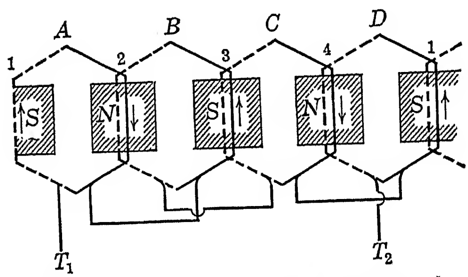
Figure 143(*a*) shows a single-phase single-layer half-coil winding for a 4-pole alternator having 4 slots, making 1 slot per pole. This

winding is called a *half-coil* winding, because there is but one half-coil or coil group per pole, or one-half as many coils or coil groups as there are poles. The two coils are shown connected in series, and T_1, T_2 are the terminals of the winding.

In Fig. 143(b) is shown a development of the winding, two coils B and D being added. There are now four coils and four poles so that there is the same number of coils or coil groups as poles. Hence, this is a *whole-coil* winding. Also, one side of each coil, shown as a solid



(a) Single-layer, half-coil winding, 1 slot per pole



(b) Two-layer, whole-coil winding, 1 slot per pole

FIG. 143.—Barrel-type single-phase windings with diamond-shaped coils.

line, lies in the top of a slot, and the other side, shown dotted, lies in the bottom of a slot. Hence this is a two-layer winding. The winding of (a) may be obtained by swinging coil B into the plane of coil A and coil D into the plane of coil C. The method of connecting the coils should be noted, the connections of coils B and D being reversed with respect to those of coils A and C, so that their emfs are additive, as is indicated by the arrows.

One slot per pole is almost never used, as thus the surface of the armature is not used economically, and, in addition, a poor voltage wave results. In Fig. 144 is shown a single-phase winding, similar to that of Fig. 143(b) except that there are 2 slots per pole. Since the coils of each coil group are connected in series before being connected

to the next group, the winding is *lap-connected.* The winding is also two-layer. In practice, a much larger number of slots per pole is used, two being shown in Fig. 144 in order to illustrate the principle.

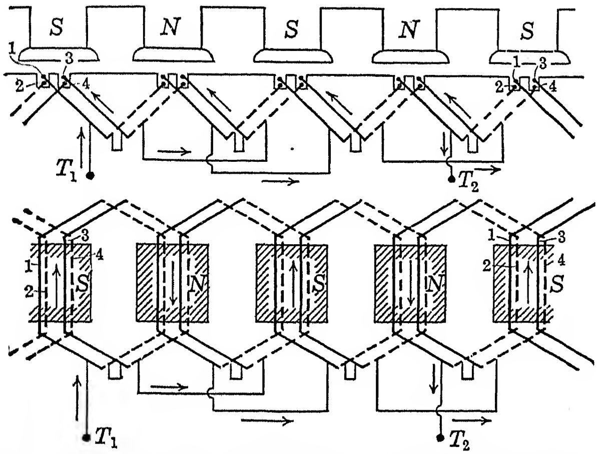


FIG. 144.—Barrel-type single-phase lap winding, 2 slots per pole.

105. Two-phase Full-pitch Lap Winding.—A 2-phase full-pitch lap winding may be obtained by placing on the armature two windings of the type shown in Fig. 144, spaced 90 electrical space degrees apart (see Figs. 129, 130, p. 147). Such a winding is shown in Fig. 145, in which there are 8 slots per pole, making 4 slots per pole per phase.

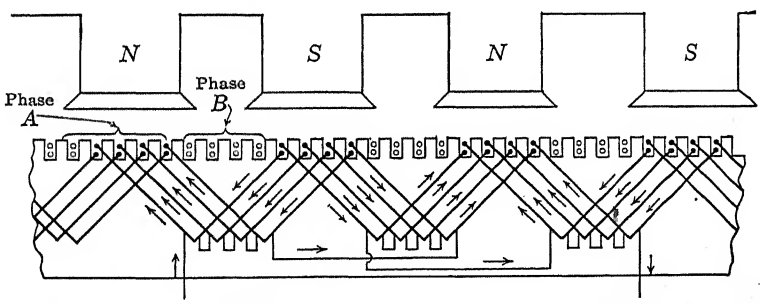


FIG. 145.—Two-phase full-pitch two-layer lap winding, 4 slots per pole per phase.

This is a full-pitch winding, the coil pitch being 8 slots, which is the number of armature slots per pole. The connections of phase B are omitted for the sake of clearness, as they are identical with those of phase A. Note that both coil sides in any one slot are always of the same phase, which is not the case with fractional-pitch windings. The reversed connection of the center phase belt should also be noted.

106. Three-phase Full-pitch Lap Winding.—A 3-phase full-pitch lap winding may be obtained by placing on the armature three windings of the type shown in Fig. 144, each winding being spaced 120 electrical space degrees from the two adjacent windings. A typical winding of this type is shown in Fig. 146, in which there are 12 slots per pole, or 4 slots per pole per phase. For each pole, therefore, there are 4 slots devoted to each phase. Since one pole represents 180 electrical space degrees, the slot pitch corresponds to, 180°/12, or 15 electrical space degrees. In the armature, Fig. 146, there are three phases, *A*, *B*, *C*; for clearness the connections of the *A*-phase only are shown. The connections of the *B*- and *C*-phases are identical with those of *A*. Since this is a full-pitch winding, the pitch of each coil must be 12 slots. For example, if the left-hand side of a coil lies in

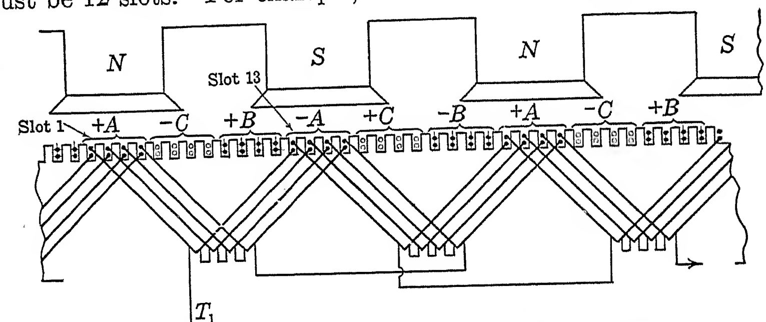


FIG. 146.—Three-phase full-pitch two-layer, lap winding.

the top of slot 1, the right-hand side must lie in the bottom of slot 13. The conductor belt $+A$ corresponds to the shaded side a of the coil a_1a, Fig. 110(a) (p. 128). The $+B$-belt, corresponding to the shaded side b of the coil, Fig. 110(a), must be displaced 120 electrical space degrees from the $+A$-belt. Since each slot corresponds to 15 electrical space degrees, the $+B$-belt must begin 120°/15°, or 8 slots, from the beginning of the $+A$-belt, as shown in Fig. 146.

Likewise, the $+C$-belt must begin 8 slots to the right of the beginning of the $+B$-belt. Note that the $-C$-belt, which is only 60° to the right of the $+A$-belt, corresponds to the c_1 side of the coil c_1c, Fig. 110(a), which is displaced 60° from the a side of coil a_1a. That is, in Fig. 146, coil sides $+A$, $+B$, $+C$ correspond to coil sides a, b, c, Fig. 110, and coil sides $-A$, $-B$, $-C$ correspond to coil sides a_1, b_1, c_1. Note that in this type of winding the two coil sides in any one slot are of the same phase, which is true of all such full-pitch windings.

107. Fractional-pitch Windings.—In a fractional-pitch winding the coil span is less than 180 electrical degrees. For example, in Fig. 147 is shown a five-sixths-pitch 3-phase winding. A coil, instead of

having a pitch of 12 slots, now has a pitch of 10 slots, so that its spread is no longer equal to a full pole pitch. Aside from the pitch, this winding is in every way similar to the winding shown in Fig. 146.

Note that the top layer, Fig. 147, is in every way identical to the top layer, Fig. 146. (The letters A, B, C with (+) and (−) signs denoting phase belts apply to the top layer only.) The bottom layer, Fig. 147, is similar to that shown in Fig. 146 but is slid two slots to the left. This results in there being in each phase belt only two slots containing conductors of the same phase.

The advantages of this type of winding are that it improves the wave form, there is an appreciable saving of copper in the coil ends, and the inductance of the winding is reduced because of the lesser mutual inductance between those conductors which lie in slots containing also conductors of either of the other two phases (see Fig.

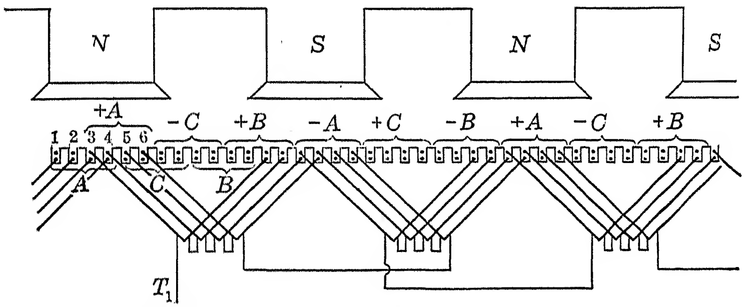


FIG. 147.—Three-phase five-sixths-pitch two-layer lap winding, 4 slots per pole per phase.

147). The coil-end inductance is also reduced because of the lesser length of coil ends. Such windings generate slightly less emf than full-pitch windings under the same conditions, since the two coil sides do not lie under corresponding parts of the poles at any given instant and hence the phase displacement of their emfs is slightly less than 180°. This is illustrated in Fig. 148, in which E_1 is the emf induced in the conductors comprising one side of a coil and E_2 is the emf induced in the conductors comprising the other side of the coil. E_1 is equal to E_2 numerically, as each is induced by the same number of conductors cutting the same flux at the same speed. Figure 148(a) gives the relation of the induced emfs E_1 and E_2 in the two coil sides when a full-pitch coil is used. When one side of a coil is under an N-pole, the other side is in a corresponding position under an S-pole. The induced emfs differ by 180° in phase, but the coil connection is such that these emfs add, their sum being E as shown in Fig. 148(a).

When a five-sixths pitch is used, the coil spread is equal to $\frac{5}{6}$ of 180°, or 150 electrical space degrees. The emfs E_1 and E_2 will differ

in phase by 30 electrical time degrees, as shown by the angle β, Fig. 149(b). The total emf E, which is their vector sum, is slightly less than when a full-pitch coil is used.

(a) Full pitch (b) Fractional pitch

FIG. 148.—Relation of coil-side emfs in full-pitch and fractional-pitch windings.

The ratio $E/(E_1+E_2)=E/2E_1$ is the pitch factor k_p.

A study of Fig. 148(b) shows that

$$k_p = \frac{E}{2E_1} = \frac{2E_1 \cos \beta/2}{2E_1} = \cos\frac{\beta}{2}. \tag{138}$$

For example, for five-sixths pitch, $\beta = 30°$,

$$k_p = \cos 15° = 0.966 \text{ (see p. 178).}$$

The pitch factors for the harmonics are considerably less than that for the fundamental so that the harmonics are reduced much more,

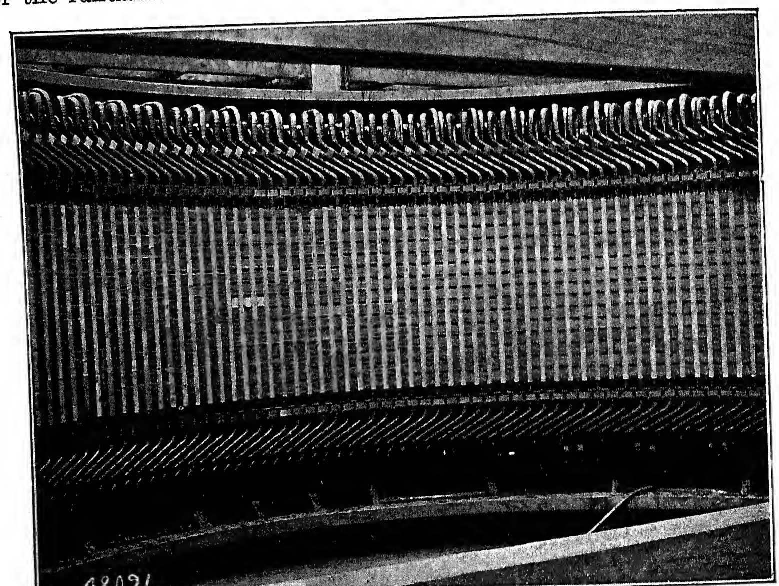


FIG. 149.—Showing winding and end connections of alternator armature.

proportionately, than the fundamental. For example, a two-thirds pitch will eliminate the third harmonic, a four-fifths pitch the fifth, etc. Hence, with a fractional-pitch winding the wave form is improved.

Note that, with the fractional-pitch winding of Fig. 147, only two of the slots of each phase under a pole contain coil sides of the same phase. In the other slots the two coil sides are of different phases. For example, slots 1 and 2 contain both phase A and phase B conductors; slots 3 and 4 contain phase A conductors only; slots 5 and 6 contain both phase A and phase C conductors. Of this group, slots 3 and 4 contain phase A conductors only. The fact that certain slots contain conductors of different phases reduces slightly the inductance of the winding, as has already been pointed out (p. 163).

In Fig. 149 is shown a portion of a finished lap winding of a water-wheel-driven alternator. The end connections, the binding down of

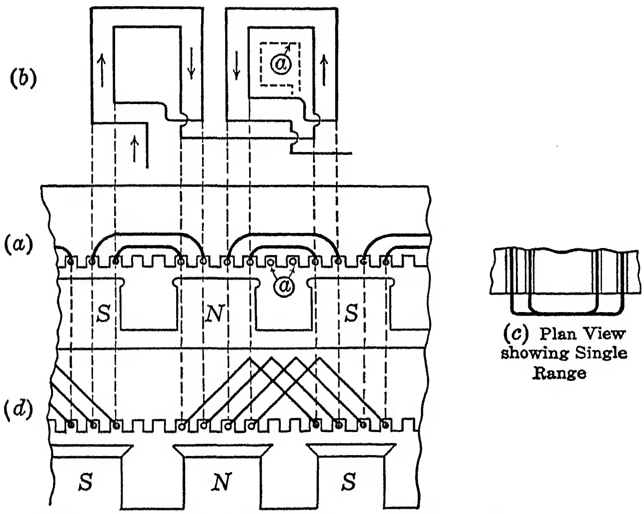


FIG. 150.—Single-phase single-range spiral winding and equivalent barrel-type winding.

the coil ends, the slot wedges, and the ventilating ducts are clearly shown.

108. Spiral and Chain Windings.—Instead of making the coils lap one another, the winding may be placed on the armature in the manner shown in Fig. 150(a). This is called a *spiral* winding, since the coils of each group are connected to form a spiral winding, as is shown in (b). Note that the coils themselves have a pitch of less than 180 electrical space degrees. Notwithstanding this lesser pitch, the winding is not considered as having the properties peculiar to a fractional-pitch winding. The slot conductors may be reconnected by barrel-type end connections, as shown in (d), without changing the electrical characteristics of the winding. This gives a full-pitch half-coil *barrel-type* winding. The differential action of the coil sides

of Fig. 150(a), owing to their not having a full pitch, is taken into consideration in (d) by the belt-factor constant (see Sec. 114, p. 176).

An inside coil, shown dotted, at a may be added to the winding under each pole, but it contributes so little to the induced emf, because of its small pitch, that to use it is wasteful. As the ends of the coils may be bent so that they all lie in a single vertical plane, Fig. 150(c), the winding in (a) is a *single-range* winding.

In Fig. 151 are shown spiral windings connected to form a 3-phase chain winding, in which there are 6 slots per pole, making 2 slots per pole per phase. This is a two-range winding, for the coil ends in order to pass one another must lie in two different planes perpendicular to the shaft. If the number of coil groups per phase is odd, which

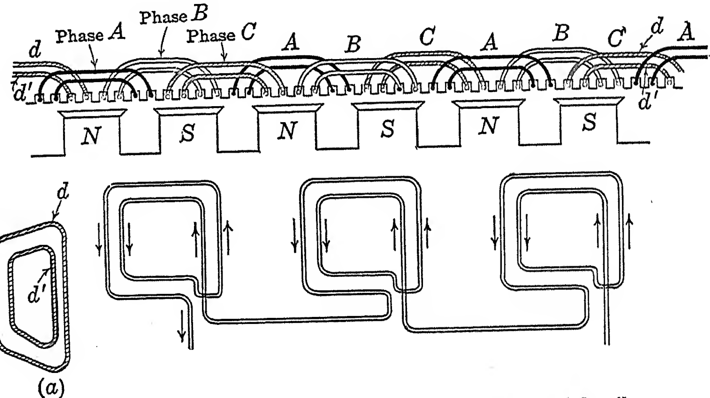


FIG. 151.—Three-phase chain winding, requiring special coils.

occurs if the number of poles is not a multiple of 4, coils having one long side and one short side, or trapezoidal in shape, such as coils dd' in Fig. 151(a), must be used in order that they may pass the coil ends of one phase such as A and complete the winding.

Since there are 6 slots per pole, the slot pitch represents 180/6, or 30, electrical space degrees. If the poles, Fig. 151, are assumed to move from left to right and the phase sequence is A-B-C, the left-hand sides of the coils of phase B must start 120/30, or 4, slots to the right of the left-hand sides of the coils of phase A, as shown in the figure. The coils of phase C have a similar relation to those of phase B.

The chief advantage of a chain winding is the considerable space between the coil ends, so that there is little opportunity for electrical breakdown at these points. They are admirably adapted, therefore, to high-voltage machines. Although coils of several different sizes must be kept in stock as spares, a coil may be replaced more easily than in the lap winding, where it is necessary to remove a large num-

ber of coils in order to replace a single coil. In the United States at present the chain winding is scarcely ever used, having been replaced by the lap winding.

ALTERNATOR CONSTRUCTION

109. Types of Alternators.—The general design and construction of alternators are roughly divided into three classes, depending on the type of prime mover. The direct-connected engine-driven (steam or internal-combustion) type must operate necessarily at low speeds. In order to obtain the desired uniformity in angular speed, considerable flywheel effect[1] (WR^2) is necessary. This may be obtained by the use of a separate flywheel or by incorporating sufficient flywheel effect in the rotating element of the alternator. The speeds of water-wheel alternators vary over a wide range, from near 60 to 500 rpm, the lower speeds being used at the lower heads. Both the engine-driven and the water-wheel-driven types of alternators have salient poles, Fig. 153 (p. 169).

Owing to the considerable interval that may elapse from the time a water-wheel alternator loses its load to the closing of the gates, such alternators are designed to operate safely at approximately twice the rated speed. The turbine-driven type of alternator (unless driven through a reduction gear as is sometimes done with small units) runs at very high speed (750 to 3,600 rpm). Because of high windage losses and high centrifugal stresses, the rotors are of the smooth-cylindrical type in which the field turns are embedded in slots, Fig. 160 (p. 175). Water-wheel-driven alternators are made in both vertical and horizontal types. Because of the much better mechanical arrangement of alternator above and water wheel beneath, the vertical type is by far the most common. However, Pelton-wheel-driven alternators are usually of the horizontal type.

In the early days turbine-driven alternators were made in the vertical type, but because of balance and vibration this type has been practically superseded by the horizontal type.

110. Stator or Armature.—The stator or stationary member of the alternator is almost always the armature, the field structure being the rotating member or rotor. When the machine is in operation, the armature iron is continuously cut by the flux of the rotating field and must be laminated in order to reduce eddy-current losses. In machines of small diameter, each lamination is usually a single circular punching.

In the larger sizes of rotating machinery the stator iron is built up

[1] In the United States, flywheel effect is expressed by WR^2, where W is the weight of the rotor in pounds and R its radius of gyration in feet.

of overlapping circular segments either dovetailed or bolted to the frame. In Fig. 152(*a*) is shown a segmented stator punching of the bolted type for a medium-speed machine, and in (*b*) is shown a segment of the dovetailed type for a turbine-driven alternator. The large depth of iron behind the slot should be noted. In (*c*) is shown a ventilating segment to be used with segments like that shown in (*b*) in order to secure ventilating ducts through the stator core, Fig. 155.

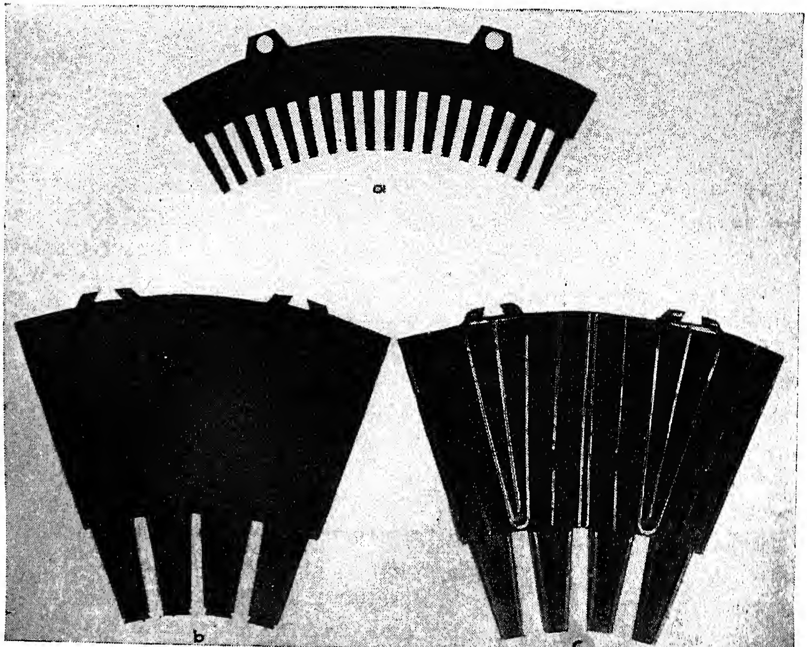


FIG. 152.—Segmented stator punchings: (*a*) slow speed; (*b*) turbine driven; (*c*) ventilating segment. (*Allis Chalmers Mfg. Co.*)

Frequently the laminations such as are shown in (*b*) are perforated to produce longitudinal air ducts (see Fig. 157).

Engine-driven alternators must rotate at comparatively low speeds and must have a large number of poles, and the armatures must be of comparatively large diameter. The pole pieces are made up of laminations riveted together and are dovetailed to the field spider, Fig. 153. The armature is built up of small overlapping segments, dovetailed to the frame of the machine in much the same manner as the armature of engine-driven direct-current generators is assembled (see Vol. I, Chap. XI) except that in the alternator the armature laminations are a part of the stationary member. Figure 153 shows the general construction of such an alternator. The frame itself may be a hollow box

casting, Fig. 153, or it may consist of fabricated steel plates between which the laminations are bolted, Fig. 154. Both constructions give

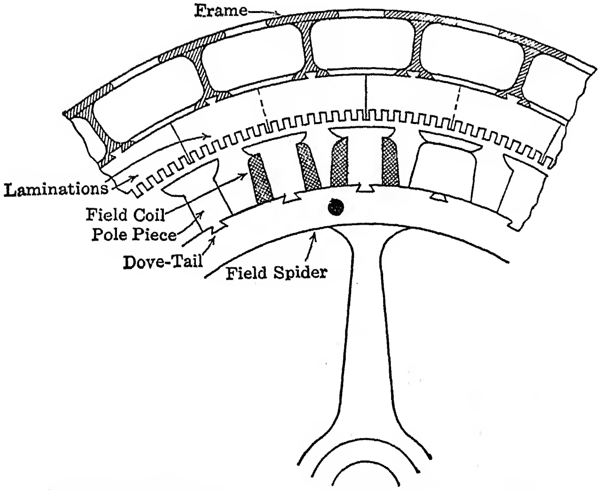


FIG. 153.—Cross section of engine-driven alternator.

the necessary mechanical stiffness with small weight, and with either frame there is ample opportunity for the discharge of the cooling air, which passes out through the cooling ducts.

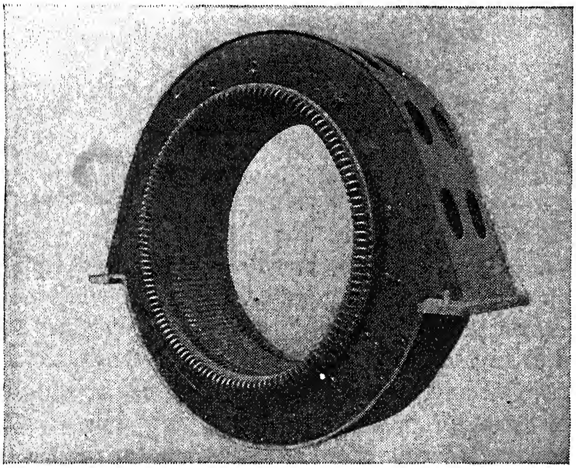


FIG. 154.—Completely wound stator of engine-driven alternator. (*Westinghouse Electric Corp.*)

In Fig. 155, is shown the stator of a high-speed turbine-driven alternator without the winding. Because of their low reactance the short-circuit current of turbine-driven alternators is abnormally large,

and the mechanical stresses due to these currents are extremely great. (Such electromagnetic stresses are proportional to the current *squared.*) Unless well supported, the coil ends are likely to be pulled out of position under short circuit. Braces for supporting such coil ends are shown in Fig. 155. Some of the bracing is removed to show the manner in which the spiral ventilating ducts are alternated so as to provide uniform temperature throughout the stator [also see Fig. 152(*c*)].

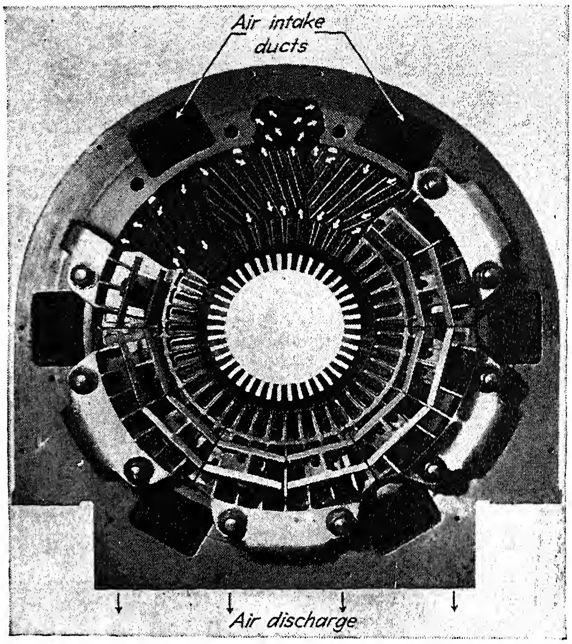


FIG. 155.—End view of turbogenerator stator showing manner in which the spiral ventilating ducts are alternated so as to provide uniform temperatures throughout the stator. (*Allis-Chalmers Mfg. Co.*)

111. Slots.—Two general types of slot are used for alternating-current machines, the open slot and the semiclosed slot. The open slot, shown in Fig. 156(*a*), is the more common, because the coils can be form-wound and insulated prior to being placed in the slots, giving the least expensive and most satisfactory method of winding.

The semiclosed, or overhung, type of slot, shown in Fig. 156(*b*), is often necessary, especially in induction motors. The larger area of tooth face reduces the air-gap reluctance and also reduces the tufting of the flux, which tends to produce ripples in the emf wave. It is usually necessary to place the conductors in the slot one at a time, which is expensive and uneconomical of slot space. It is also difficult to apply insulation.

In both types of slot, the conductors are usually held in the slot by fiber wedges, Fig. 156. The effect of the semiclosed slot may be obtained by the use of open slots and magnetic wedges. These wedges are only partly of iron, so that the slot is not entirely closed.

Coil insulation is divided into two general classes, A and B. Class A insulation, such as paper and cambric, is of organic material and when impregnated with varnishes or fillers has a limiting operating temperature of 100°C, as measured with an embedded detector (see Vol. I, Chap. XIV). Class B insulation, of which mica tapes and fiber glass (fabric woven with glass fibers) are examples, can operate at a limiting temperature of 120°C, as measured with an embedded detector. Until recently, low-temperature organic varnishes were used to bind the mica films and to impregnate the fiber glass, which is usually applied in the form of tape. Recently, new silicone varnishes

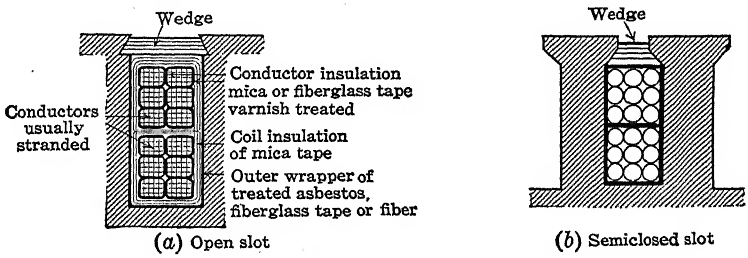


FIG. 156.—Cross sections of typical a-c machine slots.

have been developed that have very much higher operating temperatures. The limiting temperatures of mica and fiber-glass insulation employing this material are as yet uncertain. The slot in Fig. 156(a) is shown as being insulated with class B insulation, which is common for alternators, since the output increases with the operating temperature. If the mica and fiber glass are replaced with varnished cambric or cotton, class A insulation results.

The conductors or parts of conductors nearer the top of the slots have lesser self-inductance than those nearer the bottom of the slots (see pp. 171 and 332). Hence, the current tends to flow in the top portions of each conductor. To prevent such unequal distribution of current the large conductors in alternator armatures are stranded, as indicated in Fig. 156(a), and each strand is insulated with enamel. Each conductor is made up so that all strands occupy top, intermediate, and bottom positions for equal distances, thus equalizing their self-inductances.

In order to prevent corona formation in the voids between the conductors and the grounded laminations, such as would occur when

the voltage is high, the slot insulation is covered with a semiconducting paint, and the surfaces are connected to ground.

112. Ventilation.—The problem of ventilating slow-speed salient-pole alternators is not a difficult one. The length of embedded conductor is not large, the exposed dissipating surface is large, and the fan action of the salient poles provides circulating air. However, the output of turbine-driven alternators is so great for their size, the length of embedded conductor is so large, and so little ventilating action is obtained from the smooth cylindrical rotor that the problem of proper ventilation is a difficult one. Also, since ventilating ducts in the solid-steel rotor are impracticable, all ventilating gas must flow in axially through the air gap or through axial ducts in the stator laminations. In the stator, special ventilating ducts must be provided, Fig. 157 (also see Fig. 155). Totally enclosed systems of ventilation are now used. In addition to eliminating the accumulations of dirt

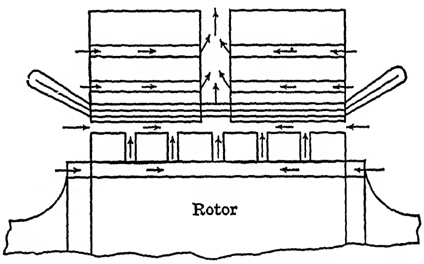


FIG. 157.—Passage of ventilating air through the ducts of a turboalternator.

in the ventilating system that would result if outside air were circulated, this method minimizes fire hazard by eliminating the available supply of oxygen. Means are also provided for releasing carbon dioxide if fire should start. The cooling medium, which may be either air or hydrogen, is cooled by passing over pipes through which cooling water is circulated; the cooling medium itself circulates continuously through the alternator ventilating system.

Nearly all the turbine-driven alternators and large-sized synchronous motors that are now being built are hydrogen-cooled. The primary reason for using hydrogen rather than air is to reduce windage loss to about one-tenth the value with air, which increases the efficiency 0.6 per cent or more at rated load, and to provide better cooling, which increases the rating about 20 per cent. Other advantages of hydrogen are the reduction of oxidation of the insulation and the reduction of fire hazard and of windage noise. The properties of hydrogen that make it advantageous as a cooling medium are that its density is only 7 per cent that of air, hence the decreased windage loss, and that

it has 7.5 times the thermal conductivity of air and for a given tem- perature difference will transfer 30 per cent more heat units from a given surface than air. Because of its explosive nature, it must be used in a gastight enclosure. Since synchronous condensers have no protruding shaft, they were the first type of machine to employ hydrogen cooling, Fig. 340 (p. 412). With alternators, a special oil seal in the bearings has been developed, and the loss of hydrogen by leakage is thus negligible. The danger of explosion is practically eliminated by maintaining the hydrogen pressure slightly above atmospheric. Also, the explosive range of hydrogen-air mixtures is between 5 and 75 per cent hydrogen, and the normal percentage under operating conditions is 95 to 98 per cent. This mixture also will not support combustion.

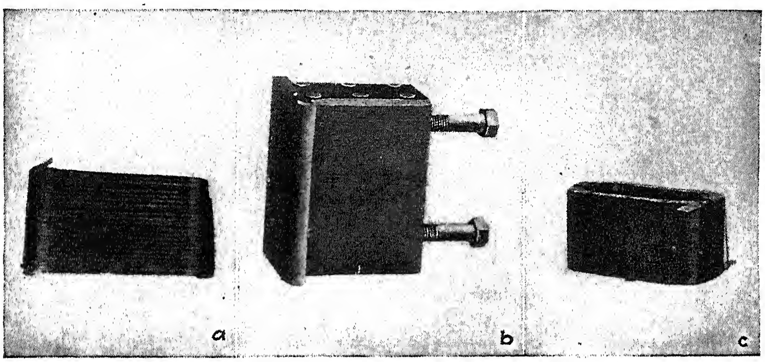


FIG. 158.—Salient-pole piece with two types of coil.

In Fig. 157 is shown in simplified form the longitudinal system of ventilation. The cooling air flows longitudinally or axially through the air gap and the perforations in the laminations and is discharged radially through the ventilating ducts. The paths of the cooling air or hydrogen for a radial ventilating system are indicated in Fig. 155.

113. Rotating-field Structure.—In order to reduce pole-face losses and at the same time to facilitate construction and mounting, the cores of practically all salient poles are made of laminations riveted together, Fig. 158. With slow-speed alternators, these are either dovetailed, Fig. 153, or bolted, Fig. 159, to the rotor spider. The spider may be of cast iron or steel, or it may be of fabricated steel construction, Fig. 159.

The field coils of the smaller capacity machines are usually wound with wire of rectangular section, cotton-covered, and thoroughly impregnated; with machines of larger ratings, edge-wise-wound strap field coils are used, and these are frequently insulated with high-tem-

perature bonded mica strip. Both types of coil are shown in Fig. 158.

In order to damp any pulsation or hunting, particularly when the alternator is driven by a reciprocating prime mover, cage dampers are built into the pole faces, as shown in Fig. 159 (see Sec. 226, p. 403).

The nonsalient-pole or cylindrical type of rotor is necessary for direct-driven turbine-driven alternators, which run at high speed. The rotor is a cylindrical solid-steel forging, Fig. 160, in which longitudinal slots for holding the field winding are milled. The rotor shown in Fig. 160 is for two poles. The narrow longitudinal slots cut along the pole faces are for purposes of dynamic balance. Were it not for

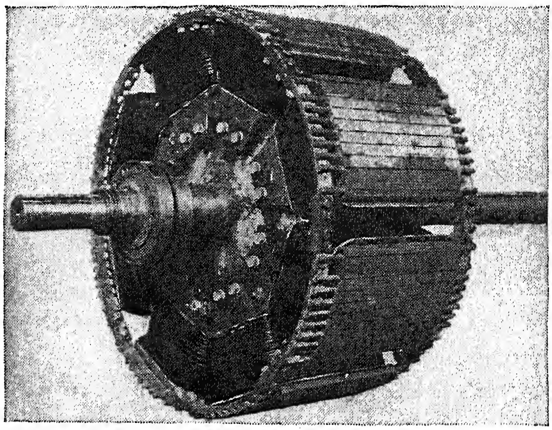


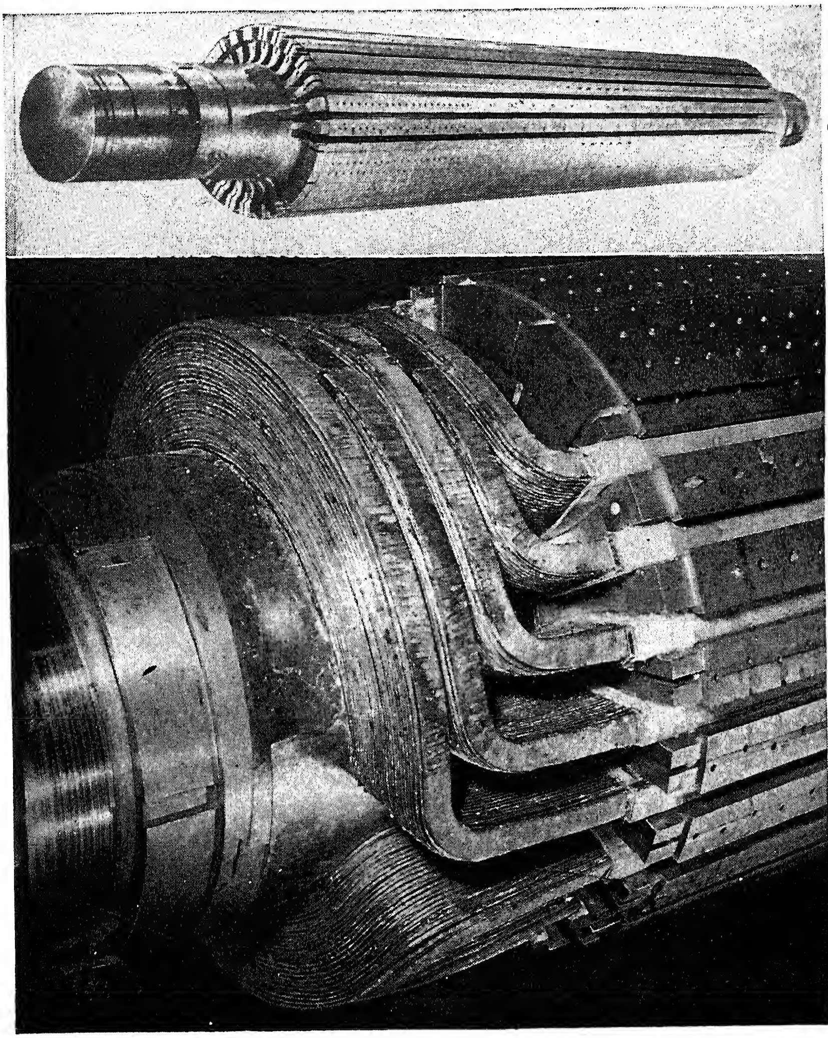
FIG. 159.—Salient-pole rotor with dampers in pole faces. (*Westinghouse Electric Corp.*)

such slots the rotor would be more resilient along the pole axis, because of the material removed for the slots, than along an axis at right angles to the pole axis. The two different degrees of resiliency would produce vibration.

The rotor is wound with strip copper. Figure 160(*b*) is a close-up view of one end of the rotor and the field winding. The copper end connections must be supported by metallic end flanges to resist centrifugal force. In order to minimize windage losses, which tend to be high at turbine speeds, the surfaces of the finished rotor are made as smooth as possible.

Excitation.—The excitation voltage is usually 120 or 250 volts and in the larger stations is supplied by an individual exciter driven directly or through a gear reduction or by a motor or is supplied by bus bars devoted to excitation only. The excitation bus is usually supplied by a

motor-generator set, which takes its energy from the main station bus. In smaller installations, the exciter is mounted directly on the alternator



(a)

(b)

(*a*) Before winding.
(*b*) Details of end connections.
Fig. 160.—Two-pole rotor for 43,750-kva 13,800-volt 80 per cent power factor 3,600-rpm turbine-driven alternator. (*Allis Chalmers Mfg. Co.*)

shaft or else is belt-driven from the alternator shaft. Large central stations usually have a storage battery floating on the exciter bus and, in addition, may have steam-driven exciters for emergencies.

ALTERNATOR ELECTROMOTIVE FORCES AND OUTPUTS

114. Induced Electromotive Force.—Figure 161(*a*) shows the magnetic flux between the armature surface and a north and a south pole of an alternator. Assume that the flux distribution is sinusoidal, Fig. 161(*b*), the flux density being a maximum under the center of the pole. Let B' be the average value of the flux density. B' is equal to $2/\pi$ times the maximum value B (see p. 13). Let a be a conductor cutting this flux with a velocity of v cm per sec. Assume this conductor a to have a length l cm perpendicular to the plane of the paper.

The maximum emf induced in conductor a occurs when it is

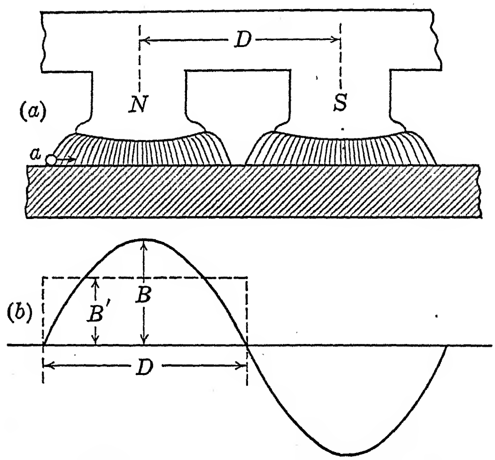


FIG. 161.—Generation of alternating emf.

directly under the center of the pole in the maximum flux density B (See Vol I, Chap. XII). That is, in the cgs system

$$e_m = Blv10^{-8} \text{ volts.} \tag{I}$$

Let D be the pole pitch in centimeters and f the frequency in cycles per second.

The time in seconds necessary for the conductor a to move the distance D is $1/2f$ sec. Therefore,

$$v = \frac{D}{1/2f} = 2fD \text{ cm per sec.} \tag{II}$$

The total flux cut per pole is

$$\phi = B'lD = \frac{2}{\pi} BlD \text{ maxwells,}$$

$$B = \frac{\pi\phi}{2lD} \text{ gausses.} \tag{III}$$

The rms emf is $1/\sqrt{2}$ times the maximum for a sine wave. The *rms* induced volts per conductor, by substituting (II) and (III) in (I), is

$$E_c = \frac{e_m}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\pi\phi}{2lD}\right) l(2fD)10^{-8} \text{ volts.}$$

If there are Z conductors in series per phase, the rms emf per phase is

$$E = 2.22Z\phi f 10^{-8} \text{ volts} \tag{139}$$

[2.22 = 2 times 1.11, the form factor for a sine wave (see p. 13)].

If the emf wave is not a sine wave, the form factor should be correspondingly changed.

If the mks[1] system is used, (139) becomes

$$E = 2.22Z\Phi f \text{ volts,} \tag{140}$$

where Φ is the flux in webers (1 weber = 10^8 maxwells).

Owing to the fact that the emfs in the different coils of a phase belt are not in time phase with one another, Fig. 164, the conductor emfs do not add algebraically. A factor k_b, therefore, called the *breadth factor* or *belt factor*, must be introduced to correct for this relative phase displacement. This factor is unity for a concentrated winding and less than unity for a distributed winding. Its value is readily determined.

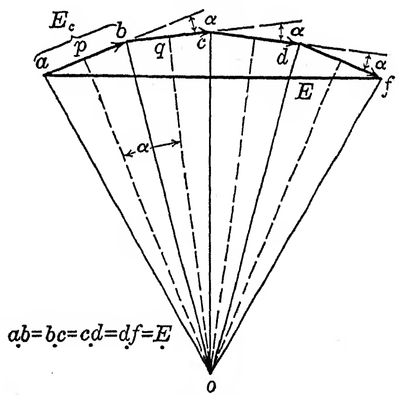


FIG. 162.—Determination of breadth or belt factor.

In Fig. 162 let E_c be the emf per coil side and n the number of slots per pole per phase or the number of coil sides per phase belt. ($n = 4$ in Fig. 162.) If the electrical angle between slots is $\alpha°$, the resultant emf E is found by vector addition of the coil-side emfs ab, bc, cd, df.

Draw perpendiculars at the centers p, q, etc., of the vectors ab, bc, etc.

These perpendiculars will intersect at o. Draw radii oa, ob, etc. $\angle poq = \alpha$; $\angle pob = \alpha/2$. $E_c = 2oa \sin(\alpha/2)$.

$$E = 2\left(oa \sin\frac{n\alpha}{2}\right).$$

$$k_b = \frac{E}{nE_c} = \frac{\sin(n\alpha/2)}{n \sin(\alpha/2)}. \tag{141}$$

[1] Meter-kilogram-second system. See Vol. I, 4th ed.

Example.—Determine k_b for a 3-phase winding in which there are 12 slots per pole.

$$n = 4, \alpha = \frac{180°}{12} = 15°.$$

$$k_b = \frac{\sin \frac{4 \cdot 15}{2}}{4 \sin (15°/2)} = \frac{\sin 30°}{4 \sin 7.5°} = \frac{0.5}{0.522} = 0.958. \quad \textit{Ans.}$$

The table gives values of k_b for a few typical windings.

VALUES OF BREADTH FACTOR k_b

Slots per pole per phase	Single-phase	2-phase	3-phase
1	1.000	1.000	1.000
2	0.707	0.924	0.966
3	0.667	0.910	0.960
4	0.653	0.907	0.958

If fractional pitch is used, the emfs in the two coil sides are out of phase, as shown in Fig. 148(*b*) (p. 164). This again reduces the emf. Correction for this may be made by multiplying the voltage equation by k_p, the *pitch factor*. Equation (138) (p. 164) may be written as follows:

$$k_p = \cos \frac{180°(1 - p)}{2}, \tag{142}$$

where p is the pitch, expressed as a fraction.

For example, with five-sixths pitch,

$$k_p = \cos \frac{180°(1 - 5/6)}{2}$$
$$= \cos 15° = 0.966.$$

VALUES OF PITCH FACTOR k_p

Pitch	9/10	6/7	5/6	4/5	3/4	2/3
k_p	0.988	0.974	0.966	0.951	0.924	0.866

Inserting k_b and k_p in (139), (140), gives the complete emf equations

$$E = 2.22 k_b k_p Z \phi f 10^{-8} \text{ volts.} \tag{143}$$
$$E = 2.22 k_b k_p Z \Phi f \text{ volts.} \tag{144}$$

Example.—A 6-pole 3-phase 60-cycle alternator has 12 slots per pole and four conductors per slot. The winding is five-sixths pitch. There are 2,500,000 maxwells (= 0.025 weber) entering the armature from each north pole, and this flux is sinusoidally distributed along the air gap. The armature coils are all con-

nected in series. The winding is Y-connected. Determine the open-circuit emf of the alternator.

The total number of slots is 72.

The series conductors per phase, therefore, are

$$Z = \frac{4 \cdot 72}{3} = 96.$$

Slots per pole per phase $= 72/(6 \cdot 3) = 4$. k_b (from table) $= 0.958$. $k_p = 0.966$.

The total induced emf per phase is

$$E = 2.22 \cdot 0.958 \cdot 0.966 \cdot 96 \cdot 2{,}500{,}000 \cdot 60 \cdot 10^{-8} = 296 \text{ volts.}$$

As the winding is Y-connected, the terminal voltage is

$$296\sqrt{3} = 513 \text{ volts.} \quad \textit{Ans.}$$

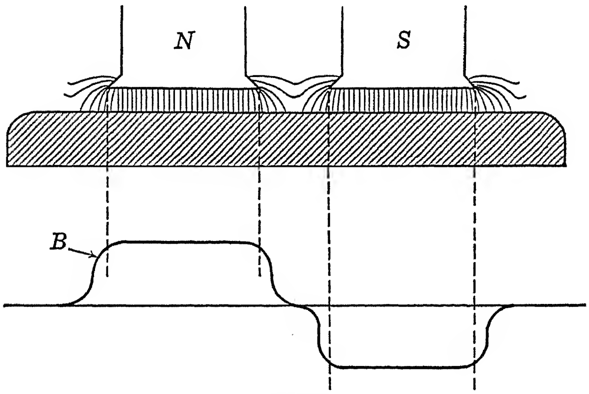


FIG. 163.—Flux density in air gap of salient-pole machine.

115. Wave Shape.—Ordinarily, the flux distribution in a generator is not sinusoidal, especially with salient-pole machines, the flux wave at no-load being flat-topped, as shown in Fig. 163. The emf wave *per conductor* has the same shape as the flux-density curve, B. This follows from the fact that the induced emf $e = Blv10^{-8}$ volts; at constant frequency v is constant, hence e is proportional to B (also see Vol. I, Chap. XII). If the coil is a full-pitch coil, the emfs in the two sides of each coil will be 180° out of phase in space but electrically in phase, and of the same magnitude, as at any instant these coil sides both lie under corresponding parts of opposite poles. Therefore the emf wave induced in each coil will have the same shape as the emf induced in each coil side. If but one slot per pole per phase is used, the resulting emf wave will have the same shape as the flux-density curve, which may be flat-topped, as shown in Fig. 163.

Figure 164(a) shows a phase belt, consisting of four coils, of a 3-phase alternator having 12 slots per pole or 4 slots per pole per phase. The shape of the emf wave for each of the four full-pitch coils forming

one phase of the winding is the same as the shape of the flux-density curve, Fig. 164(b), at No. 1, 2, 3, 4. As 12 slots represent 180 electrical space degrees, 180/12, or 15, is the interval in electrical space degrees between successive slots. The four emfs, therefore, are 15 electrical time degrees apart, as shown in Fig. 164(b). As the coils are connected in series, the resultant emf is found by adding the ordinates of the four waves. The resultant emf wave, E, instead of being flat-topped, like the emf wave of the individual coil, is very nearly a sine wave. This is the reason why a distributed winding gives a better wave shape than a concentrated winding.

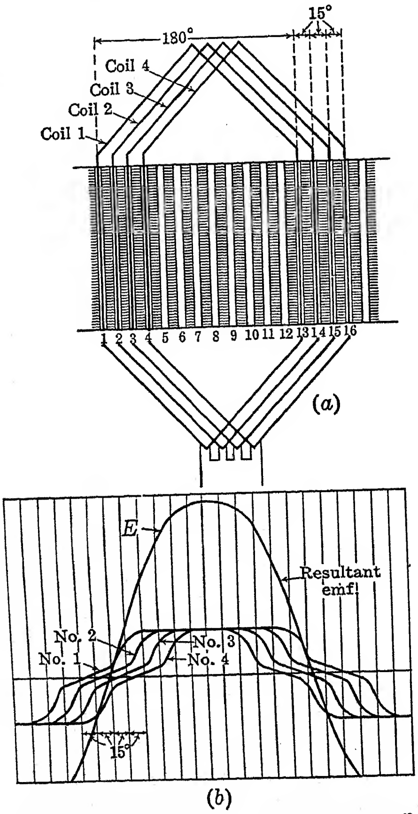


FIG. 164.—Resultant emf wave in four-coil phase belt.

The approach to a sine curve of the resultant emf wave may also be considered as due to a much greater proportionate reduction in the harmonics, which are substantial in the coil-side emf waves No. 1, 2, 3, 4, Fig. 164(b). The angle between adjacent coil sides for the fundamental is 15°, but the angle α_3 for the third harmonic will be $3 \cdot 15°$, or 45°; for the fifth the angle α_5 will be $5 \cdot 15°$, or 75° (see Fig. 164). Hence the breadth factor for the harmonics is very much less than for the fundamental, so that they are materially reduced in the resultant emf wave.

With a fractional-pitch winding, the emf in *each coil side,* Fig. 148 (p. 164), must first be added graphically to obtain the coil emf. The coil emfs are then added as in Fig. 164(b) to obtain the belt emf. As a result, the emf wave form with fractional pitch is more nearly sinusoidal than with full pitch.[1]

[1] For a more complete discussion of wave form, see R. R. LAWRENCE, "Principles of Alternating Currents" and "Principles of Alternating-current Machinery"; also, "Standard Handbook," 7th ed., Sec. 7.

(In Fig. 148, $\beta_3 = 3\beta$, $\beta_5 = 5\beta$, etc., where β_3, β_5 are the angles for the third and fifth harmonics. Hence a fractional-pitch winding will make a greater proportionate reduction in the harmonics than in the fundamental.)

116. Magnetomotive Force of Distributed Field Windings.—The pole winding for a nonsalient-pole rotor such as is used with turbine-driven alternators is usually of the form shown in Fig. 165(*a*), although the number of slots per pole is frequently greater than the six shown in the figure (see Fig. 160). This type of winding gives a flux-density curve that much more nearly approaches a sine wave than does that of the salient-pole rotor with a uniform air gap, Fig. 163. Consider

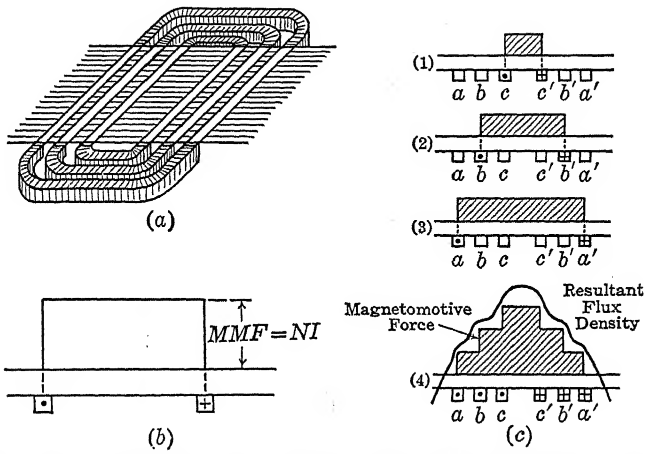


FIG. 165.—Distributed field winding and resulting mmf and flux waves.

Fig. 165(*b*), which shows the section of a single coil embedded in the surface of a field pole. If the current is considered as concentrated at the centers of the conductors, the mmf of the coil is a rectangle, its height being equal to the ampere-turns of the coil.

Figure 165(*c*) shows cross-sectional views of the coils in (*a*), consisting of *aa′*, *bb′*, and *cc′*. The mmf of coil *cc′* acting alone is shown in (1); that of coil *bb′* acting alone is shown in (2); that of coil *aa′* acting alone is shown in (3). In (4) all three mmfs are active, and they are combined to form the resultant mmf wave, which is "stepped." Actually, the current is not concentrated at the centers of the conductors as assumed so that the mmf rectangles are actually trapezoids. Also, the resulting flux will fringe at the tooth tips. Both effects cause the flux-density curve to be much smoother than the stepped mmf curve indicated in (4). Also, with the large number of slots per pole such as occurs in practice the effect of the "steps" on the flux-density curve is hardly noticeable.

Hence the induced emf wave in each armature conductor will be nearly sinusoidal. Any irregularities that do occur in the coil emfs will be almost entirely eliminated when the emfs of the belt are combined, as in Fig. 164(b).

Thus a field winding may be distributed in the same manner as an armature winding [compare with Fig. 150(a), p. 165]. As a matter of fact, the armature current itself produces mmf waves similar to those shown in Fig. 165(c), although their amplitudes vary with the time. This constitutes the *armature reaction* of the alternator[1] (see Sec. 126, p. 194). Moreover, if, in the usual 3-phase armature winding, direct current flows between any two terminals or between any one terminal and the other two connected together, north and south poles for which the flux distribution is nearly sinusoidal will be produced in the air gap. Some distributed field windings therefore are wound in the same manner as 3-phase armatures.

117. Phasing Alternator Windings.—Three-phase alternator windings may be connected in either Y or delta. However, owing to the fact that, with the delta connection, third-harmonic voltages and multiples thereof are short-circuited in the winding, and also no neutral connection is available, the Y-connection is used almost universally with alternators. In practice, instances often occur where six leads come from the machine, these leads being the three pairs of terminals from the three phases. The proper phase relations must be observed in making the connections, whether they are to be in Y or delta.

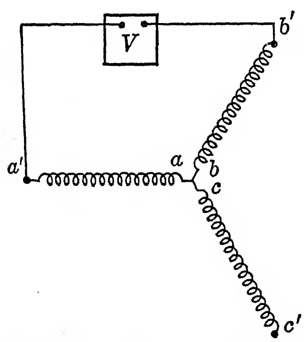


FIG. 166.—Connecting alternator coils in Y.

Let aa', bb', cc', Fig. 166, be the three coil windings of a 3-phase alternator.

Assume that these three windings are to be connected in Y. First, connect ends a and b together. Measure $E_{a'b'}$, the emf across their open ends. This should equal $\sqrt{3}$ times the coil emf. It may be equal to the coil emf, in which case one coil should be reversed. Next, connect the end c of coil cc' to point ab. The emfs $E_{b'c'}$ and $E_{a'c'}$ should each be $\sqrt{3}$ times the coil emf. If not, the coil cc' should be reversed.

If it is desired to connect the coils in delta, the ends a and b', Fig. 167, should first be connected. The emf $E_{a'b}$ across their open ends

[1] See Vol. I, Chap. XII, the section on Armature Reaction in Multipolar Machines.

should be equal to the coil emf. If not, one of these two coils should be reversed. End c' of coil cc' should then be connected to b. The emf $E_{ca'}$ across the open ends should be zero, as shown by the vector diagram in (b) (see Sec. 92, p. 134). If this emf is practically zero, the two ends c and a' may be closed. The emf $E_{ca'}$ may be twice the coil emf, as shown in (c). If this is the case, coil cc' should be reversed.

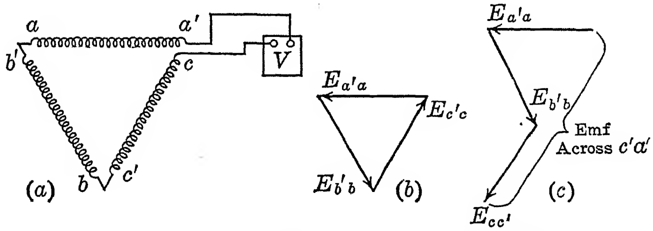


FIG. 167.—Connecting alternator coils in delta.

118. Rating of Alternators.—The rating of electric machinery is determined, in general, by its temperature rise. This temperature rise is caused by the losses in the machine. The I^2R-loss in the armature, due to the load current, limits the output of a machine. This loss depends on the value of the armature current and is independent of power factor. For example, 100 amp in a single-phase 200-volt generator will produce the same I^2R-loss if the load power factor be unity, 0.4, or any other value. The output in *kilowatts*, however, is proportional to the power factor. If the above generator is limited to 100 amp, its output will be 20 kw at unity power factor but only 8 kw at 0.4 power factor. The rating is 20 kva regardless of power factor.

For the foregoing reasons alternators are ordinarily rated in *kva.* If a machine is rated in kilowatts, unity power factor is assumed unless otherwise specified. In stating the output of a machine, it is always well to state the power factor.

The rating of the prime mover driving an alternator is determined entirely by the *kilowatt* load. The same turbine could be used to drive a 200-kva alternator operating at 0.5 power factor or a 100-kva alternator operating at unity power factor, although the first alternator would have double the kva rating of the second.

CHAPTER VII

ALTERNATOR REGULATION AND OPERATION

119. Alternator Regulation.—It is shown in Vol. I (Chap. XII) that the terminal voltage of a shunt generator drops as load is applied. This is due to three causes—the I_aR_a-drop in the armature, armature reaction, and the drop in field current that results from the decrease in terminal volts. As commercial alternators are excited from a separate source, there is no decrease of field current due to the drop in the alternator terminal voltage. Both the I_aR_a-drop in the alternator armature and armature reaction, however, ordinarily cause a drop of terminal voltage as load is applied. Another factor that causes the alternator voltage to drop with application of load is the *leakage reactance* of the alternator armature. This will be discussed later.

The regulation of direct-current generators is inherently better than the regulation of alternators. For example, shunt generators of commercial size regulate very closely, and it is usually possible to compound a shunt generator so that its terminal voltage is practically constant at all loads. In the alternator, the armature leakage-reactance drop, which is not present in the direct-current generator, and the greater effect of armature reaction together result in poorer regulation. In addition, alternators cannot be compounded readily.

The regulation of the alternator depends not only on the magnitude of the current but on the power factor as well. A knowledge of the regulation of an alternator at various power factors is usually essential, since the amount by which the voltage varies with the load has an important bearing on the operation of the system as a whole. If the alternator supplies incandescent lamps, it must regulate very closely, or else special regulators are necessary on the lighting circuits. Alternators, moreover, may regulate well at unity power factor, while at low power factors the regulation may be very poor, even if the *current* be the same in the two cases.

In the larger types of alternator, the large values of current that result from short circuit may cause serious damage to the machine and to the system. The value of this short-circuit current is closely related to the regulation of the alternator, so that a knowledge of the regulation is helpful in designing the circuit breakers, switches, power-limiting reactances, etc. Furthermore, the loads at which power

systems become unstable, that is, drop their load entirely or pull out of synchronism, is determined in part by the regulation characteristics of the alternators. Hence, engineers investigate carefully these characteristics in selecting the alternators that are best adapted to a power-system project (see p. 456).

The excitation power and the rating of the exciter also depend on the regulation. These are also important.

It is very desirable, therefore, to understand the factors and the reactions that affect the regulation and the operation of alternators. As it is usually impossible to obtain the requisite loads for testing an alternator under actual load conditions, it becomes necessary, in determining the regulation, to employ methods that do not require actual loading. These methods will be described later.

120. Armature Leakage Reactance.—When current flows in the conductors of an alternator armature, it produces magnetic flux which links these conductors.

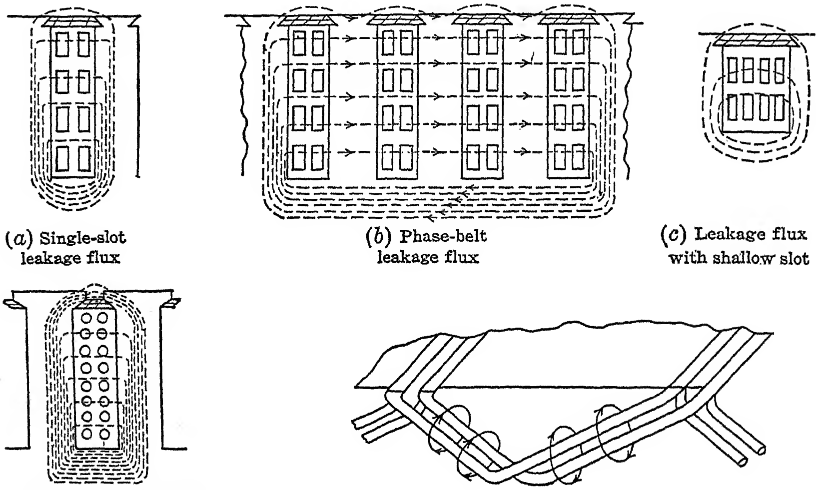
The magnetic leakage flux linking with the current gives inductance to the armature conductors. This inductance when multiplied by 2π times the frequency gives the reactance of the conductors. Alternating current in the conductors, therefore, encounters not only resistance but reactance as well. In modern alternators, the conductors are embedded in slots; and since the iron surrounding the slots is a path of low reluctance, the leakage flux is relatively large. Therefore, armature conductors have considerable self-inductance. In Fig. 168(*a*) is shown the slot leakage flux with a single slot. The path of the flux is almost directly across the slot and around through the iron behind the slot. The reluctance of this local magnetic circuit lies almost entirely in the slot itself, as the reluctance of that part of the path which lies in the iron is practically negligible. In (*b*) is shown the slot leakage flux in a phase belt. The magnetic lines go transversely through all the slots and complete their circuit through the iron behind the slots. A deep, narrow slot, such as is shown in (*a*) and (*b*), has lower reluctance than a shallow and wider slot, such as is shown in (*c*), so that the flux per ampere conductor will be greater.

However, such shallow slots are seldom used since, with the reduced slot section, the maximum amount of copper cannot be applied to the armature.

In (*d*) the leakage flux of a single semiclosed slot is shown. Owing to the low reluctance at the overhanging tooth tips, the slot leakage flux per ampere conductor is much greater than in the open slot, such as is shown in (*a*) and (*b*), other conditions being the same. It is to be noted that the conductors nearer the bottom of the slot, which are

linked by all the flux crossing the slot above them, have greater flux linkages and hence higher self-inductance than the conductors nearer the top of the slot. Hence, in such armature conductors the current density is greatest at the top of the conductor. It is for this reason that, in the larger alternators, the armature conductors are composed of insulated, transposed strands.

In addition to the foregoing slot leakage flux, there is additional leakage flux about the coil ends, as is indicated in (*e*). Whereas, in the slot, leakage may be 10 maxwells per amp-in. of conductor in the slot, the coil-end leakage may be 1 or 2 maxwells per amp-in.



(*a*) Single-slot leakage flux

(*b*) Phase-belt leakage flux

(*c*) Leakage flux with shallow slot

(*d*) Leakage flux with semi-closed slot

(*e*) Coil-end leakage flux

FIG. 168.—Slot and coil-end leakage flux.

It is pointed out in Vol. I that inductance varies as the *square* of the number of turns. This same law applies to the conductors in alternator slots. If the number of series conductors in a slot is *doubled*, the leakage reactance per slot is four times as great, other conditions remaining unchanged.

As the leakage reactance is proportional to the frequency

$$(X = 2\pi f L),$$

the leakage reactance of a 25-cycle alternator will be considerably less than that of a 60-cycle alternator, other conditions being the same.

121. Armature Resistance.—The armature iron forms a considerable portion of the path of the flux that links the armature conductors, Fig. 168. Since this flux is alternating, it is accompanied by hysteresis

and eddy-current losses, which occur in the iron immediately surrounding the slots. As this flux is produced by the armature current, the power represented by this loss must be supplied by the *armature current.* The eddy-current loss varies as the square of the flux density, and the hysteresis loss varies as the 1.6 power of the flux density. As the leakage flux is nearly proportional to the current, the eddy-current loss varies as the square of the current and the hysteresis loss as the 1.6 power of the current, practically. The combined loss varies nearly as the square of the current.

The effect of these local iron losses is to increase the total loss due to the flow of current through the armature. As these local losses vary nearly as the current squared, their effect is practically the same as if the resistance of the armature were increased (see Sec. 31, p. 55).

Unless the armature conductors are small in cross section, the effect of the slot leakage flux is to force the current toward the top of the slot, so that the current density in the portions of a conductor near the top of the slot is greater than in those portions near the bottom of the slot. This also increases the effective resistance of the armature.

The effective resistance of an armature, therefore, is greater for alternating than for direct current, owing to the alternating flux that accompanies the alternating current. The percentage increase depends, to a large extent, on the shape of the slots and teeth and on the size of the conductors and ranges from 20 to 60 per cent. As the armature resistance drop is very small as compared with the voltage drops due to armature leakage reactance and armature reaction, considerable error in determining the resistance introduces little error in most computations. The effective armature resistance may be determined by measuring the change in input with and without current flowing in the armature (see Sec. 141, p. 229). A more common, though less accurate, method is to measure the ohmic resistance with direct current and to increase this value by an estimated factor, such as 40 per cent, to cover the indeterminate losses.

SINGLE-PHASE ARMATURE REACTION

122. Current and Electromotive Force in Phase.—In direct-current machines, the armature ampere-turns act on the magnetic circuit of the machine in such a way as to distort the air-gap flux and to change its magnitude. For a given armature current, the direction and magnitude of this armature reaction depend on the position of the brushes (see Vol. I, Chap. XII). In an alternator, somewhat similar conditions exist. For a given armature current, the magnitude and direction of the armature reaction cannot depend on brush position

but do depend on the phase relation existing between current and voltage and, hence, on the power factor of the load.

Figure 169 shows the paths of the magnetic flux in the poles and armature of a single-phase multipolar salient-pole alternator at no-load. The armature moves from left to right. At the instant shown,

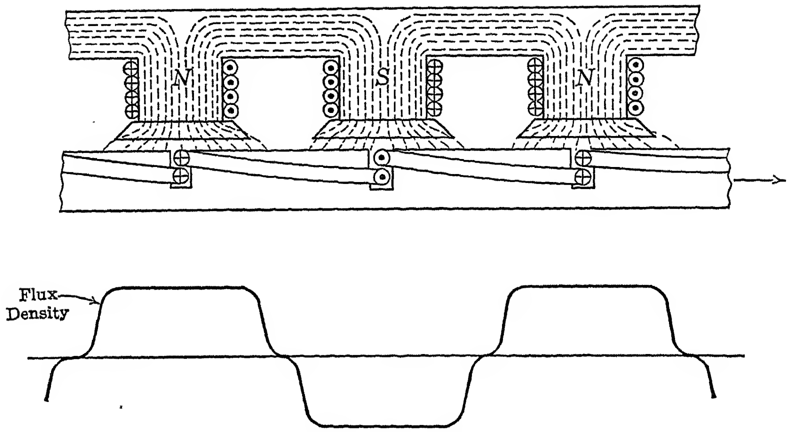


FIG. 169.—Flux distribution at no-load.

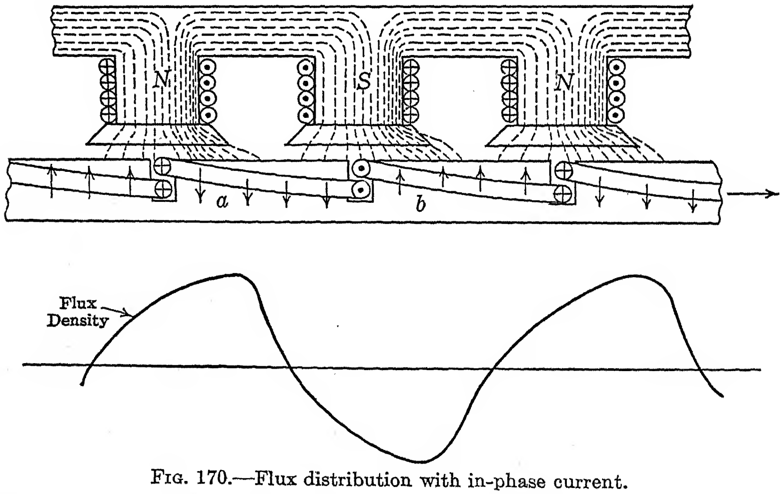


FIG. 170.—Flux distribution with in-phase current.

the coil sides are directly under the pole centers, and the induced emfs must have their maximum values. Since the armature current is zero, the armature can have no effect on the flux distribution. Hence, the flux distribution in (*a*) is determined entirely by the mmf of the field coils. The flux-density curve is symmetrical and is usually flat-

topped, similar to that of a d-c generator (see Vol. I, Chap. XII, Armature Reaction in Multipolar Machines).

If the armature circuit be closed, the armature will deliver current. If this current is in phase with the no-load induced emf, or excitation voltage, the power factor at the alternator terminals will be somewhat less than unity. Under these conditions the current will have its maximum value when the coil sides are directly under the centers of the poles, Fig. 170. The direction of the current will be inward in the conductors that lie under the N-poles. In coil a, the direction of the current is such that its mmf acts downward, as shown. On the other hand, the direction of the current in coil b is such that its mmf acts upward. The effect of the current in these coils on the main magnetic circuit is shown by the flux-density curve. The flux is increased on the right-hand side of each pole and decreased on the left-hand side. Were there no effect of saturation, the total flux would not be changed, as the increase on one side of the pole would be balanced by the decrease on the other side. This occurs also in direct-current generators when the brushes are in the geometrical neutral (see Vol. I, Chap. XII), when cross magnetization alone results.

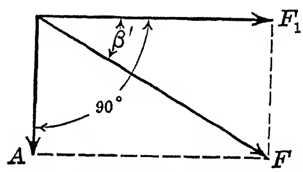


Fig. 171.—Mmf vector diagram—in-phase current.

The mmf vector diagram is shown in Fig. 171. The field mmf is represented by the vector F_1, the armature mmf vector A is at right angles to F_1, and the resultant mmf is given by the vector F displaced from F_1 by the angle β' in a clockwise direction. This vector diagram is identical with that for a d-c generator with the brushes in the geometrical neutral (Vol. I, Chap. XII).

Under the conditions of Fig. 170 the mmfs of the armature coils are acting principally on the interpolar space, whose reluctance is high. When the coils are in this position, therefore, the effect of the coil ampere-turns upon the magnetic flux of the alternator is a minimum. This does not apply to a nonsalient-pole alternator where the air gap is substantially uniform.

123. Current in Quadrature Lagging.—Figure 172 shows the conditions when the current lags the no-load emf by 90°. When the coil is in position (1), Fig. 172(a), the emf is a maximum, as in Fig. 170. The current is zero at this instant because it lags the induced emf by 90°. The current does not reach its maximum value until the coil has traveled 90 electrical space degrees farther and has reached position (2). The coil then lies directly under an S-pole. It will be noted that the mmf of this coil is *downward* and is, therefore, in direct opposition

to the magnetic flux entering the south pole, as shown in (*b*). *Therefore, when the current lags the no-load emf by 90°, its mmf acts in direct opposition to the main field.* As a result, the field is weakened by a lagging current, and this is accompanied by a reduction of the induced emf.

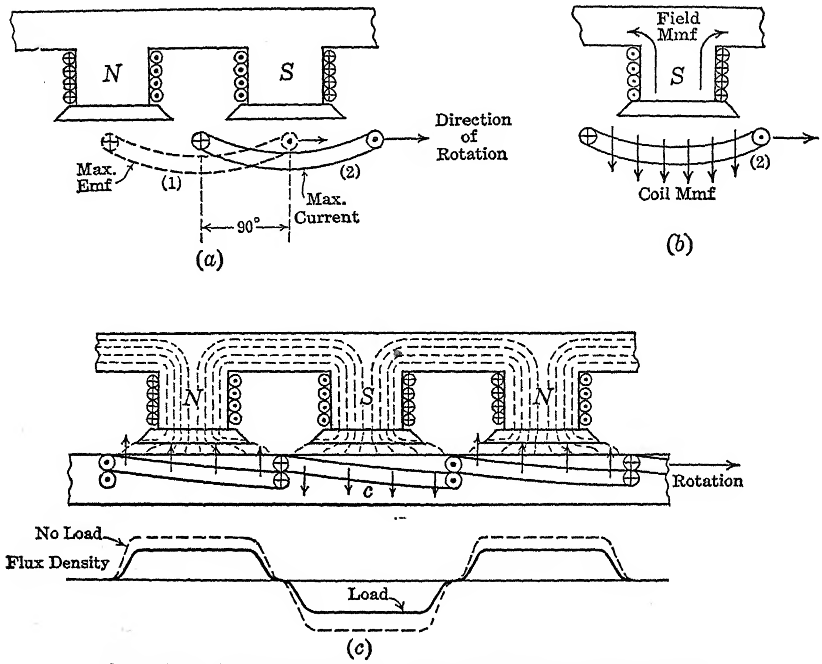


FIG. 172.—Armature reaction—current lags no-load emf by 90°.

This result is similar to the effect of moving the brushes forward 90° in a d-c generator. All the armature ampere-turns are then demagnetizing, weakening the field.

When the current is a maximum in the coils, their mmfs are acting directly on the field poles rather than on the interpolar space as in Fig. 170. Hence they are acting on a magnetic path of low reluctance, and their effect on the magnetic flux of the alternator is much greater than when the current is in phase with the no-load emf. With smooth-core or nonsalient-pole rotors such as are used in turbine-driven alternators, the air gap is essentially uniform so that a given armature mmf has practically the same effect at all positions of the armature coils.

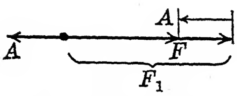


FIG. 173.—Vector diagram showing effect of armature reaction with current lagging 90°.

The foregoing effect of armature reaction may be represented by a vector diagram, Fig. 173. The vector F_1 represents the field mmf and A the armature mmf in direct opposition. Their vector sum F is the resultant mmf.

In Fig. 172(*c*) is shown the resultant, or load flux-density, curve together with the no-load curve, which is given for comparison. The total flux, represented by the area under the load flux-density curve, has been substantially reduced.

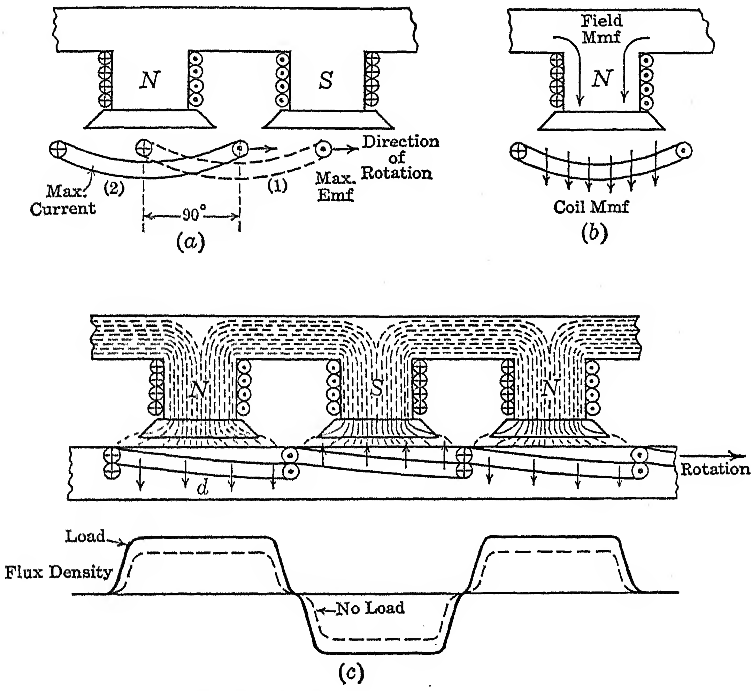


FIG. 174.—Armature reaction—current leads no-load emf by 90°.

124. Current in Quadrature Leading.—Figure 174 shows the conditions existing when the current leads the no-load emf by 90°. As before, the emf reaches its maximum value when the coil sides are directly under the pole centers [position (1), Fig. 174(*a*)]. The current, however, reaches its maximum value 90 electrical space degrees *ahead* of this position, or at (2). The ampere-turns of the coil now assist or strengthen the main field, as they are acting in conjunction with it. This is illustrated in (*b*), in which the coil is directly under an N-pole and its mmf is acting in conjunction with the mmf of the N-pole. As with the current lagging 90°, the coil is in the most favorable position so far as its effect on the magnetic circuit of the alternator is concerned.

This effect of armature reaction may be represented by a vector diagram, Fig. 175. The mmf of the field is F_1, that of the armature is A, and the resultant mmf is their sum F, because the two are acting in the same direction.

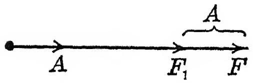


FIG. 175.—Vector diagram showing effect of armature reaction with current leading by 90°.

In Fig. 174(*c*) is shown the resultant, or load flux-density, curve together with the no-load flux-density curve, which is given for comparison. The total flux, represented by the area of the load flux-density curve, has been substantially increased.

125. Pulsation of Single-phase Armature Reaction.—The armature mmfs acting in the alternator field, such as are shown in Figs. 170, 172, 174, are not steady but pulsating. This is due to the fact that not only are the armature coils moving in space but simultaneously the current in them is changing with time. Pulsating armature reaction may be explained by considering the conditions that exist when a single armature coil rotates with angular velocity ω in a bipolar field, Fig. 176. The current is assumed to be in phase with the no-load or excitation voltage, and the current varies sinusoidally with time. In its initial position the plane of the coil lies in the plane $x - x'$, perpendicular to the pole axis ($\omega t = 0$). In (*b*), the coil is shown as having turned through an angle $\omega t_2 = 90°$, and the current has reached its maximum value. Hence the armature mmf A_2 is a maximum, and its direction is downward perpendicular to the plane of the coil and to the pole axis, as shown. In (*a*), the coil has turned through an angle $\omega t_1 = 45°$, and the magnitude of the mmf, which is proportional to the current, is $A_1 = A_2 \sin 45° = 0.707A_2$. The vectors A_1 and A_2 are shown in the vector diagram in (*e*). A_1 can be resolved into two components, a in the direction of A_2, and a_1, which lies in the direction of the main field and hence strengthens it. In (*c*), $\omega t_3 = 135°$, and the mmf $A_3 = A_2 \sin 135° = 0.707A_2$. The vector A_3 is also shown in (*e*), and it too can be resolved into two components, a in the direction of A_2 and a_3 opposing the main field. Similar analysis shows that over every half-revolution, for every component, such as a_1, that aids the main field, there is an equal and opposite component, such as a_3, which opposes it. Hence the average mmf will lie along A_2. In (*d*), $\omega t_4 = 225°$, but the position of the coil and the current are identical with those in (*a*) so that the mmf vector A_4 will coincide with A_1.

Hence in one half-revolution of the coil the variable armature mmf has completed one cycle. In one revolution of the coil it will have

completed two cycles so that the frequency of the mmf is *double* that of the current. Over each half-cycle the *resultant* mmf will be perpendicular to the pole axis and will vary sinusoidally with time but of double frequency. The average over each half-cycle of the components of armature reaction that lie along the pole axis is zero. They alternately strengthen and weaken the field at double frequency,

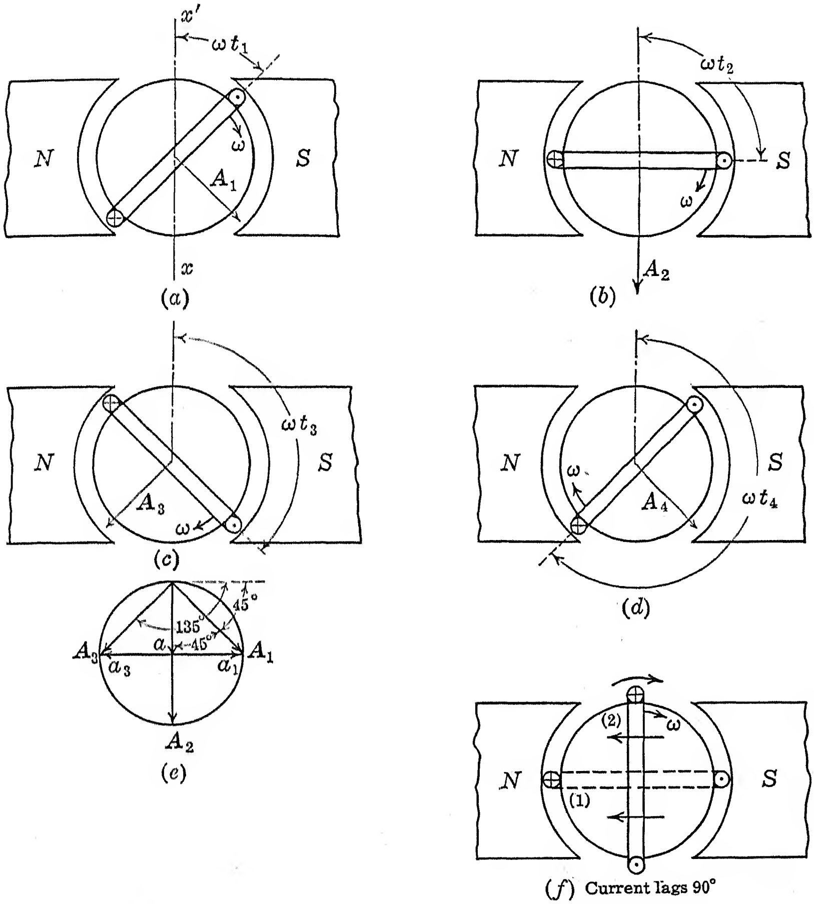


FIG. 176.—Pulsating armature reaction in a bipolar alternator.

but their net effect on the field strength is zero. These double-frequency pulsations produce hysteresis and eddy currents in the field structure. However, the induced eddy currents in the iron, particularly in the solid portions, and in the pole-face dampers, Fig. 159 (p. 174), tend to damp out the pulsations (Lenz's law). The double-frequency pulsations of flux when cut at synchronous speed by the

armature conductors cause third harmonics to be induced in the armature, but these harmonics usually are small.

In Fig. 176(f) the conditions for the current lagging the excitation voltage by 90° are shown. The induced emf is a maximum when the coil is in position (1), and the current is a maximum when the coil is in position (2). The average double-frequency mmf is in opposition to the main field, and the pulsating cross-magnetizing components, whose average is zero, act perpendicular to the pole axis (compare with Fig. 171). With the current leading the excitation voltage by 90°, the average armature mmf strengthens the field.

It will be shown in Sec. 126 that with balanced and constant polyphase currents in the armature the armature mmf is steady and with constant power factor is stationary with respect to the field.

126. Polyphase Armature Reaction.—In Sec. 125 it is shown that single-phase armature reaction pulsates at double frequency. With a constant balanced polyphase load, however, the fundamental component of armature reaction is constant in magnitude and has a constant space relation to the field poles. For example, if the field is stationary and the armature rotates, the mmf is stationary in space; if the field rotates and the armature is stationary, the armature mmf rotates synchronously with the field. Consider Fig. 177. At (a) are shown three equal 3-phase currents I_A, I_B, I_C, as functions of time. In (b) is shown a full-pitch 3-phase two-layer lap winding, similar to that in Fig. 146 (p. 162). It is assumed that when a current is positive in (a) the current is inward in the +phase belts in (b) and (c), and accordingly when a current is negative in (a) the current is outward in the +phase belts.

Consider the conditions for time (1) in (a). I_A is positive maximum, and I_B and I_C are negative and each equal to one-half its maximum value. Hence the currents will be inward in the $+A$-, $-B$-, $-C$-belts and outward in the $-A$-, $+B$-, $+C$-belts. Also, the mmfs due to currents in the A-belts will be twice those due to currents in the B- and C-belts. Consider slots a and b. The current is outward in a and inward in b, and the slot currents are equal, so that the two slots can be considered as acting like a single coil. The direction of the mmf will be upward, and the mmf can be represented by the rectangle $a'b'$ whose altitude to scale is equal to the ampere-turns (see Fig. 165, p. 181, and also Vol. I, Chap. XII, Armature Reaction in Multipolar Machines). Slots c, d act in a similar manner, and their mmf may be represented by rectangle $c'd'$. Similar relations hold for the other slots in the $+C$-, $-B$-belts. In slots e, f in the $-A$-, $+A$-belts the

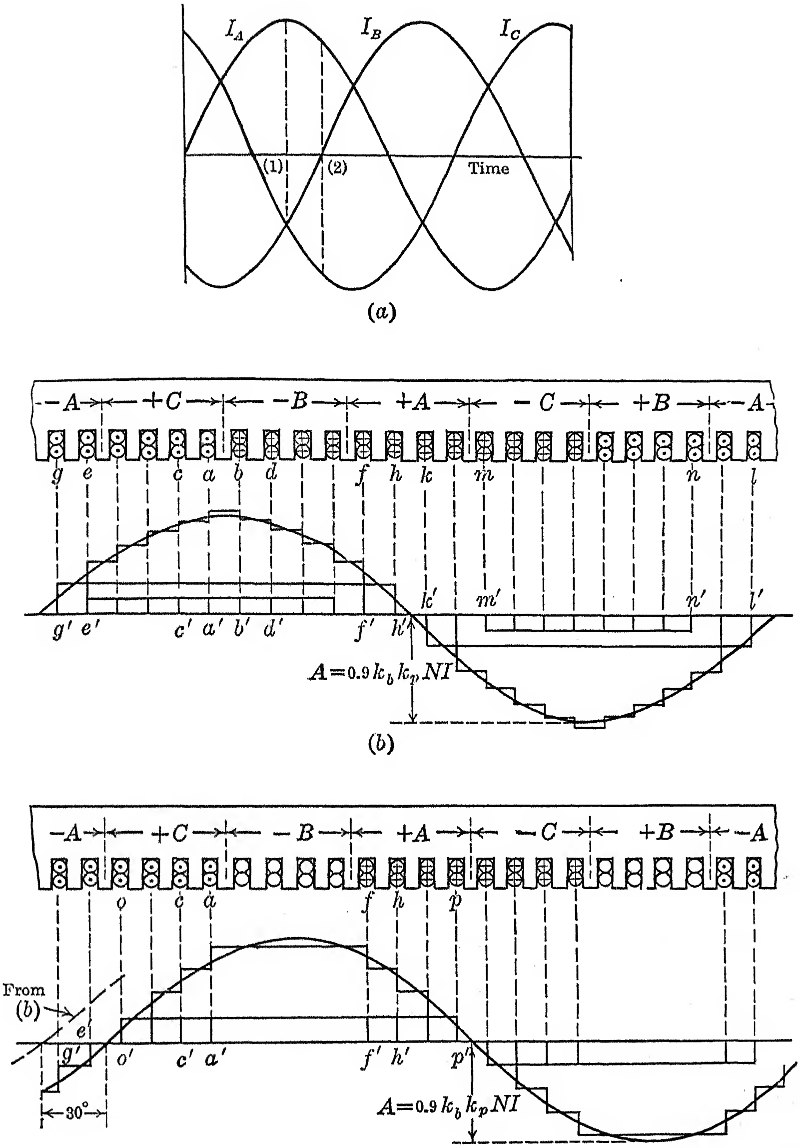


FIG. 177.—Three-phase armature mmfs.

current is twice that in the $B-$, $C-$belts; hence the rectangles $e'f'$ and $g'h'$ have twice the altitude of $a'b'$, $c'd'$, as shown.

Magnetomotive relations similar to the foregoing hold for the slots m, n and k, l, the direction of their mmfs being downward. The resultant mmf is obtained by adding the several rectangles and consists of a stepped wave, as shown. Actually, owing to the fringing of the flux, the resultant wave will not have rectangular steps but rather will have rounded ripples. The harmonics are so small that they can be neglected, and the fundamental can be represented by a smooth curve as indicated. It can be shown that its maximum value

$$A = 0.9k_bk_pNI \quad \text{ampere-turns per pole,} \qquad (145)$$

where k_b is the breadth factor (p. 177), k_p the pitch factor, N the total series armature turns per pole, and I the rms current.[1]

In (*c*) are shown the conditions occurring at time (2) in (*a*), when the electrical angle between (1) and (2) is 30°. I_A is still positive and equal to 0.866 of its maximum value; I_B is zero; I_C is still negative and also equal to 0.866 of its maximum value. Hence the currents in all the slots of the A- and C-belts are of the same polarity as in (*a*) and are now equal so that the mmf rectangles will all have the same altitude. Thus the mmf of slots a, f can be represented by rectangle $a'f'$, of slots c, h by $c'h'$, of slots o, p by $o'p'$, etc. Again the resultant mmf is a stepped wave, and its fundamental can be represented by a sine wave having the same amplitude as that in (*b*). Note that the wave also has moved 30 electrical degrees to the right, as indicated, which corresponds to the angle between (1) and (2) in (*a*). [The portion of the dotted wave is from (*b*)]. Since Fig. 177 represents a rotating-field type of alternator, the field poles must have rotated 30 electrical degrees in the interval (1–2) in (*a*). *Hence the fundamental component of armature mmf rotates synchronously with the field and has a constant geometrical relation to it.* (This rotating armature mmf constitutes the rotating field of the induction motor, p. 308.)

The fact that the armature mmfs in Fig. 177 are sine waves and those in Figs. 170, 172, 174 are irregular, or nonsinusoidal, waves may raise the question as to whether the latter can be represented by vectors to be combined with the vectors representing sine waves. As is shown on p. 178, owing to the effect of breadth and pitch (see p. 180) and also owing to the Y-connection, the harmonics in the flux wave are reduced to small values in the resultant emf wave. Since in the performance of the alternator the fundamental component only of

[1] See V. Karapetoff, "Magnetic Circuit," p. 127; R. R. Lawrence, "Principles of Alternating-current Machinery," 3d ed., p. 60.

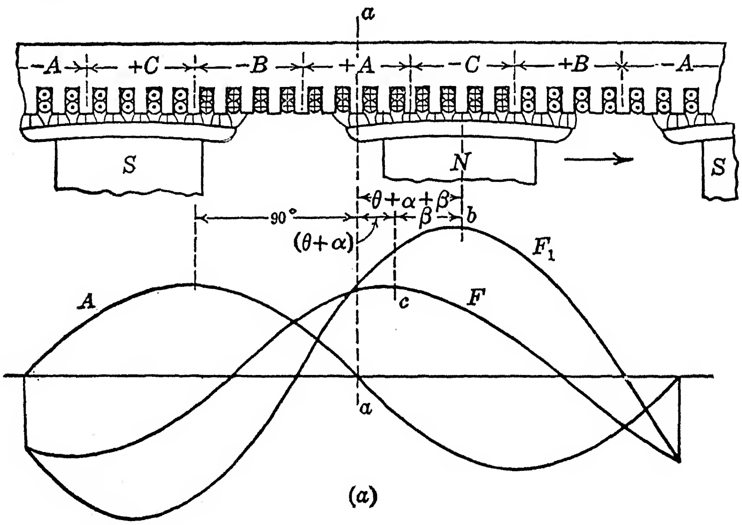
the voltage and current waves usually need be considered, it is necessary to consider only the fundamental component of the flux wave. With salient-pole alternators the variable reluctance along the air gap does distort the wave shape and introduces effects that give only approximate results in the simpler methods of analysis and that can be taken into consideration only in the more involved methods. With smooth-core field rotors the air-gap reluctance is substantially uniform, and owing to the distributed field winding the field mmf is nearly sinusoidal, Fig. 165 (p. 181). Hence, since only sine waves of mmf are involved, the performance of such alternators can be calculated quite accurately.

127. Field, Armature, and Resultant Mmfs.—The vector diagrams for single-phase armature reaction are given in Figs. 171, 173, 175, 176, for currents either in phase with the excitation voltage or lagging or leading it by 90°. These same diagrams are even more precise for polyphase alternators operating under the same conditions, since the pulsating component of armature reaction does not exist. All vector diagrams of mmfs are only approximate unless the mmfs are distributed sinusoidally, since vector operations apply only to sine or cosine waves (Sec. 12, p. 18).

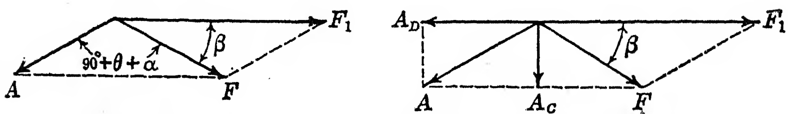
In Fig. 178 are shown the conditions in a 3-phase alternator when the current lags the *terminal* voltage by an angle θ. Under these conditions the actual emf E_a induced in the armature lags the excitation voltage by the angle β, and the terminal voltage lags the induced emf by the angle α. This is all explained with the alternator vector diagram (see Fig. 188). The armature, Fig. 178(*a*), is identical with that of Fig. 177, and the conditions correspond to those in (*a*), where the current in phase A is at its maximum value. The poles are shown in Fig. 178(*a*), and their faces are rounded so that a sinusoidal flux-density curve F_1 is obtained. F_1 could, however, be the fundamental component of nonsinusoidal flux-density curves such as are shown in Figs. 170, 172. As an approximation it is assumed that the air-gap reluctance is uniform, such as exists with smooth-core rotors. Under these conditions the mmf and flux density are proportional to each other so that to scale F_1 may be considered as representing the flux density along the air gap *at no-load.*

Under the load conditions in Fig. 178(*a*) the current is a maximum in phase A, and the armature mmf is represented by the curve A, which is derived in Fig. 177(*b*). The resultant mmf F is obtained by adding at each point the ordinates of the curves F_1 and A. In Fig. 178 it is assumed that the load power factor is cos θ, that the no-load, or excitation, voltage E leads the emf E_a induced under load by the

angle β, and that E_a leads the terminal voltage V by the angle α. Since the power factor of the load is cos θ, V must lead the current I by the angle θ (see Figs. 188, 189, pp. 209, 211). Hence the angle between the current I and the emf E_a must be $\theta + \alpha$, and the angle between I and the excitation voltage E must be $\theta + \alpha + \beta$. Refer again to Fig. 178(a). The maximum value of emf induced in a conductor occurs when the maximum value of the flux-density wave is cutting that conductor. The maximum value of excitation voltage



(a)



(b) Vector diagram (c) Cross and demagnetizing components

FIG. 178.—Field, armature, and resultant mmfs in 3-phase armature.

E occurs in phase A when the top b of the flux-density wave F_1 is at the center of the $+A$-belt at line aa. Since the current I lags E by the angle $\theta + \alpha + \beta$, the current in the $+A$-belt will not reach its maximum value until the point b has moved along the gap by the angle $\theta + \alpha + \beta$, the conditions shown in (a). It follows that the maximum value of emf E_a induced in the armature under load will occur when the center c of the resultant mmf curve F is at line aa. Since the current I lags this emf by the angle $\theta + \alpha$, F will have moved along the gap by the angle $\theta + \alpha$ before the current in the $+A$-belt is a maximum. This condition also is shown in (a).

It now becomes possible to draw the mmf vector diagram for the general condition of current I lagging terminal voltage V by the angle θ. Each of the sine waves in (a) may be represented by a vector (Sec. 12, p. 18). Hence, if F_1 is laid off horizontally to the right, F lags F_1 by the angle β and A lags F by the angle $90° + \theta + \alpha$. Hence, the resultant mmf F is the vector sum of the no-load mmf F_1 and the armature mmf A, a result that also has been obtained by adding the ordinates of the waves F_1 and A in (a).

Note that under these conditions of lagging current the direction of the armature-reaction mmf A is such that it has considerable demagnetizing effect on the magnetic circuit of the alternator. In fact, as shown in (c), A may be resolved into a cross-magnetizing component A_C (see Fig. 171) and a demagnetizing component A_D (see Fig. 173).

Were the current, Fig. 178(a), in phase with the excitation voltage, or no-load emf, the maximum point b of the wave F_1 would be directly under the center of the $+A$-belt so that the emf and current in phase A would reach their maximum values simultaneously. Hence A would lag F_1 by 90°, corresponding to Figs. 170, 171. Were the current lagging the excitation voltage by 90°, the angle $\theta + \alpha + \beta$ would be 90° and F_1 and A would be in opposition, corresponding to Figs. 172, 173. Were the current leading the excitation voltage by 90°, point b would be 90° to the left of line aa, and F_1 and A would be in conjunction, corresponding to Figs. 174, 175.

The vector diagram, Fig. 178(b), is important in that it is the basis of nearly all the methods of analysis of alternator operation.

128. Armature Impedance Drop.—In a direct-current generator, the induced armature emf is obtained by adding *numerically* the IR drop in the armature and the terminal voltage. In the alternator, the armature leakage-reactance drop IX as well as the armature resistance drop must be added to the terminal voltage in order to obtain the induced armature emf. These voltage drops must be added *vectorially* to the terminal voltage, in order to obtain the induced emf. That is, the emf induced in an alternator armature is the terminal voltage plus the armature *impedance drop*, this addition being performed vectorially.

Current in Phase with Terminal Voltage.—Figure 179(a) shows the conditions existing when the load power factor is unity. V is the generator terminal voltage, and I is the armature current in phase with V. The IR-drop in the armature is in phase with the current I, R being the *effective* resistance of the armature. The IX-drop leads the current by 90° and is laid off at the end of IR. The vector sum of these two gives the IZ-drop in the armature. This impedance drop

when added vectorially to the terminal voltage V gives the emf E_a induced in the alternator armature. The vector addition is performed by completing the parallelogram having V and IZ for its adjacent sides. The diagonal E_a is the vector sum of IZ and V and represents the induced emf.

The same result is obtained by adding the IR-drop directly to V, Fig. 179(b), and then adding the IX-drop, at right angles to I and leading, at the end of IR. The vector addition in this case is made by the use of the polygon of vectors described in Chap. I (p. 14). The impedance drop IZ is shown dotted in Fig. 179(b), as it is not used in obtaining E_a by this particular method.

It is to be noted that, with a load of unity power factor, the current is in phase with the *terminal* voltage but lags the *induced* emf by an angle α.

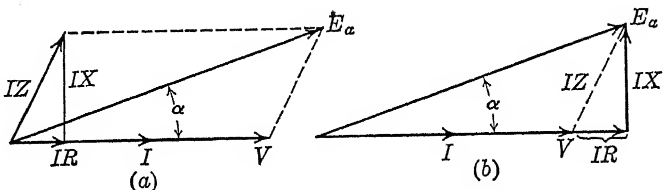


FIG. 179.—Alternator vector diagram for unity power factor.

It is a simple matter to find E_a if the other quantities are known. E_a is the hypotenuse of a right triangle of which $V + IR$ is one side and IX the other.

$$E_a = \sqrt{(V + IR)^2 + (IX)^2}. \tag{146}$$

Example.—A 60-kva 220-volt 60-cycle alternator has an effective armature resistance of 0.016 ohm and an armature leakage reactance of 0.070 ohm. Determine induced emf when the machine is delivering rated current at a load power factor of unity.

$$\text{The current } I = \frac{60{,}000}{220} = 273 \text{ amp},$$
$$IR = 273 \cdot 0.016 = 4.37 \text{ volts},$$
$$IX = 273 \cdot 0.070 = 19.1 \text{ volts},$$
$$E_a = \sqrt{(220 + 4.4)^2 + (19.1)^2} = 225 \text{ volts.} \quad \textit{Ans.}$$

Lagging Current.—When the current lags the terminal voltage by the angle θ, the same method is employed to calculate the induced emf. Figure 180(a) shows the current I lagging terminal voltage V by the angle θ. The IR-drop is in phase with the current vector I, and the IX-drop is in quadrature with I and leading, as before. The resulting impedance drop IZ is then found, being the resultant of IR and IX. This impedance drop is then added vectorially to V, giving

the armature induced emf E_a. It will be noted, Figs. 179, 180, that the position of the armature impedance triangle is determined by the *current* and *not* by the voltage. When, therefore, the current lags, this impedance triangle swings clockwise with the current.

As before, the impedance drop may be added at the end of V, if the correct phase relations are observed. The most direct method of finding the induced emf E_a is to use the method described under the triangle of vectors (p. 14). IR, which is in phase with the current, is first added vectorially at the end of the terminal voltage V, Fig. 180(b). Then the reactance drop IX, at right angles to the current and leading, is added at the end of IR. The resultant emf E_a is found by completing the polygon. The geometrical solution of the diagram Fig. 180(b), is quite simple. If IR is projected on the current vector I, a right triangle of voltages, Obd, is formed, of which E_a is the hypote-

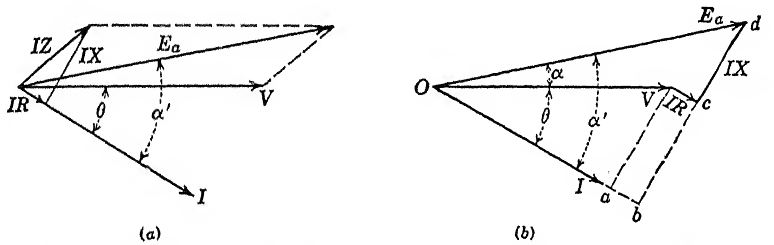


FIG. 180.—Alternator vector diagram for power factor $\cos\theta$, current lagging.

nuse. The values of the two legs of this right triangle may be found as follows:

$$Oa = V\cos\theta,$$
$$ab = IR,$$
$$aV = bc = V\sin\theta,$$
$$cd = IX,$$
$$E_a = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc + cd)^2}$$
$$= \sqrt{(V\cos\theta + IR)^2 + (V\sin\theta + IX)^2}. \qquad (147)$$

The current lags the induced emf E_a by the angle α', which can be readily determined.

$$\tan\alpha' = \frac{bd}{Ob} = \frac{V\sin\theta + IX}{V\cos\theta + IR}.$$

Example.—Determine E_a for a load in which the power factor is 0.7, current lagging, using the constants of the example on p. 200.

The rating of an alternator, as has been pointed out, depends on the current or kva rather than the kilowatts. The current rating of the generator, therefore, will remain unchanged, although the kilowatts in this example are reduced to 0.7 of their former value.

$$\cos\theta = 0.70, \qquad IR = 4.37 \text{ volts as before.}$$
$$\theta = 45.6°.$$
$$\sin\theta = 0.7145, \qquad IX = 19.1 \text{ volts as before.}$$
$$E_a = \sqrt{(220 \cdot 0.70 + 4.4)^2 + (220 \cdot 0.7145 + 19.1)^2} = 237 \text{ volts.} \quad Ans.$$

It is to be noted that the value of the induced emf is now much larger than before, although the value of the impedance drop is the same. For a fixed value of induced emf, therefore, the terminal volts become less with increasing lag of the current, even though the value of the current remains unchanged. This is due to the angle at which the impedance drop subtracts from the induced emf. It would be expected, therefore, that the regulation of an alternator would be poorer for lagging current.

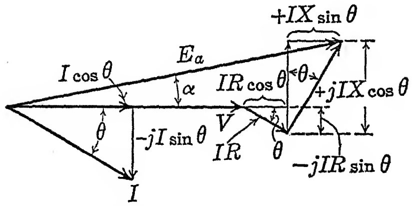


FIG. 181.—Alternator vector diagram in complex for lagging current.

At unity power factor, the armature resistance drop is the important factor in determining the value of E_a. With a lagging current the resistance drop plays but a small part, and the armature *leakage-reactance* drop becomes the important factor.

The foregoing relations also may be determined by the use of complex algebra. That is,

$$\begin{aligned} \dot{E}_a &= V + I(\cos\theta - j\sin\theta)(R + jX) \\ &= V + IR\cos\theta - jIR\sin\theta + jIX\cos\theta + IX\sin\theta. \end{aligned} \tag{148}$$

Each of these quantities is given in Fig. 181.

$$\begin{aligned} \dot{E}_a &= (V + IR\cos\theta + IX\sin\theta) + j(IX\cos\theta - IR\sin\theta) \\ &= e_1 + je_2. \end{aligned}$$

In the foregoing example,

$$\begin{aligned} \dot{E}_a &= 220 + 273(0.70 - j0.7145)(0.016 + j0.070) \\ &= 220 + 3.06 - j3.12 + j13.38 + 13.65 \\ &= 220 + 16.72 + j10.26 = 236.7 + j10.3 \text{ volts,} \\ |E_a| &= \sqrt{(236.7)^2 + (10.3)^2} = 237 \text{ volts.} \quad Ans. \end{aligned}$$

Leading Current.—Figure 182 shows the alternator vector diagram when the current *leads* the terminal voltage by an angle θ. As the current changes its phase relation to lead with respect to the voltage V, the impedance triangle swings with the current in a counterclockwise direction about the end of V. E_a is found in the same manner as in Fig. 180. The voltage drop IR, parallel to the current, is projected on the current vector.

$$
\begin{aligned}
Oa &= V \cos \theta, \\
ab &= IR, \\
aV &= bc = V \sin \theta, \\
cd &= IX,
\end{aligned}
$$

$$
E_a = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc - cd)^2} = \sqrt{(V \cos \theta + IR)^2 + (V \sin \theta - IX)^2}. \tag{149}
$$

This differs from (147) only in the sign of IX, which is now negative.

Example.—Repeat the foregoing example when the power factor is 0.7, current leading.

$$
\cos \theta = 0.70, \qquad IR = 4.37 \approx 4.4 \text{ volts.}
$$
$$
\sin \theta = 0.7145, \qquad IX = 19.1 \text{ volts.}
$$
$$
E_a = \sqrt{(220 \cdot 0.70 + 4.4)^2 + (220 \cdot 0.7145 - 19.1)^2} = 210 \text{ volts.} \quad \textit{Ans.}
$$

The induced emf in the armature is now *less* numerically than the terminal voltage. This is a condition that cannot exist in a direct-

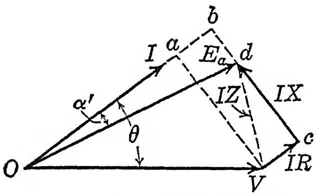


FIG. 182.—Alternator vector diagram for power factor cos θ, leading current.

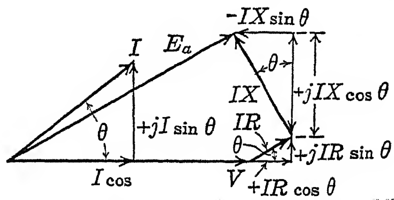


FIG. 183.—Alternator vector diagram in complex for leading current.

current generator. It results from the phase position of the IZ-drop with respect to V.

The foregoing relations may likewise be determined by complex algebra.

$$
\begin{aligned}
\dot{E}_a &= V + I(\cos \theta + j \sin \theta)(R + jX) \\
&= V + IR \cos \theta + jIR \sin \theta + jIX \cos \theta - IX \sin \theta.
\end{aligned} \tag{150}
$$

Each of these quantities is given in Fig. 183.

In the foregoing example,

$$
\begin{aligned}
\dot{E}_a &= 220 + 273(0.70 + j0.7145)(0.016 + j0.070) \\
&= 220 + 3.06 + j3.12 + j13.38 - 13.65 \\
&= 220 - 10.59 + j16.50 = 209.4 + j16.5 \text{ volts,}
\end{aligned}
$$
$$
|E_a| = \sqrt{(209.4)^2 + (16.5)^2} = 210 \text{ volts.} \quad \textit{Ans.}
$$

129. Alternator Regulation.[1]—The voltage E_a, determined in the preceding sections, is the voltage *induced* in the alternator armature

[1] The ASA (American Standards Association) standard specifies regulation as follows: In synchronous generators, the regulation is the rise in voltage when the rated load at rated power factor is reduced to zero, expressed in per cent of rated voltage. The excitation shall remain constant during the test at a value that gives rated voltage at rated current and rated power factor. (Rule 3.210, Rotating Electrical Machinery Standard C50-1943.)

under load conditions. In practice, it is a quantity difficult to measure and can be calculated only approximately. There is no simple method of making a direct measurement of the armature leakage reactance X although it may be computed or determined from the Potier diagram (p. 222). In several of the methods of computing alternator performance it is not necessary to know either the value of E_a or that of the armature leakage reactance X.

A knowledge of the voltage regulation is important, because it shows how closely a machine will maintain its voltage under the various conditions of load, from no-load to full load.

If there were no armature *reaction*, E_a would be the no-load voltage of the alternator, just as in a separately excited direct-current generator the induced emf under load would be equal to the no-load emf if there were no armature reaction. As has just been shown the effect of armature reaction is to change the value of the magnetic flux, and this is accompanied by a corresponding change in the value of the induced emf E_a. The effect of armature reaction on the operation of the alternator is analyzed in the methods for determining regulation.

It is usually impossible to find the regulation of an alternator by actual loading, particularly in the larger sizes, until after the machine has been put into service, and even then it may be difficult to secure the desired adjustment of the load. To make an actual load test of an alternator, a machine of about equal capacity for driving purposes is essential, and usually considerable power must be supplied and then absorbed.

With polyphase alternators, there is the added difficulty of obtaining a balanced load.

The regulation of an alternator, however, may be calculated with sufficient accuracy from data obtainable from open-circuit and short-circuit tests. These tests involve very little power supply and do not require any power-absorbing devices. There are five common methods for determining regulation—the *general method;* the *synchronous-impedance,* or *emf method;* the *mmf method;* the 1925 *AIEE* (American Institute of Electrical Engineers) *method;* and the *ASA method.* The application and limitations of each method will be discussed in some detail; but before this can be done, an understanding of the space and time relations among alternator mmfs, fluxes, and emfs is necessary.

130. Space and Time Vectors.—The terminal voltage of an alternator depends on the flux cut by the armature conductors, the armature resistance drop, the armature leakage-reactance drop, and the power factor. As has just been shown, the flux is the resultant of two

mmfs, one produced by the field ampere-turns and the other produced by the armature ampere-turns (armature reaction). Moreover, as the armature or the field rotates, the space values of the flux, the armature current, and the induced emf are all closely related.

Consider Fig. 184 which shows two positions of an armature coil with relation to a pair of N- and S-field poles. A sine distribution of flux along the air gap is assumed. The line ab is the coil axis. When the coil axis lies along the axis oo of the N-pole, as shown in (a), the flux *linking* the coil is a maximum. When the coil axis ab reaches

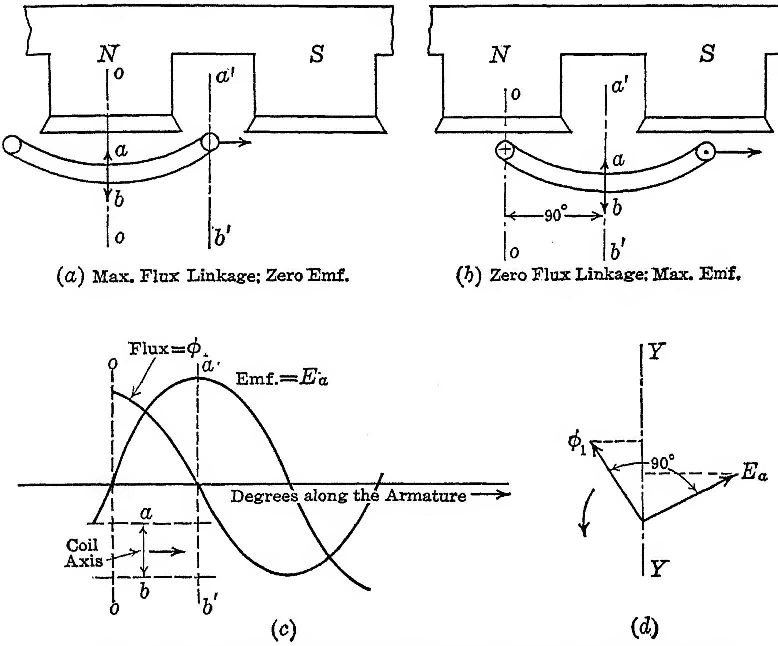


Fig. 184.—Relation of flux linking alternator to induced emf in coil.

position $a'b'$ midway between pole centers, as shown in (b), the flux *linking* the coil is zero. The flux linking the coil varies, therefore, with the time as the coil moves along the air gap. The frequency at which this flux varies is the same as the frequency of the induced emf. In position (a), the flux linking the coil is a maximum, and the induced emf is zero. In position (b), the flux linking the coil is zero, and the induced emf is a maximum. It is seen that the emf induced in the coil reaches its maximum value 90 electrical space degrees later than the maximum flux linking the coil and, therefore, later in time. The flux linking the coil may be said to *lead* by 90° the emf that it induces.

This relation of flux and emf, as the coil moves along the gap, is shown graphically in Fig. 184(c).

When the coil axis ab lies along the pole axis oo, the flux linking the coil is a maximum and the induced emf E_a is zero. As the coil axis ab moves to the right, the flux ϕ_1 linking the coil decreases sinusoidally and the induced emf E_a increases sinusoidally. When the coil axis ab reaches $a'b'$, midway between pole centers, the flux linking the coil is zero and the induced emf E_a is a maximum. Therefore, in an alternator, the flux wave, which represents the flux linking the coil at each instant, *leads* the induced emf wave by 90°, as shown.

These space relations may also be shown by rotating vectors, Fig. 184(d). The vector ϕ_1 is equal to the maximum value of the flux linking the coil, and the vector E_a is equal to the maximum value of the induced emf. Each position of these two rotating vectors represents a different position of the armature coil relative to the field poles. The instantaneous value of either quantity, ϕ_1 or E_a, is found by projecting its vector on the vertical axis YY. It is seen that the flux ϕ_1 reaches its maximum value 90 space degrees in advance of the emf E_a.

Figures 184(c), (d) are *space-phase* diagrams. Figure 184(c) shows the flux linking the coil and the induced emf in the coil for different *space* positions of the coil as it moves relative to the field poles. Figure 184(d) shows these same quantities as rotating vectors.

Although ϕ_1, the flux linking the armature coil, and E_a, the induced emf in the coil, vary with the *space* position of the coil, they vary also with the *time*. When the coil moves through 360 electrical degrees in *space* with respect to the poles, the emf wave passes through 360 electrical degrees in *time*. The time of doing this is $1/f$ sec, where f is the frequency in cycles per second. The time required, therefore, for the coil to pass through a given number of electrical *space* degrees is equal to the time required for the emf to pass through an equal number of electrical *time* degrees. For this reason, a *space-phase* diagram and a *time-phase* diagram may often be combined, just as the angular variation of emf, Fig. 3 (p. 4), can be changed to a time variation of emf, Sec. 3 (p. 6). The space-phase diagrams of Fig. 184(c), (d) may also be considered as time-phase diagrams.

131. Space and Time Vector Diagram.—Consider the conditions when the current is in phase with the induced emf E_a. In Fig. 171, which gives the mmf vector diagram when the current is in phase with the *no-load* emf E, the armature mmf vector A lags the no-load mmf F_1 by 90°. When there is current I in the armature, however, the flux is displaced by armature reaction from its no-load direction by the angle β' in the direction of lag, Fig. 171. Hence the emf E_a induced

under load reaches its maximum value β'° in the direction of lag from the no-load emf E. If the current is in phase with E_a, the current will also reach its maximum value by an angle β later in time. The angle β will differ somewhat from β' since the current is now in phase with E_a rather than E. Hence the vector F in Fig. 171 and its corresponding flux ϕ will be displaced from F_1 and ϕ_1 by some angle β

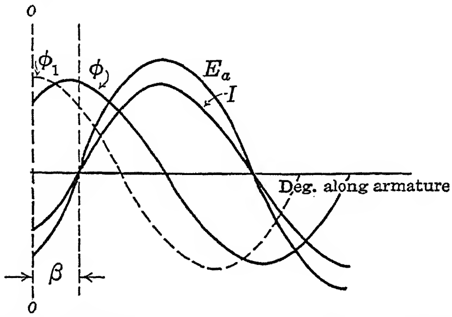


FIG. 185.—Relation among fluxes, emfs, and current.

in a clockwise direction, as is shown in Figs. 185 and 186. In Fig. 185, ϕ_1 is the no-load flux as in Fig. 184(c), and ϕ is the resultant flux displaced β° to the right of ϕ_1. The induced emf wave E_a now lags ϕ by 90°, and the current wave I, under the assumed conditions, is in phase with E_a. Hence the current I also lags ϕ by 90°.

In the vector diagram, Fig. 186, the mmfs F_1 and F produce fluxes ϕ_1 and ϕ, Fig. 185. The emf E_a lags ϕ by 90° and hence lags F by 90°.

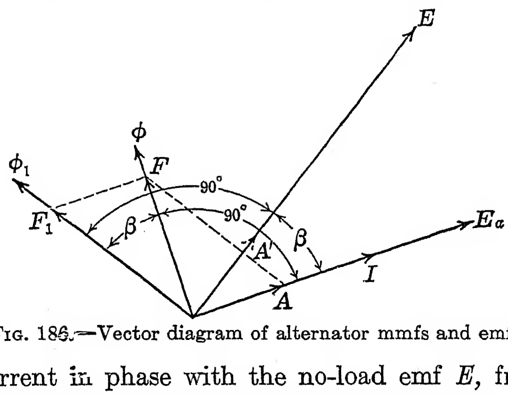


FIG. 186.—Vector diagram of alternator mmfs and emfs.

Were the current in phase with the no-load emf E, from Fig. 171, the armature reaction would lag F_1 by 90°, as shown dotted at A', Fig. 186. However, the armature mmf A now lags the resultant mmf F by 90°, as shown. This brings the space position of A in phase with E_a. I as a time vector is also in phase with E_a. Hence A is in phase with I. I may also be considered as being a *space* vector.

That is, the current wave I, Fig. 185, may be considered as giving the instantaneous values of current for different positions of the coil along the armature, as is done with E_a, Fig. 184. Also, the flux produced by the armature mmf A acting alone links any one armature coil as a function of time, as does ϕ_1, Fig. 184. Hence, A also may be considered as a *time* vector.

When the current lags the induced emf E_a by 90°, the armature reaction is in exact opposition to the resultant field, Figs. 172, 173 (p. 190). Figure 187 shows the mmf and emf vector diagram for this condition.

F_1 is the no-load mmf and A the armature mmf in direct opposition (see Fig. 173). The resultant mmf is F, found by adding F_1 and A vectorially. The fluxes corresponding to F_1 and F are omitted, but

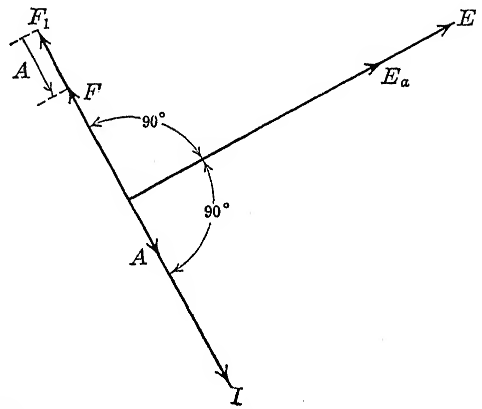


FIG. 187.—Relation of induced emfs to alternator field mmfs, current lagging by 90°.

they may be assumed to be proportional to and in phase with F_1 and F. From Sec. 130 the induced emfs E and E_a lag F_1 and F by 90°, and they are in phase with each other. The current I is assumed to lag E_a by 90°. Hence, again I is in phase with A.

Figures 186, 187 show that in alternator vector diagrams the armature mmf vector A is in phase with the current vector I.

Also, in Figs. 186, 187, F_1, F, and A constitute a *space diagram of mmf vectors* such as Figs. 173, 175, 178(*b*). E_a is also a space vector under the conditions of Figs. 184(*c*), (*d*), 185, where its value is a function of the *space* position of the coil. As the linking of the resultant flux ϕ with the armature coils also varies with time, as described on p. 205, ϕ, and hence its mmf vector F, may be considered as *time* vectors also. E_a is also a *time* vector, just as I and E are time vectors, so that E_a may be combined with them. Hence, E_a and F or ϕ may be considered as connecting links between the *space* diagram of mmfs

and the *time* diagram of currents and emfs. The space and the time diagrams, therefore, may be combined in one diagram as is done in Figs. 186 and 187.

In Figs. 177 and 178 a rotating-field structure is assumed, whereas in several other figures the armature is assumed to be the rotating member. The assumption made in each case seems best adapted to the particular reaction being analyzed. However, as explained on p. 157, it is immaterial which member rotates, since the operation of the alternator depends only on the *relative* motion of field and armature. Hence the conclusions that have been reached apply equally to the rotating-field and to the rotating-armature type of alternator.

132. General Method.—In this method the values of the armature reaction and the armature leakage reactance must be known. The armature reaction A may be computed quite accurately from Eq. (145) (p. 196).[1] The armature leakage reactance may be computed or measured. Both the armature reaction and leakage reactance may be determined from the Potier diagram (p. 222).

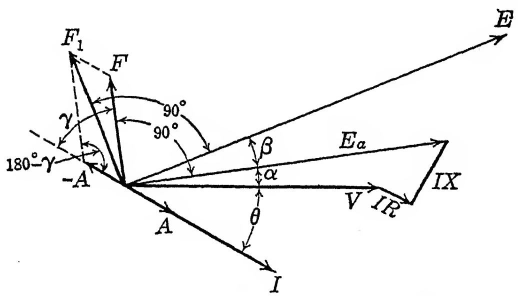


FIG. 188.—Vector diagram for general method.

Consider Fig. 188, in which the armature terminal voltage V, the power factor $\cos\theta$, and the current I, usually the rated value, are given. The vectors V and I are therefore determined. The IR- and IX-drops are added vectorially to V, Fig. 180 (p. 201), to give the load induced emf E_a. The flux that will induce the emf E_a leads E_a by 90°; in the diagram the mmf F, which produces this flux, is found on the saturation curve corresponding to E_a (also see Fig. 186). The armature reaction mmf A is in phase with I, Figs. 186, 187. The impressed field F_1 is found by adding $-A$ vectorially to F. The calculated no-load emf E is found from the saturation curve, its value corresponding to the ampere-turns F_1, and E lags F_1 by 90°, Fig. 186. If F_1, A, and F are expressed in ampere-turns, they are readily converted into

[1] LAWRENCE, R. R., "Principles of Alternating-current Machinery"; and also "Standard Handbook," 7th ed., Sec. 7.

terms of field current by dividing each by N_f, the field turns per pole. The regulation then is $(E - V)/V$ (p. 203).

The value of X may be determined experimentally as follows: The alternator is operated at short circuit, Fig. 192(b), the current being I_1' amp. The corresponding value of armature reaction is A_1, Fig. 191, and the value of the field mmf is F_1'. On open circuit with the field current unchanged, the induced emf is E_1. Hence the impressed field F_1' leads E_1 by 90°. On short circuit, however, the armature reaction reduces the mmf acting on the field, and the resultant field becomes F', the vector sum of F_1' and A_1. Hence the *actual* induced emf at short circuit is E_a' lagging F' by 90°. To find F', A_1 is computed by (145), p. 196, and F', the resultant of F_1' and A_1, then is determined. For most practical purposes, $F' = F_1' - A_1$ numerically. E_a' is found on the saturation curve, corresponding to F'; the armature impedance,

$$Z = E_a'/I_1'; \qquad X = \sqrt{Z^2 - R^2}.$$

Although this method gives more precise results than the synchronous-impedance or the mmf method, these less precise methods are preferred in many instances because of their greater simplicity.

133. Synchronous-impedance Method, or Electromotive-force Method.—This method is often called the *pessimistic method*, because it gives a value of the regulation *poorer* than the actual regulation.

An inspection of Fig. 188 shows that, with lagging current, both the armature leakage-reactance drop and the armature reaction operate to reduce the terminal voltage. Under ideal conditions, that is, if saturation is neglected and the air gap is uniform, as occurs with smooth-core rotors, the armature leakage-reactance drop and the armature reaction are both proportional to the armature current. Also, under these conditions, the phase position of armature reaction is such that it has the same effect on the voltage relations as the armature leakage-reactance drop does. This makes it possible to combine the effect of armature *reaction* with armature *leakage reactance*. Accordingly, in this method armature reaction as such is omitted, but its effect is retained by increasing the armature reactance by an appropriate amount over its actual value.

Consider Fig. 189, the solid lines of which are identical with the alternator diagram of Fig. 188. If the same constant of proportionality exists between each emf and the field that produces it, $E_a/F = E/F_1$. Under these conditions, the point b at the terminal of E lies at the intersection of IX extended and E. Now consider A as acting alone. The flux that it produces will induce an emf Oa' lagging A by 90°. Under the foregoing conditions ba is equal to Oa' and is in phase with it. Hence, ba may be *considered* as being an emf in phase with IX and tending to reduce the terminal voltage of the

alternator, thus taking the place of the armature reaction, which causes an equal diminution in terminal voltage by reducing the field. Thus ba is a *fictitious* emf that replaces the effect of armature reaction on the main flux of the alternator.

It is also evident that if IX be increased in value to IX_s, where $IX_s = IX + ab$, E may be computed without E_a being known. This assumes that the emf ab is always proportional to the armature current, which is not strictly true.

The foregoing is the principle of the emf, or *synchronous-impedance*, method. X_s is called the *synchronous reactance* of the alternator. The corresponding impedance $Z_s = \sqrt{R^2 + X_s^2}$ is called the *synchronous impedance* of the alternator.

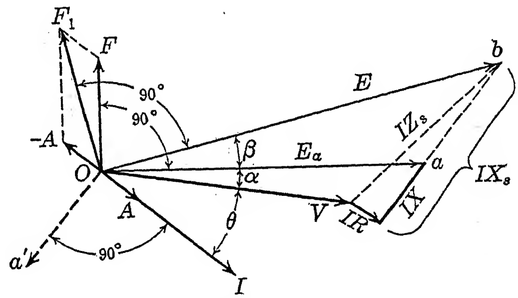


FIG. 189.—Complete vector diagram for synchronous-impedance method.

The mmf vectors F_1, F, A, $-A$ are not necessary to the method and may be omitted. They are given merely to assist in the development of the method.

134. Determination of Synchronous Reactance.—The synchronous reactance is determined experimentally as follows. The saturation curve of the alternator, E vs. I_f, is first determined in the usual manner and the curve plotted, Fig. 190. The field then is made very weak, and the alternator armature is short-circuited through an ammeter. The field is then gradually strengthened, and a curve of armature current I vs. I_f is determined. The field is increased until the armature current is about twice its rated value. These two curves are shown plotted in Fig. 190.

Consider some value of field current I'_f. On open circuit, this field current produces an emf E_1. On short circuit, the terminal voltage of the machine is practically zero. The voltage E_1 does not actually exist in the armature at short circuit, because of armature reaction. (The voltage actually induced is E'_a, Fig. 191.) If, however, the effect of the armature reaction is replaced by an armature-reactance drop, the voltage E_1 may be considered as used entirely in

sending the current I_1' through the synchronous impedance of the armature. That is,

$$E_1 = I_1'Z_s,$$

where Z_s is the *synchronous impedance* of the armature. This short-circuit condition is represented vectorially in Fig. 191, where I_1' is the short-circuit current and E_1 the *assumed* internal emf of the armature. The synchronous-impedance drop is made up of two components,

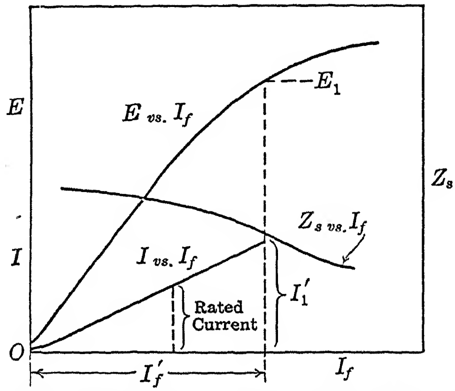


FIG. 190.—Open-circuit and short-circuit characteristics of alternator.

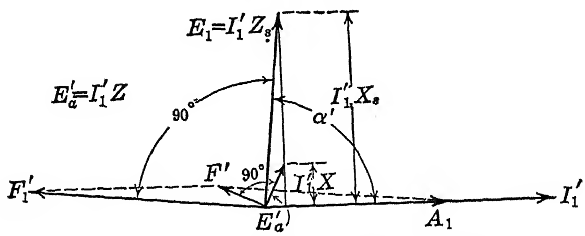


FIG. 191.—Short-circuit vector diagram of alternator.

$I_1'R$, where R is the effective resistance of the armature, and $I_1'X_s$, where X_s is the synchronous reactance of the armature.

Obviously,

$$Z_s = \frac{E_1}{I_1'}, \tag{151}$$

and

$$X_s = \sqrt{Z_s^2 - R^2}. \tag{152}$$

In practice, R is small compared with Z_s and they combine almost in quadrature, so that

$$X_s = \frac{E_1}{I_1'}, \qquad \text{very nearly.} \tag{153}$$

The value of the synchronous reactance depends to a large extent on the degree of saturation of the iron. For example, at low flux densities the armature mmf will have a much greater effect on the magnetic circuit than if the iron were saturated. Under short-circuit conditions, therefore, where the iron is operating at low flux density, the synchronous reactance will be *too large*. The variation of synchronous impedance with field current is shown in Fig. 190. As the iron becomes more saturated, the synchronous impedance *decreases*. Under operating conditions, the iron is considerably more saturated than it is under short-circuit test conditions. In order to approach as nearly as possible to operating conditions, *it is desirable to obtain the synchronous impedance at the highest possible value of armature current, as at* I_1', Fig. 190. Also, the synchronous impedance is determined at very low power factor, corresponding to short-circuit conditions, as shown by Fig. 191, where the angle α' between the current and the emf E_1 is nearly 90°. The armature current is a maximum, therefore, when the axes of the armature coils are almost opposite the pole centers, as shown in Fig. 172 (p. 190). Under these conditions and with salient poles, the permeance of the magnetic circuit is a maximum so that the value of the synchronous impedance so determined is too large and substantially greater than for other positions of the coil, as shown, for example, in Fig. 170 (p. 188).

From the foregoing it follows that the value of synchronous impedance determined at short circuit is *too large* and will make the calculated value of regulation too large. The synchronous-impedance method, therefore, is called the *pessimistic* method. It is a safe method to use in making a guaranty, because the alternator always regulates better than the computed values indicate.

The following example will illustrate the use of this method:

Example.—A 50-kva 550-volt single-phase alternator has an open-circuit emf of 300 volts when the field current is 14 amp. When the alternator is short-circuited through an ammeter, the armature current is 160 amp, the field current still being 14 amp. The ohmic resistance of the armature between terminals is 0.16 ohm. The ratio of effective to ohmic resistance may be taken as 1.2. Determine (*a*) synchronous impedance; (*b*) synchronous reactance; (*c*) regulation at 0.8 power factor, current lagging.

The rated current $I = 50{,}000/550 = 91$ amp.

(*a*) The synchronous impedance $Z_s = {}^{300}\!/_{160} = 1.87$ ohms.

The effective resistance $= 1.2 \cdot 0.16 = 0.192$ ohm.

(*b*) $X_s = \sqrt{(1.87)^2 - (0.192)^2} = 1.86$ ohms.

(*c*) $\cos\theta = 0.8, \qquad \sin\theta = 0.6.$

Applying Eq. (147) (p. 201),

$$E = \sqrt{[(550 \cdot 0.8) + (91 \cdot 0.192)]^2 + [(550 \cdot 0.6) + (91 \cdot 1.86)]^2}$$
$$= \sqrt{209{,}000 + 249{,}000} = 677 \text{ volts.}$$

As the *synchronous* reactance was used in computing E, the armature reaction was taken into consideration, so that the no-load voltage of the alternator is presumably 677 volts. The regulation (p. 203), therefore, is

$$\frac{677 - 550}{550}100 = \frac{127}{550}100 = 23.1 \text{ per cent.} \quad \textit{Ans.}$$

It is to be noted in the foregoing example that the synchronous impedance Z_s is practically equal to the synchronous reactance X_s, and in most cases it may be assumed as being equal to it without appreciable error.

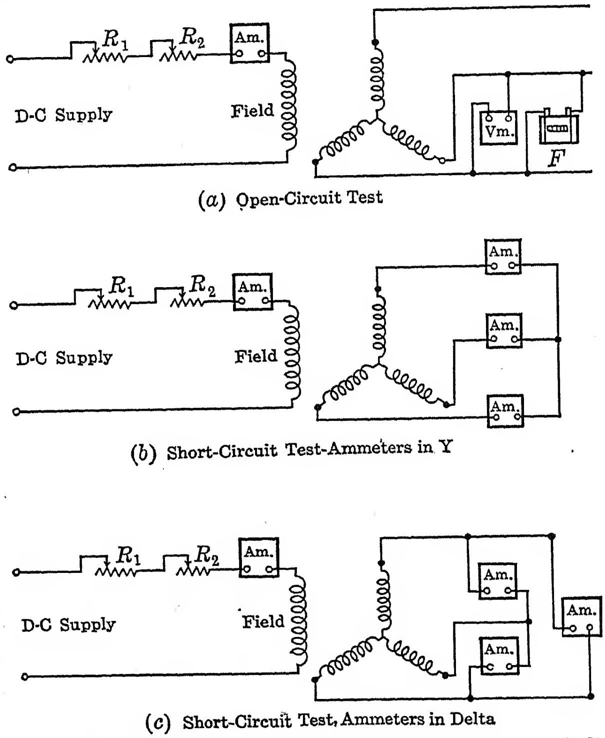


Fig. 192.—Connections for making open- and short-circuit tests of alternator.

135. Three-phase Application.—The methods of determining alternator performance give much more satisfactory results with polyphase than with single-phase. This is due to the fact that, with a constant balanced load, polyphase armature reaction is steady and has constant relation to the field poles (Sec. 126). On the other hand, single-phase armature reaction is pulsating and causes a double-frequency pulsation of the flux (Sec. 125).

Following are the applications of the synchronous-impedance method to 3-phase alternators.

In Fig. 192 are shown the connections for determining the synchronous impedance and armature reactance. Although the alternator is shown as Y-connected, it is immaterial whether the alternator actually is delta- or Y-connected.

In (*a*) are shown the connections for making the open-circuit test of a 3-phase alternator. This is substantially the same method as is used with direct-current generators. The field is excited from some direct-current source, and the field current is measured with an ammeter. The armature is driven at the rated or synchronous speed, and the open-circuit emf is measured for different values of field current. The emf of one phase only need be measured, as the phase voltages should all be equal. A frequency indicator F may be used for determining the speed of the alternator. An additional resistance R_1 in the field circuit is often necessary for obtaining the points on the lower part of the saturation curve.

In the short-circuit test, all three phases must be short-circuited. There are two methods of connecting the ammeters. They may be connected in Y, Fig. 192(*b*), in which case the ammeters read the *line* current directly, or they may be connected in delta, Fig. 192(*c*), in which case the line current is obtained by multiplying the ammeter reading by $\sqrt{3}$ or 1.73. With the delta connection, the ammeters need have only about half the range (1/1.73 or 0.58) necessary for the Y-connection. The average of the ammeter readings is usually taken, although there should be little difference in the three readings.

In calculating the regulation of a 3-phase alternator, only one of its three phases is considered in making computations. The regulation, efficiency, etc., of one phase is determined; the alternator being symmetrical, the other phases have similar characteristics. Only the single-phase calculations already described are necessary. Two conditions arise, one when the alternator is considered as Y-connected, and the other when it is considered as delta-connected. In each case, only coil values of current and voltage are used.

136. Regulation of Y-connected Generator.—It is impossible to determine whether an alternator is Y-connected or delta-connected unless the winding itself be inspected. Fortunately, it makes no difference, so far as calculation of the regulation is concerned. It may be assumed to be either, and the result is the same if the work is consistent.

If the alternator is considered as Y-connected, the coil voltage is equal to the line voltage divided by $\sqrt{3}$. The coil current and the line current are the same. The method of dealing with such a problem is illustrated by the following example.

Example.—Figure 193 shows the open- and short-circuit characteristics of a 1,500-kva 2,300-volt 60-cycle alternator. Terminal volts and line current are plotted as ordinates with values of field current as abscissas. Assume that the machine is Y-connected. The resistance between each pair of terminals as measured with direct current is 0.12 ohm. Assume that the effective resistance is 1.5 times the ohmic resistance. Determine the synchronous reactance of the alternator and its regulation at 0.85 power factor, current lagging.

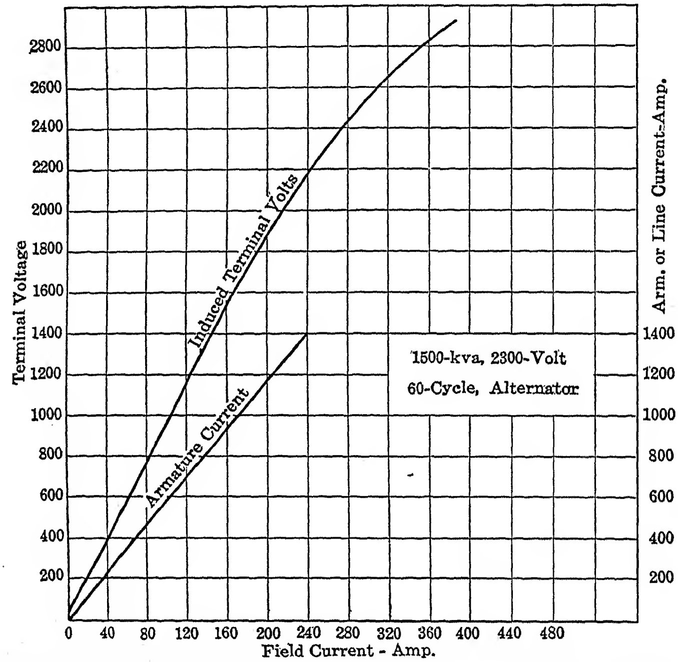


FIG. 193.—Open- and short-circuit characteristics of 1,500-kva alternator.

From Fig. 193, the maximum value of the short-circuit current is 1,400 amp, which is equal to the coil current, since the Y-connection is assumed. This corresponds to 240 amp in the field, and at 240 amp field current the open-circuit terminal emf is 2,180 volts. The corresponding coil emf is

$$\frac{2,180}{\sqrt{3}} = 1,260 \text{ volts},$$

$$Z_s \text{ (per coil)} = \frac{1,260}{1,400} = 0.90 \text{ ohm} = X_s, \text{ nearly.}$$

If the resistance between terminals is 0.12 ohm, it includes two coils in series, as the Y-connection is assumed, so that the ohmic resistance per coil is

$$\frac{0.12}{2} = 0.06 \text{ ohm.}$$

The effective resistance per coil is equal to $1.5 \cdot 0.06 = 0.09$ ohm.

$$\text{Rated current} = \frac{1{,}500{,}000}{2{,}300\sqrt{3}} = 376 \text{ amp per terminal.}$$

$$\text{Rated emf per coil} = \frac{2{,}300}{\sqrt{3}} = 1{,}330 \text{ volts.}$$

$$\cos\theta = 0.850, \qquad \theta = 31.8°, \qquad \sin\theta = 0.527.$$

No-load emf per coil is found by applying Eq. (147) (p. 201).

$$E = \sqrt{[(1{,}330 \cdot 0.850) + (376 \cdot 0.09)]^2 + [(1{,}330 \cdot 0.527) + (376 \cdot 0.90)]^2}$$
$$= 1{,}560 \text{ volts.}$$

$$\text{Percentage regulation per coil} = \frac{1{,}560 - 1{,}330}{1{,}330}100 = 17.4 \text{ per cent.} \quad \textit{Ans.}$$

$$\text{Open-circuit terminal emf} = 1{,}560\sqrt{3} = 2{,}700 \text{ volts.}$$

$$\text{Percentage regulation using this value} = \frac{2{,}700 - 2{,}300}{2{,}300}100 = 17.4 \text{ per cent.}$$

Ans.

Or, applying Eq. (148) (p. 202),

$$\dot{E} = 1{,}330 + 376(0.85 - j0.527)(0.09 + j0.90)$$
$$= 1{,}537 + j269,$$
$$|E| = \sqrt{(1{,}537)^2 + (269)^2} = 1{,}560 \text{ volts.}$$

137. Regulation of a Delta-connected Generator.—In the delta-connected alternator, the line voltage and the coil voltage are equal, but the coil current is the line current divided by $\sqrt{3}$. The ammeters connected in delta, as shown in Fig. 192(c), measure the coil current directly.

Let it be assumed in the example of the preceding section that the alternator is delta-connected. If 240 amp, the same value of field current as before, is used the coil emf in the open-circuit test is now 2,180 volts and the corresponding coil current in the short-circuit test is $1{,}400/\sqrt{3} = 808$ amp.

The synchronous impedance per coil

$$Z_s = \frac{2{,}180}{808} = 2.70 \text{ ohms,}$$

or three times its previous value.

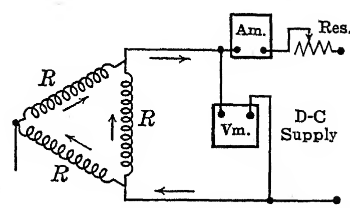


FIG. 194.—Measurement of delta coil resistance with direct current.

Figure 194 shows the circuits of the delta when the ohmic resistance is measured with direct current. Let the resistance per coil be R and the resistance measured between any two terminals be R_0. The circuit consists of two parallel branches, one of R ohms and the other of $2R$ ohms.

Therefore,

$$\frac{1}{R_0} = \frac{1}{R} + \frac{1}{2R},$$
$$R = \tfrac{3}{2}(R_0).$$

Therefore, the ohmic resistance per coil

$$R = \tfrac{3}{2} \cdot 0.12 = 0.18 \text{ ohm},$$

or three times its previous value. This must be increased 50 per cent, in order to obtain the effective resistance.

$$1.5 \cdot 0.18 = 0.27 \text{ ohm effective resistance.}$$

Rated *coil* current of the machine $= 376/\sqrt{3} = 217$ amp.

Applying (147),

$$E = \sqrt{[(2{,}300 \cdot 0.85) + (217 \cdot 0.27)]^2 + [(2{,}300 \cdot 0.527) + (217 \cdot 2.7)]^2}$$
$$= 2{,}700 \text{ volts},$$

which checks the result obtained on the assumption that the alternator is Y-connected.

Therefore, an alternator may be assumed to be either Y- or delta-connected when it is desired to calculate the regulation.

138. Magnetomotive-force Method.—In the synchronous-impedance method of determining regulation, a voltage is substituted for

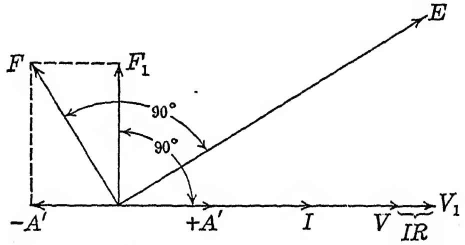


FIG. 195.—Vector diagram for mmf method at unity power factor.

armature reaction or for a mmf. In the mmf method, a mmf is substituted for a voltage, this voltage being the IX-drop in the armature of the alternator. In other words, the armature *leakage reactance* is considered as being zero, but the armature reaction is increased a sufficient amount to compensate for this.

The method involves a short-circuit and an open-circuit test and in this respect is similar to the synchronous-impedance method. Figure 195 shows the principle of the method. This diagram is constructed for unity power factor. V is the terminal voltage. To this is added the IR-drop, giving the voltage V_1. A certain field mmf F_1 is required to produce this voltage V_1. The value of this mmf in terms of the field current is found on the saturation curve, Fig. 196. Corresponding to the value of V_1, the field current F_1 is found. F_1 is laid off at right angles to V_1 and leading it, as a mmf leads by 90° the emf that its flux induces. In the short-circuit test, the field current is adjusted until the rated current flows. The corresponding value of field current A', Fig. 196, is then read. The mmf represented by this

field current is necessary to send rated current through the armature leakage reactance and at the same time overcome the armature reaction, if the resistance be neglected. This mmf A' replaces the combined effect of the armature *leakage reactance* and the armature *reaction.* It is laid off 180° from the current, as shown at $-A'$, Fig. 195. (The total mmf that is assumed to produce the total voltage drop is $+A'$. The component that must balance this mmf is $-A'$.) The resultant mmf is F, which, at unity power factor, is the square root of the sum of the squares of F_1 and $-A'$. F is the mmf that

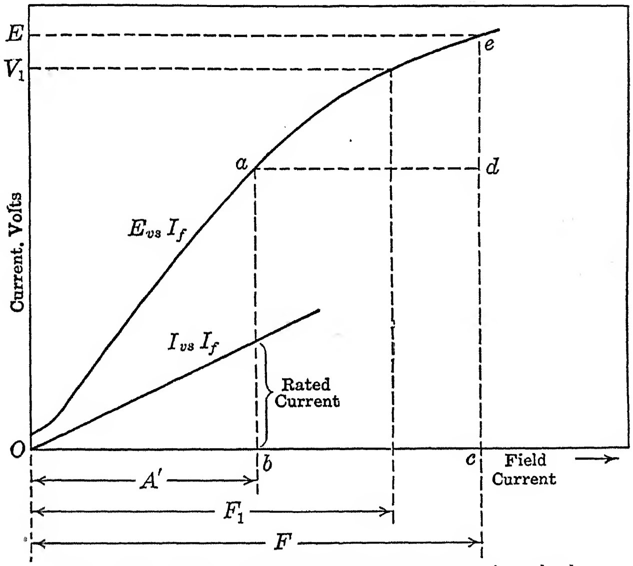


FIG. 196.—Open- and short-circuit tests, mmf method.

exists at no-load under the assumptions made. The no-load emf E lags F by 90°, Fig. 195, and is found on the saturation curve corresponding to field current F, Fig. 196.

To summarize the method at unity power factor, the IR-drop is added to the terminal voltage, and the field current corresponding to this sum is found on the saturation curve. The alternator is then short-circuited, and the field current necessary to send rated current through the armature is determined. The square root of the sum of the squares of these field currents then is found. The value of emf on the saturation curve corresponding to this resultant field current is assumed to be the no-load emf of the alternator.

When the power factor is less than unity, the diagram is similar to that shown in Fig. 197.

The voltage V_1 is the vector sum of V and IR. Its value is readily found by projecting these voltages on the current vector. Thus,

$$V_1 = \sqrt{(V \cos \theta + IR)^2 + (V \sin \theta)^2}. \tag{154}$$

In most cases, a numerical addition of V and IR is sufficiently accurate.

The value of the angle α may be found by finding the angle β.

$$\sin \beta = \frac{V \sin \theta}{V_1},$$
$$\alpha = \theta - \beta.$$

α is usually so small that it may be neglected.

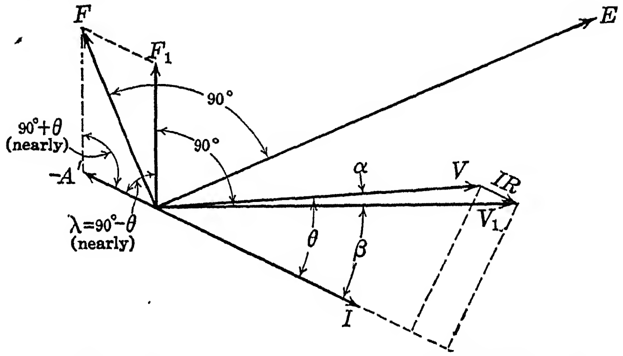


FIG. 197.—Vector diagram for mmf method, lagging current.

The vector F_1 leads V by $90° - \alpha°$, but α is so small that it may be neglected. The armature reaction vector $-A'$ is 180° from the current vector. By geometry, the angle between $-A'$ and F_1 is

$$\begin{aligned}\lambda &= 180° - (90° + \theta - \alpha)\\ &= 180° - (90° + \theta) \text{ nearly}\\ &= 90° - \theta \text{ nearly.}\end{aligned}$$

By the cosine law,

$$F^2 = F_1^2 + A'^2 - 2F_1A' \cos (90° + \theta). \tag{155}$$

The emf E corresponding to F and found from the saturation curve, Fig. 196, is the no-load emf of the alternator.

Example.—Consider the example of Sec. 136. The exact method will be used first. The alternator will be considered as Y-connected.

$$\text{Coil emf} = \frac{2{,}300}{\sqrt{3}} = 1{,}330 \text{ volts.}$$

$$IR\text{-drop is } 376 \cdot 0.09 = 33.8 \text{ volts.}$$

$$\cos \theta = 0.85, \qquad \sin \theta = 0.527.$$

$$V_1 = \sqrt{[(1{,}330 \cdot 0.85) + (34)]^2 + [1{,}330 \cdot 0.527]^2} = 1{,}359 \text{ volts.}$$

Algebraic addition would have given 1,364 volts.

$$\sin \beta = \frac{1{,}330 \cdot 0.527}{1{,}359} = 0.516,$$
$$\beta = 31.1°, \ \theta = 31.8°,$$
$$\alpha = 31.8° - 31.1° = 0.7°, \text{ which is negligible.}$$

From Fig. 193, the field current corresponding to 1,359 coil volts, or 2,350 volts on the saturation curve ($2{,}350 = 1{,}359\sqrt{3}$), is

$$F_1 = 266 \text{ amp.}$$

The rated coil current is 376 amp. Corresponding to this current, Fig. 193, the field current is 64 amp from the short-circuit test.

$$F^2 = \overline{266^2} + \overline{64^2} - 2 \cdot 266 \cdot 64 \cos (90° + 31.8°),$$
$$F^2 = 92{,}840, \quad F = 305 \text{ amp.}$$

From the saturation curve, the terminal voltage corresponding to 305 amp field current is 2,580 volts across the terminals, or 1,490 coil volts.

$$\text{Regulation} = \frac{1{,}490 - 1{,}330}{1{,}330} = 0.120, \text{ or } 12.0 \text{ per cent.} \quad \textit{Ans.}$$

Because of the low saturation on short circuit, a given mmf will produce a greater increase of flux than an equal mmf will produce under operating conditions, where the saturation of the iron is greater. The emf corresponding to a given increase in mmf at short circuit will be much greater, therefore, than the emf corresponding to an equal increase in mmf taken higher up on the saturation curve. This is illustrated in Fig. 196. On short circuit, the emf *ab* corresponds to the mmf A'. The additional emf *de* corresponds to a mmf *bc* equal to A' but taken higher up on the saturation curve. The emf *de* is obviously much less than the emf *ab*. Hence, that part of the mmf A' which replaces an emf is too small under load conditions. The no-load emf E found on the saturation curve is, therefore, too low, and the regulation as determined by this method is ordinarily less than the actual regulation. For this reason, this method is often called the *optimistic* method. This is illustrated by the foregoing example, where the regulation as obtained by the synchronous-impedance method is 17.4 per cent, whereas that obtained by the mmf method is 12.0 per cent.

That part of the mmf A' which actually is armature reaction is too high on short circuit, owing to low saturation and to the favorable position of the armature coils with respect to the field poles. As in the synchronous-impedance method, this factor tends to give too high a value of regulation. These two sources of error tend to offset each other in the mmf method, whereas they both produce errors in the same direction in the synchronous-impedance method. The mmf

method, therefore, usually gives results closer to the actual regulation than does the synchronous-impedance method. The actual value of the alternator regulation ordinarily lies between the two values just determined. Were the saturation curve a straight line, both methods would give nearly the same result.

139. Potier Diagram.—On p. 213 it is shown that the value of synchronous reactance, determined at short circuit, is too large, since it is obtained at low saturation of the magnetic circuit of the alternator. For this reason, synchronous reactance determined under these conditions is termed *unsaturated* synchronous reactance. There are methods for determining synchronous reactance that in large measure take saturation into consideration. Among these is the Potier method. In Fig. 198 is shown the alternator vector diagram for very low power factor, the current I lagging the terminal voltage

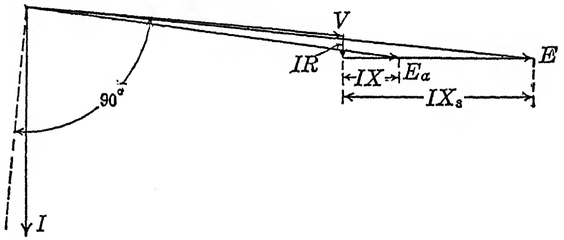


FIG. 198.—Alternator vector diagram at low power factor.

V by nearly 90°. Note that the terminal voltage vector V, the induced-emf vector E_a, and the excitation-emf vector E are nearly in phase with one another. The resistance-drop vector IR is small and is practically at right angles to the other voltage vectors. Hence it has negligible effect on their sums and differences so that

$$IX_s = E - V$$

and $IX = E_a - V$, practically.

In the Potier method a no-load saturation curve OAG and a saturation curve at or near zero power factor EBF, Fig. 199, and usually at rated current, are determined. The low-power-factor curve may be obtained with an underexcited synchronous motor as load, the motor operating satisfactorily to well below 50 per cent rated voltage. If the synchronous motor can be driven mechanically to supply its losses, the entire characteristic to zero voltage and at zero power factor may be obtained. The two saturation curves, Fig. 199, must be similar since the magnetic circuit is the same for both. This is illustrated for the salient-pole alternator in Fig. 172 (p. 190), which shows that at zero power factor the armature mmf acts directly on the poles them-

selves and is in direct opposition to the field mmf. Hence, the two curves, Fig. 199, must be similar, being displaced horizontally by the mmf of armature reaction (in terms of field current).[1] It is therefore not necessary to obtain low values for the low-power-factor characteristic since the upper portions of the two characteristics may be superposed and the lower portion of the low-power-factor characteristic is identical with the corresponding portion of the no-load characteristic. As a matter of fact, since the two characteristics are similar and

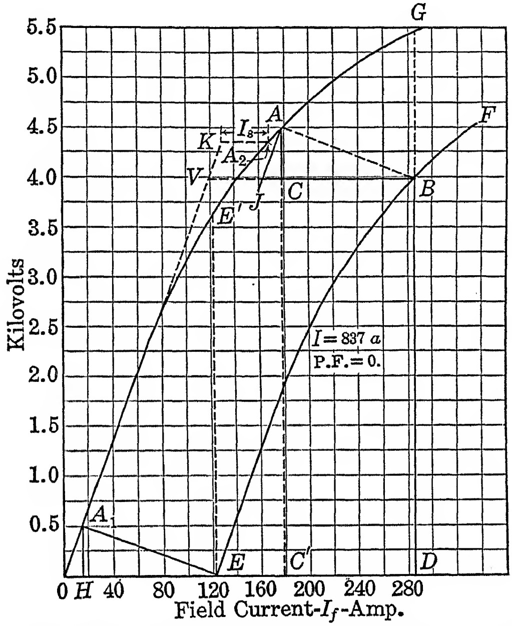


FIG. 199.—Open-circuit and low-power-factor characteristics of alternator.

parallel, two points or so on the upper portion of the low-power-factor characteristic will permit the location of the no-load characteristic when superposed and the low-power-factor characteristic may then be traced. Also, the lower portion of the low-power-factor characteristic is essentially a straight line, so that it can be drawn quite accurately. Referring to Fig. 199, which gives the two characteristics for a 10,000-kva 6,900-volt 514-rpm 60-cycle water-wheel alternator, note that OE is the field current which produces rated-load current at short circuit (see Figs. 191 and 196).

Let point A on the no-load characteristic OAG and B on the low-

[1] Unless otherwise stated, armature mmfs will be given in terms of field current. If A is the armature mmf, then $A = N_f I_f$, where N_f and I_f are the field turns and field current. Hence, $I_f = A/N_f$.

power-factor characteristic *EBF* correspond to the same degree of saturation. That is, if the curve *EBF* were moved so that point *B* coincided with *A*, the coordinate axes remaining parallel, the two curves would coincide. Draw *BC* parallel to the horizontal axis and *AC* parallel to the vertical axis. Since both points *A* and *B* correspond to the same degree of saturation, the net mmf acting on the magnetic circuit must be the same for both. The total field mmf corresponding to point *B* is *OD*, and that corresponding to *C* is *OC′*. Since the net mmfs must be the same, the mmf $BC = DC'$ must be the demagnetizing armature mmf *A*. Curve *EBF* gives *terminal voltage*, and curve *OAG* gives induced emf. If points *B* and *C* are made to coincide by moving curve *EBF* to the left parallel to itself, armature reaction is eliminated. The terminal voltage at zero power factor would be $DB = C'C$. Since the corresponding mmf is *OC′*, the induced emf corresponding to terminal voltage *CC′* is *C′A*. The difference *AC* between the induced emf and the terminal voltage therefore must be the *IX*-voltage drop, Fig. 198. Hence, with a Potier triangle such as *ABC*, it is possible to determine the armature reaction and armature leakage reactance for any point of operation on the saturation curve.

To determine points *A* and *B*, which correspond to the same saturation, the curve *EBF* may be traced on thin paper and superposed to coincide with curve *OAG*. A pin point over point *A* will locate *B*. This method frequently is not accurate, particularly if the saturation is low, for the coincidence of the curves is not critical. Another method is as follows: Since the curves *OAG* and *EBF* are parallel, Potier triangles *ABC* and A_1EH must be equal. A_1O, being at the bottom of the saturation curve, is essentially a straight line. Draw *BJ* equal to *EO*, and through *J* draw *JA* (*A* not being known) parallel to the lower part of the saturation curve. The intersection of *JA* with curve *OAG* locates point *A*. For accuracy, point *A* should be well up on the saturation curve.

Example.—Referring to Fig. 199, which shows the no-load and low-power-factor characteristics for a 10,000-kva 6,900-volt 514-rpm Y-connected 0.8-power-factor 60-cycle water-wheel alternator, the curves give voltages to neutral. The voltages *DB* and *C′C* are the terminal voltages to neutral, 3,980 volts.

The effective resistance of the armature is 0.06 ohm per phase, and the field voltage is 240 volts. Determine by means of the Potier diagram (*a*) armature leakage reactance; (*b*) armature reaction in terms of field current; (*c*) induced emf E_a at 0.8 power factor, lagging current; (*d*) regulation at 0.8 power factor, lagging current.

$$\text{Rated current } I = \frac{10,000}{6,900\sqrt{3}} = 837 \text{ amp.}$$

(a) Rated terminal voltage to neutral $V = \dfrac{6{,}900}{\sqrt{3}} = 3{,}980$ volts; Distance $AC = 500$ volts; $X = {}^{500}\!/_{837} = 0.597$ ohm. *Ans.*

(b) Distance $BC = 107$ amp $= A$. Using Eq. (148) (p. 202),

$$\begin{aligned}
E_a &= 3{,}980 + 837(0.8 - j0.6)(0.06 + j0.597) \\
&= 3{,}980 + 40.2 - j30.1 + j399.5 + 299.5 \\
&= 4{,}320 + j369.4 \text{ volts,}
\end{aligned}$$

$$|E_a| = \sqrt{(4{,}320)^2 + (369.4)^2} = 4{,}330 \text{ volts, or } 4.330 \text{ kv (shown at } A_2\text{).} \quad \textit{Ans.}$$

From Fig. 199, the field current corresponding to 4,330 volts $= 167$ amp $= F_1$

$$\tan\alpha, \text{ Fig. 181 (p. 202)} = \frac{369.4}{4{,}320} = 0.0855; \qquad \alpha = 4.9°;$$

$$\cos\theta = 0.80; \qquad \theta = 36.9°; \qquad \theta + \alpha = 41.8°.$$

Referring to Fig. 188 (p. 209),

$$\gamma + 90° + \alpha + \theta = 180°; \qquad \gamma = 90° - (\alpha + \theta) = 90° - 41.8° = 48.2°.$$

Applying the law of cosines (p. 605) to the mmf diagram, Fig. 188, $-A$ being considered a positive magnitude,

$$\begin{aligned}
F_1^2 &= F^2 + A^2 - 2FA\cos(180° - \gamma) \\
&= \overline{167}^2 + \overline{107}^2 - 2 \cdot 167 \cdot 107 \cos 131.8° \\
&= 27{,}890 + 11{,}450 + 35{,}740 \sin 41.8° = 66{,}000. \\
F_1 &= 251 \text{ amp.}
\end{aligned}$$

From the no-load characteristic, Fig. 199, for $I_f = 251$ amp, $E = 5{,}230$ volts, or 9,060 terminal volts.

$$\text{Regulation} = \frac{5{,}230 - \overset{3980}{\cancel{1{,}330}}}{\underset{3980.}{\cancel{1{,}330}}} = 0.314, \text{ or } 31.4\%. \quad \textit{Ans.}$$

Note that point A corresponds to an induced emf of 4,500 volts, whereas the computed E_a is 4,330 volts, shown at A_2. Hence, strictly speaking, another triangle having A_2 at 4,330 volts should be used and the computation repeated, which again would give an emf slightly different for A_2. However, the result obtained from the first recomputation will differ only slightly from that obtained originally, and usually the general precision of the method does not warrant recomputation.

140. American Standards Association Method.—Referring to Fig. 199, DB is the terminal voltage V at zero power factor for field current OD, and DG is the corresponding no-load emf E. Hence, from Fig. 198, $GB = E - V$ is equal to IX_s for this degree of saturation. Hence, the saturated synchronous reactance $X_s = GB/I$. For example, in Fig. 199, $GB = 1{,}470$ volts. Hence,

$$X_s = \frac{1{,}470}{837} = 1.76 \text{ ohms.}$$

The unsaturated synchronous reactance such as is obtained in the synchronous-impedance method also may be determined. OE gives the value of field current with rated armature current at short circuit.

The open-circuit emf corresponding to OE is EE'. Hence the unsaturated synchronous reactance $X_s = EE'/I = 3{,}670/837 = 4.39$ ohms, or approximately 2.5 times the value of the saturated reactance. This illustrates the errors that saturation produces in the different methods. The armature IR-drop, which is very small and at right angles to the reactance drop, has been neglected.

In the method given in the AIEE Standard of 1925, the saturated synchronous reactance was determined from the difference of the curves, Fig. 199, as described above, and then was used to obtain the regulation. This method has been superseded by the ASA method, first given in an ASA standard in 1936 and later in 1943.[1] The method is in reality the mmf method applied to a nonsaturated condition, resistance being neglected, and then correction made for saturation.

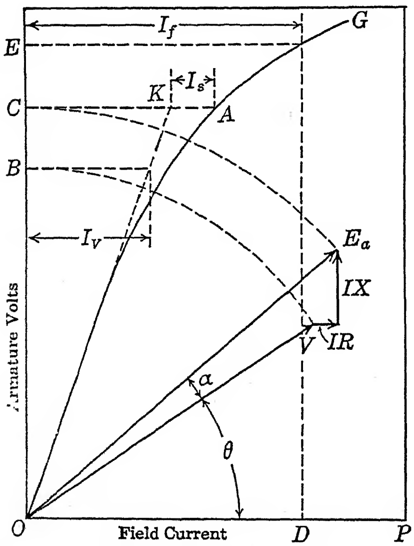


FIG. 200.—Saturation curve and vectors in ASA method.

In Fig. 200, OP is the armature-current axis as well as the field-current axis, and the terminal voltage V is laid off $\theta°$, where $\cos\theta$ is the power factor of the load. Then IR and IX are added vectorially to give the internal induced emf E_a, X being the leakage reactance determined from AC in the Potier diagram, Fig. 199. OG is the no-load saturation curve. OK is tangent to the straight portion of the saturation curve and is called the *air-gap line*. The horizontal difference between the air-gap line and the saturation curve at any emf is a measure of the saturation at that emf. Arcs with the origin as a center are swung to make $OB = V$ and $OC = E_a$ on the voltage axis. I_V is the field mmf necessary to produce V, were there no saturation. (In the discussion of this method mmfs will be given in terms of field current.) Line CA is drawn parallel to the axis of abscissas. The distance $KA = I_s$ is the increase in mmf that is necessary to take into account the saturation at emf E_a.

[1] American Standard 50, Rotating Electrical Machinery, American Standards Association, Mar. 29, 1943.

In Fig. 201, I'_f is the field current necessary to produce rated armature current at short circuit, in Fig. 199 is equal to OE, and in Fig. 196 is equal to A'. I_V is laid off at an angle θ to the right of a perpendicular to I'_f. I_r is the resultant of I'_f and I_V. I_s, Fig. 200, is added in phase with I_r, giving $I_f = F$. The emf OE corresponding to I_f or OD, Fig. 200, is the no-load emf E. The regulation then is $(E - V)/V$.

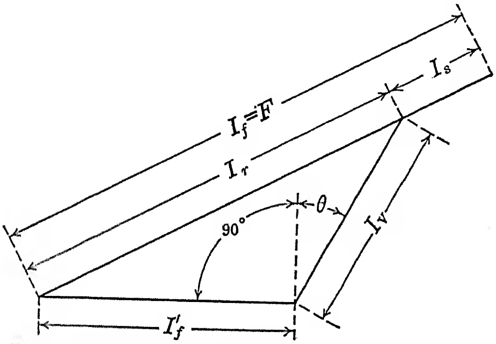


Fig. 201.—Mmf vector diagram for ASA method.

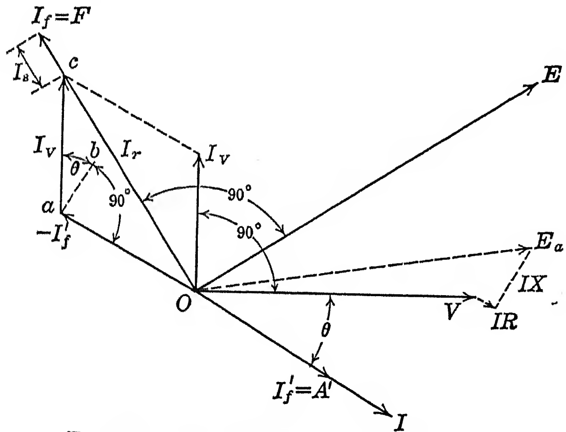


Fig. 202.—Vector diagram for ASA method.

The basis of the ASA method is given in Fig. 202. The terminal voltage V, the current I lagging V by the angle θ, the field current $I'_f = A'$ necessary to produce rated current at short circuit are shown vectorially. The vectors IR, IX, E_a are shown dotted since they are not involved directly. As in Fig. 201, I_V, which leads V by 90°, is added vectorially to $-I'_f$. Since $ac = I_V$ is perpendicular to V and ab is perpendicular to I, angle $bac = \theta$. I_r is the resultant of I_V and $-I'_f$, as in Fig. 201. The vector I_s is added to I_r and in phase with it to

give I_f, or F, the resultant mmf. Hence the vector diagram Oac and I_s is similar to the vector diagram of Fig. 201, except that I_V is added to $-I'_f$ rather than to $+I'_f$ making the direction of I_r and I_f upward to the left rather than to the right.

Example.—Determine, by the ASA method, the regulation at 0.8 power factor, lagging current, of the 10,000-kva 6,900-volt 60-cycle alternator (Sec. 139).

In Fig. 199, $OE = I'_f = 122$ amp; the field current corresponding to the terminal voltage V, on the air-gap line, $I_V = 118$ amp. Applying the cosine law (p. 605) to the diagram, Fig. 201,

$$I_r^2 = I_f'^2 + I_V^2 - 2I'_fI_V \cos (90° + \theta),$$

where $\theta = \cos^{-1} 0.80 = 36.9°$.

$$\begin{aligned} I_r^2 &= \overline{122}^2 + \overline{118}^2 - 2 \cdot 122 \cdot 118(-\sin \theta) \text{ [see Eq. (29), p. 604]} \\ &= 14{,}880 + 13{,}920 + 17{,}270, \\ I_r &= 215 \text{ amp}, \\ I_s&, \text{ Fig. 199}, = 38 \text{ amp}, \\ I_r + I_s &= 215 + 38 = 253 \text{ amp} = I_f. \end{aligned}$$

On curve OAG, Fig. 199, corresponding to 253 amp, $E = 5{,}250$ volts.

$$\text{Regulation} = \frac{5{,}250 - 3{,}980}{3{,}980} = 0.319, \text{ or } 31.9\%. \quad \textit{Ans.}$$

This is in close agreement with the 31.4 per cent obtained in Sec. 139.

From the foregoing it follows that for a given current the regulation depends on the *power factor*. The highest values of regulation occur at low power factors, lagging current. At unity power factor the values of regulation are nominal. With leading current, the terminal voltage tends to *rise* as load is applied, and the regulation may become zero or even *negative*.

Figure 203 shows three typical load curves of an alternator, one being taken at unity power factor, the second at 0.8 power factor, lagging current, and the third at 0.8 power factor, leading current. The regulation in each case is

$$\text{Regulation} = \frac{ac - ab}{ab}. \tag{156}$$

It should be kept in mind that for a fixed *kilowatt* output the regulation with lagging current is even poorer than the values obtained for fixed *current* output.

The Potier and ASA methods minimize errors due to saturation. However, they do not take into consideration the wide variation of air-gap permeance occurring in salient-pole alternators. In the Blondel, or two-reaction, method this is taken into consideration. The armature mmf is resolved into two components, a direct component acting

directly along the pole axis as in Fig. 172 (p. 190) and a quadrature component acting at right angles (in electrical space degrees) to the direct component or acting on the interpolar space midway between pole axes, Fig. 170 (p. 188). Because of the much lower permeance associated with the quadrature component, it is multiplied by a coefficient taking this factor into consideration. This method has been developed further by Doherty and Nickle in a method that bears their names. One source of error, difficult to eliminate by simple methods, is the fact that when the mmf curves are nonsinusoidal they cannot be represented correctly by vectors.[1]

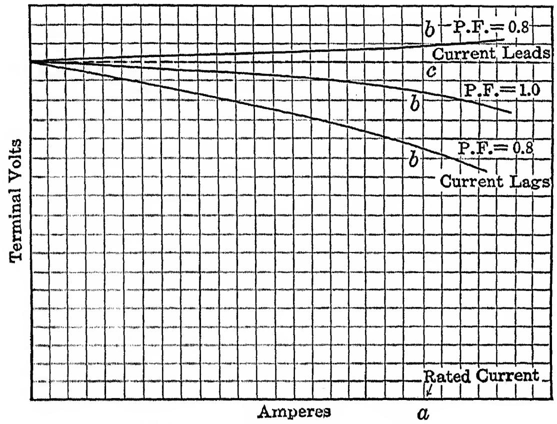


FIG. 203.—Characteristics of alternator at different power factors.

141. Efficiencies of Alternators.—Just as with direct-current generators, the input to alternators is not readily measured (see Vol. I, Chap. XIV). The direct measurement of efficiency by actual loading is accompanied by the difficulties of providing the necessary power and finding suitable load. Hence, the efficiencies of alternators are usually determined from their losses.

In ASA Standard 50[2] (p. 31) the losses are divided as follows: (*a*) field loss $I_f^2R_f$; (*b*) rheostat loss $I_f^2R_r$; (*c*) brush-contact (electrical) loss in field collector-ring brushes; (*d*) exciter losses; (*e*) friction and windage; (*f*) brush-friction loss; (*g*) ventilation loss; (*h*) core loss; (*i*) I^2R-loss in armature windings; (*j*) stray-load loss, due to eddy currents in copper and additional core losses in the iron produced by distortion of the magnetic flux by the load current.

[1] LAWRENCE, R. R., "Principles of Alternating-current Machinery," and LANGSDORF, A. S., "Theory of Alternating-current Machinery," McGraw-Hill Book Company, Inc.

[2] See footnote 1, p. 226.

The foregoing losses are corrected to 75°C. (*a*) Field loss ($= P_f$) may be obtained by multiplying the field current by the field *winding* voltage and is equal to $I_f^2R_f$, where I_f is the field current and R_f the resistance of the field *winding;* (*b*) the rheostat loss is chargeable to the plant and not to the alternator; (*c*) usually neglected; (*d*) as with (*b*), exciter loss is chargeable to the plant and not to the alternator; (*e*) friction and windage loss ($= P_{fw}$) is caused by mechanical friction and the fanning action of the alternator rotor. Windage loss is high in high-speed turbine-driven alternators, although it is greatly reduced by hydrogen cooling (p. 172). It is defined as the mechanical power required to drive the alternator at rated speed with no excitation. It can be measured by driving the alternator with an auxiliary motor and measuring the input to the latter with and without the alternator being mechanically connected, correction being made for the change in losses in the driving motor. (*f*) This is included with (*e*); (*g*) ventilation loss ($= P_v$) is the power required to circulate the cooling air, in addition to windage loss. If there are long ducts external to the alternator, the losses in these are not included. (*h*) Core loss ($= P_{cl}$) is due to eddy currents and hysteresis caused by the main magnetic field. It is the difference in power required to drive the alternator with and without the field excited. In ASA Standard 50, Rule 2.112, it is stated that the alternator shall be excited so that the voltage at the terminals corresponds to the calculated internal emf which is equal to the rated terminal voltage corrected for resistance drop only. Since the air-gap flux is determined by the internal induced emf (Secs. 128, 129, pp. 199 and 203) this emf would seem to be a more correct criterion.

(*i*) Armature I^2R-loss ($= P_R$) is defined as the sum of the I^2R-losses in all the armature current paths, the resistance R being measured with *dc* and corrected to 75°C. Thus, R is *not* the effective resistance. (*j*) Stray-load losses ($= P_s$) are defined as the difference between the mechanical power input and the sum of the friction and windage loss (P_{fw}) and the I^2R-loss (P_R) at the temperature of the winding during the test, when the alternator is driven at rated speed with excitation adjusted to cause in the short-circuited armature a value of current corresponding to the load at which the loss is to be determined. Note that no correction is made for the core loss due to the *resultant* field, which acts to induce the short-circuit current. However, this field F', Fig. 191 (p. 212), and the corresponding loss are relatively small. The sum of the losses in (*i*) and (*j*) gives the loss due to the *effective* armature resistance (Sec. 121, p. 186).

A convenient method for measuring the mechanical input in (*e*),

(h), (j) is to use a small d-c motor. Its armature input may be measured for each condition of operation of the alternator and the output found by subtracting the armature copper loss and the stray power. If the friction and windage loss are known, the core loss (h) may be determined by measuring the input to the alternator operating as a synchronous motor and subtracting the friction and windage loss and the effective armature resistance loss.

The efficiency becomes

$$\eta = \frac{nVI \cos \theta}{nVI \cos \theta + P_f + P_{fw} + P_v + P_{cl} + P_R + P_s}, \qquad (157)$$

where n is the number of phases; V and I, coil voltage and current; $\cos \theta$, power factor; P_f, field loss; P_{fw}, friction and windage loss; P_v, ventilation loss; P_{cl}, core loss; P_R, armature d-c resistance loss; P_s, stray load loss.

The following tables give percentage efficiencies and other data for typical synchronous generators.

Characteristics of Three-phase 60-cycle Generators
Single continuous rating, 50°C, 80% power factor, 240, 480, 600, 2,400 volts
Manufactured by Westinghouse Electric Corporation

Rating, kva	Poles	Speed, rpm	Exciter kw at 125 volts	Load: Half	Load: Three-quarters	Load: Full	Net weight, lb
				Efficiency at 80% power factor			
Horizontal engine-driven type							
62.5	24	300	5.0	84.6	86.2	87.3	2,590
125	24	300	5.0	87.5	88.8	89.7	3,750
500	40	180	15	90.6	91.5	92.0	15,690
1,000	52	138	25	91.9	92.6	93.0	28,080
Horizontal high-speed coupled type							
62.5	6	1,200	1.5	85.1*	88.4	89.9	2,445
				86.9†	88.0	89.7	
125	6	1,200	2.0	87.9*†	90.5	91.6	3,125
500	10	720	7.5	91.6†	93.1	93.7	7,920
1,125	10	720	10.0	93.3†	94.5	95.0	16,300
2,188	10	720	15.0	94.6†	95.4	95.7	19,950

* 240 to 480 volts.
† 2,400 volts.

Turbine-driven direct-connected type							
Armatures, 60° rise. Fields, 25° rise							Cu ft of air per min
1,000‡	2	3,600	15	93.7	95.8	95.6	3,500
2,000‡	2	3,600	20	94.7	95.7	96.1	5,000
5,000‡	2	3,600	35	94.7	95.9	96.5	12,000
15,000‡	2	3,600	100	95.1	96.2	96.8	36,000

‡ Kilowatts.

HYDROGEN-COOLED TURBINE-DRIVEN DIRECT-CONNECTED AIEE–ASME STANDARD RATINGS

85% power factor, 0.85 short-circuit ratio, ½ psi* H_2

All 2 poles, 3,600 rpm, 13,800 volts†

Rating (½ psi), kw	Rating (½ psi), kva	Turbine capability, kw	Rating at 15 psi		Exciter, kw	Efficiency at ½ psi		
			Kw	Kva		½	¾	Full
20,000	23,529	22,000	23,000	27,058	110	97.8	98.0	98.1
30,000	35,294	33,000	34,500	40,588	145	97.8	98.0	98.1
40,000	47,058	44,000	46,000	54,117	155	98.0	98.1	98.3
60,000	70,588	66,000	69,000	81,176	200	98.0	98.5	98.5
			(15% above ½ psi)					

* Psi H_2 = pounds per square inch of hydrogen.

† Machines over 100,000 kw are usually 1,800 rpm, 4 poles.

142. Voltage Regulators.—Voltage regulators for d-c generators are described in Vol. I (Chap. XII). With alternators, such regulators are more necessary even than with d-c generators, since the regulation of alternators is much greater than for d-c generators, as is illustrated by the examples (pp. 216 and 228). Furthermore, there is no satisfactory method of compounding alternators. The Tirrill regulator described in Vol. I, with a few changes to adapt it to alternating current, is also applicable to alternators. The principle of operation is the same, that is, the exciter field rheostat is intermittently short-circuited by vibrating relay contacts, the duration of the short-circuit period depending on how far the voltage has departed from its prescribed value.

A number of regulators have been developed having some advantages over the Tirrill type. Typical of such regulators is the Silverstat, developed by the Westinghouse Electric Corporation. A diagram is shown in Fig. 204. The main control element consists of an open **C**-type magnet with an iron armature capable of being drawn into the air gap. The armature system consists of a pivoted arm that swings

on an axis, and the spring S, by lever action, tends to pull the armature out of the air gap.

The regulating system consists of a number of leaf springs mounted close together but insulated from each other. A connection is made from the fixed end of each spring to a tap on the regulating resistance in series with the shunt field of the exciter. The tap connections are

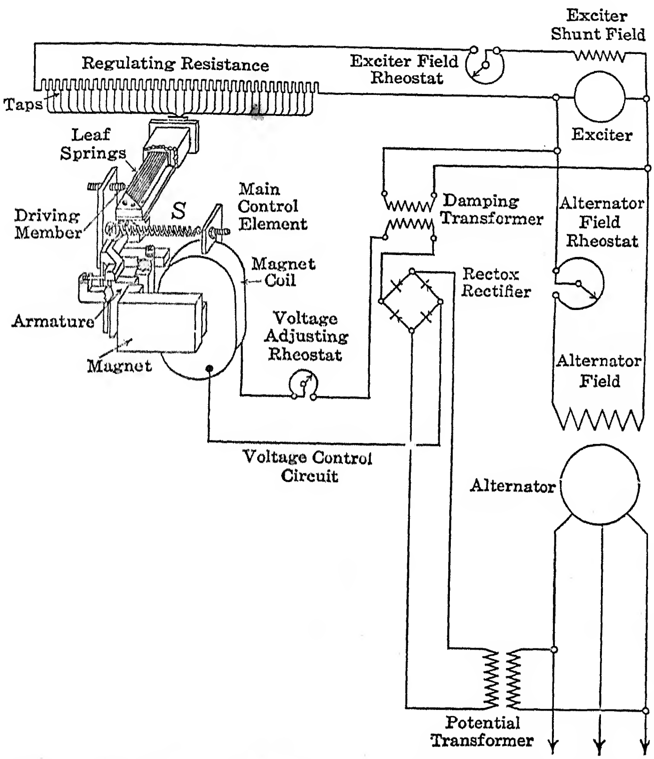


FIG. 204.—Silverstat voltage regulator. (*Westinghouse Electric Corp.*)

made consecutively. Silver contact buttons are mounted on the free ends of the springs, so arranged that when the driving member is exerting little or no pressure the buttons are all out of contact and there is no shunting of the regulating resistance. When the driving member exerts its greatest pressure, the buttons are all in contact, thus short-circuiting the entire regulating resistance.

Hence, as the moving arm moves through its travel, depending on its direction, it closes or opens the contacts of the silver buttons in sequence, cutting out or inserting resistance by small steps in the exciter shunt-field circuit. Since the resistance between buttons

depends on the pressure, the resistance in the field circuit is actually varied in almost infinitesimal steps.

The voltage control circuit is connected across one phase of the alternator whose voltage is to be controlled, a potential transformer being used if the voltage exceeds 125 volts. A Rectox rectifier, bridge-connected to give full-wave rectification (p. 558), converts the alternating current to direct current. This current flows through the magnet coil, the voltage-adjusting rheostat, and the secondary of the damping transformer all in series. Thus the control mechanism is operated with direct current.

If the voltage of the alternator rises, the armature is drawn further into the air gap of the magnet, causing the driving member to reduce the pressure on the leaf springs, opening some of the short-circuiting silver buttons, thus inserting more resistance into the exciter field, and bringing the voltage back to its correct value. If the voltage of the alternator drops, the process is reversed. To stabilize the regulated voltage and prevent excessive oscillations or swinging with excitation change, a damping transformer is connected with its primary across the field circuit of the generator being regulated and its secondary in series with the regulator coil. When the excitation changes, a transfer of energy by induction occurs between the primary and secondary circuits. Because of the direction in which the two windings are connected, the energy exerts a damping action on any tendency toward oscillations in the two circuits. The transformer is of special design, having a short air gap in its laminated magnetic circuit. As the transformer is connected between d-c circuits, it does not operate when the system is in a balanced condition.

The advantages of this regulator are as follows: It is simple and direct acting. There are no vibrating contacts, the only moving contacts being the silver buttons supported by the leaf springs. Since the moving element has little inertia and the maximum travel of the driving member is only a fraction of an inch, the regulator functions quickly, and the maintenance is low.

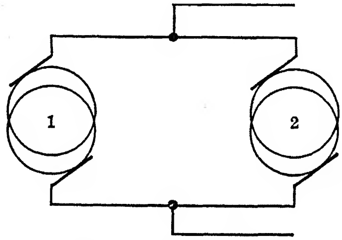
143. Parallel Operation of Alternators.—The same reasons that make it necessary to operate direct-current generators in parallel (see Vol. I, Chap. XIV) apply to alternators. Since there are no commutation difficulties, alternators are made in units of very much greater rating than is possible for direct-current machines. The largest single alternating-current unit at the present time has a rating of 200,000 kva (see p. 2).

In order to operate satisfactorily in parallel, direct-current generators must have drooping voltage characteristics. In order that

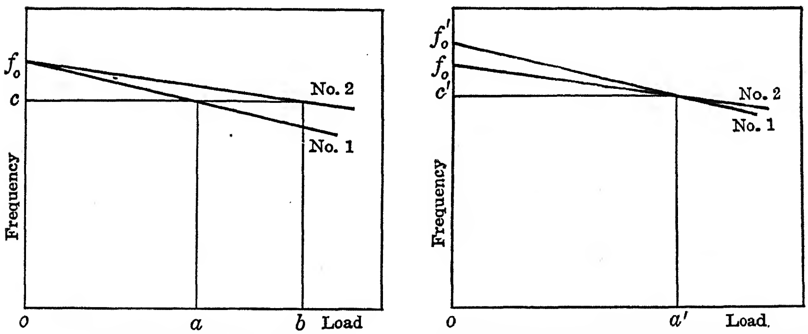
alternators may operate satisfactorily in parallel, their *prime movers* must have *drooping* speed-load characteristics. The reason for this is as follows:

Two alternators 1 and 2, Fig. 205(*a*), which for simplicity are shown as single-phase, are operating in parallel. If they are operating in parallel, the terminal voltage and frequency are necessarily the same.

Figure 205(*b*) shows the speed-load characteristic of each of the prime movers driving the alternators. (Instead of plotting speed in



(*a*) Alternators in parallel



(*b*) Speed-load characteristics (*c*) Change of speed-load characteristic of No. 1

FIG. 205.—Parallel operation of alternators.

rpm, the frequency or electrical speed is plotted. For example, a 6-pole alternator running at 1,200 rpm would have the same electrical speed as an 8-pole alternator running at 900 rpm.)

The load is given in terms of the output of the alternators, the difference between prime-mover output and alternator output being the small losses in the alternators, exclusive of field loss. For clarity, the change of speed is exaggerated.

The speed-load curves of the prime movers are determined by their governors, if they are steam-, water-, or gas-driven units. If they are motor-driven the speed-load characteristics depend on the motor speed-load characteristics.

The prime-mover governors are adjusted, Fig. 205(*b*), so that the no-load frequency f_0 of the two alternators is the same. Under all conditions of load the alternators, being in parallel, must operate at the same frequency.

Let *oc*, Fig. 205(*b*), be the frequency at which the system is operating. By projecting horizontally to intersect the speed-load curves, the load taken by each alternator at this frequency is obtained. *oa* is the load on alternator 1, and *ob* is the load on alternator 2, as both alternators must be operating at system frequency. Let the field of 1 be strengthened by means of its field rheostat. At the same time, weaken the field of 2 so that the line voltage does not change. If these were direct-current generators, generator 1 would immediately take more load. But 1 *cannot take more load* because its prime mover can deliver only the load *oa* at this frequency. Alternator 2 cannot drop any load because its prime mover can deliver only the load *ob* at this frequency. Both alternators must always operate at the same frequency, which is not true of direct-current generators. *Therefore, the kilowatt load delivered by alternators in parallel cannot be shifted appreciably by means of the generator fields.*

To change the kilowatt load of either alternator, the speed-load characteristic of its prime mover must be changed. In engine- and turbine-driven units, this is done by changing the tension in the governor spring or altering in some manner the governing device. Assume, in Fig. 205(*c*), that it is desired to make alternator 1 take the same load as 2. The governor spring of 1 is adjusted so that the characteristic of 1 is raised, as shown in Fig. 205(*c*). Both alternators now deliver the same load *oa′* at a frequency *oc′*. Under the conditions shown, Fig. 205(*c*), the frequency *oc′* is higher than the original frequency *oc* in (*b*).

If the original frequency is to be maintained, the speed-load characteristic of 2 must be lowered at the same time that the characteristic of 1 is raised. Therefore, to adjust the power load between alternators in parallel, the speed-load characteristics of the prime movers must be changed. If the alternators are driven by shunt motors, the speed-load characteristics of the motors may be changed by adjusting the motor field rheostats. It will be noted, in Fig. 205(*c*), that the loads of the two alternators are equal at one frequency only. Also, the no-load frequencies are now different, that of 1 being f_0' and that of 2 still being f_0.

If the speed-load characteristics of the prime movers were flat, the operation of the alternators in parallel would be unstable. That is, very small disturbances or changes of frequency would cause very

large fluctuations in the kilowatt load delivered by each alternator. This condition would result in serious operating difficulties.

144. Synchronizing Power.—It has been shown that direct-current shunt generators operating in parallel are in *stable equilibrium* (see Vol. I, Chap. XIV). That is, any circumstance that tends to throw machines out of parallel is counteracted by reactions opposing this tendency. In the same way, any action tending to throw alternators out of parallel is opposed by reactions that tend to prevent the alternators pulling out. This is most clearly illustrated by the conditions

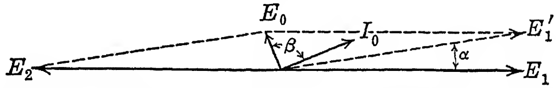


FIG. 206.—Synchronizing current of alternators in parallel.

existing when neither alternator is supplying external load. If the two alternators are considered as a local series circuit, their emfs are in *opposition*. These emfs are represented in Fig. 206 by E_1 and E_2. E_1 and E_2 are equal and opposite, so that the net emf acting in the local circuit of the two alternators is zero. There is, therefore, no current flowing between the alternators, just as there is no current circulating between two batteries having equal emfs and with terminals of like polarity connected together.

Assume that the prime mover of generator 1 speeds up temporarily. The internal induced voltage of generator 1 will advance an angle α with respect to E_2. That is, E_1 will advance to position E_1'. The vector sum of the two alternator emfs E_1' and E_2 will no longer be zero; but, owing to the change in their phase relation, the vector sum of E_1' and E_2 will be E_0.

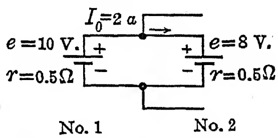


FIG. 207.—Batteries in parallel.

The result is the same as with the two batteries of Fig. 207. Battery 1 has an emf of 10 volts, and 2 has an emf of 8 volts. If the load current is zero, the current circulating between these batteries is found by dividing the sum of the two emfs, giving each the proper sign, by the sum of the resistances of the two batteries. That is,

$$I_0 = \frac{10 + (-8)}{0.5 + 0.5} = 2 \text{ amp.}$$

In the same way, the current circulating between the two alternators is given by the *resultant* voltage divided by the sum of the impedances of the two alternators.

$$\dot{I}_0 = \frac{\dot{E}_1' + \dot{E}_2}{\dot{Z}_1 + \dot{Z}_2}, \qquad |I_0| = \frac{E_0}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}}, \qquad (158)$$

where Z_1, Z_2; R_1, R_2; X_1, X_2 are the impedances, resistances, and reactances of the two machines. As the resistance of an alternator armature is very small compared with its reactance, this circulatory current will lag by an angle β, nearly 90°, with respect to the emf E_0 producing it, as shown in Fig. 206. This causes I_0 to be nearly in phase with the emf E_1'. It therefore puts a power load on alternator 1, which tends to slow it down. On the other hand, I_0 is nearly 180° from E_2, that is, it is acting in opposition to E_2. I_0 therefore develops motor action in alternator 2, as the induced emf acts in opposition to the current. This motor action tends to speed up alternator 2. *Therefore, if two alternators in parallel attempt to pull out of step, a current is developed that circulates between the two machines. This current tends to accelerate the lagging alternator and to retard the leading alternator and so acts to prevent the alternators from pulling out of synchronism.*

If the alternators are operating under load, I_0 merely puts more load on the alternator that tends to lead and takes load from the alternator that tends to lag. The lagging alternator will not ordinarily operate as a motor, as it did under no-load conditions, but as its load is reduced, its angular position will be advanced.

Because I_0 tends to hold the two generators in synchronism, it is called the *synchronizing current.*

145. Reactive Power.—It is stated in Sec. 143 that changing the field current does not vary appreciably the distribution of power load between two alternators. It does, however, affect the current and the reactive volt-amperes (vars) delivered by the two alternators. Figure 208(a) shows the vector diagram for two similar alternators having a common terminal voltage V. The alternators are delivering equal currents I_1 and I_2, which are in phase with the terminal voltage V. The resultant load current is their sum I', which is in phase with V. As the alternators have equal resistances and reactances, their internal emfs E_1 and E_2 are equal. (In this diagram, the alternators are treated with reference to the *external* circuit, in which case the voltages and currents are acting in *conjunction.*)

Let the field of alternator 1 be weakened and that of 2 be strengthened. It has been shown already that this cannot affect appreciably the division of the kilowatt load between the alternators. When the field of alternator 1 is weakened, its *internal emf decreases;* and when the field of 2 is strengthened, its *internal emf increases.* The alternators must continue to have equal *terminal* voltage. It has been shown already that, if an alternator delivers a leading current, its internal emf is less than when it delivers a lagging current (see Sec. 128, p. 199).

Through armature reaction, a leading current in an alternator tends to *strengthen* the field, and a lagging current tends to *weaken* the field.

For alternator 1 to operate with a reduced internal emf, it must deliver a leading current, making E_1, Fig. 208(*b*), less in magnitude than its previous value, Fig. 208(*a*). On the other hand, E_2, Fig. 208(*b*), is greater in magnitude than in Fig. 208(*a*), because alternator 2 now delivers a lagging current. Also, through armature reaction, the leading current in generator 1 tends to strengthen its field, and the lagging current in generator 2 tends to weaken its field. In both cases, the change of flux produced by change in field current is *opposed* by armature reaction. The load current I' cannot change in phase or in magnitude, as the phase and magnitude of I' are determined entirely by the character of the system load. Therefore I_1 and I_2 will adjust

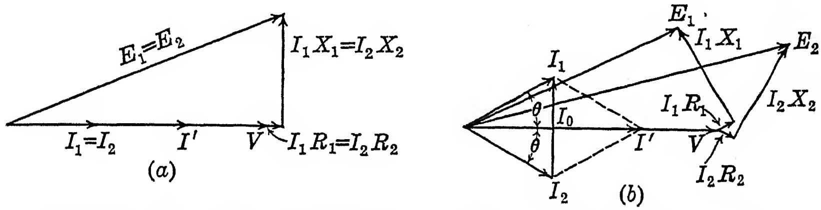


FIG. 208.—Vector diagram of voltages and currents with alternators in parallel.

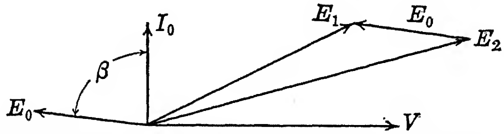


FIG. 209.—Vector diagram showing effect of excitation on alternator circulatory current.

their magnitudes and phase relations so that their vector sum is equal to I' in phase with V. If I_1 and I_2 are equal, as shown in (*b*), they must make equal angles θ with V so that their resultant I' will still lie along V.

It will be noted that each alternator is delivering a larger current than it did before and yet the kilowatt output of each has not changed. This means that the heating (I^2R) loss in each alternator has been increased with a corresponding decrease in efficiency and power rating. This is not, therefore, the best condition of operation.

Figure 209 shows the diagram of Fig. 208(*b*) with the voltage drops eliminated. E_0 is now the *difference* of E_1 and E_2, and I_0, the circulating current, lags E_0 by nearly 90°, as in Fig. 206. Note that I_0, the difference of I_1 and I_2 in Fig. 208(*b*), is nearly in quadrature with the terminal voltage V, so that it transfers practically no power from one alternator to the other. This substantiates what has been demon-

strated already, that changing the field current cannot transfer appreciable power load from one alternator to the other. It does, however, transfer lagging (negative) vars from the overexcited to the underexcited alternator, causing the latter to *deliver* leading, or positive vars.

In the preceding discussion the reactions incident to parallel operation have been analyzed with respect to the leakage-reactance drop IX and the internal induced emf E_a. Synchronous reactance X_s may be substituted for the leakage reactance X if the effect of armature reaction is omitted. The no-load, or excitation, emf E is then obtained, rather than the internal emfs E_1 and E_2.

146. Synchronizing.—Before direct-current generators can be connected safely in parallel, two conditions must be fulfilled. The two terminal voltages must be equal, or substantially so, and the proper polarity must be observed.

The same two conditions must be fulfilled when alternators are connected in parallel. The equality of voltages can be readily determined by connecting a voltmeter first to one alternator and then to the other. The voltmeter, when so connected, does not give any indication of the instantaneous polarity, as the indications of an alternating-current voltmeter are independent of polarity.

Lamps, however, can be used to determine the correct polarity. Figure 210 shows the connections for phasing a 3-phase alternator with the bus bars. A lamp is connected across each pole of the 3-pole switch that connects the alternator to the bus bars. Strictly speaking, the voltage rating of the lamps should be 15 per cent greater than that of either alternator or bus bars. For example, if the system is 220 volts, two 115-volt lamps in series may be used across each pole, although these lamps will be subjected to overvoltage during a part of the synchronizing period. If the incoming alternator is properly connected, the three lamps should all become bright and dim together. If they brighten and grow dim in sequence, this means that the phase rotation of the alternator is opposite to that of the bus bars, so that any two of the leads from the alternator must be reversed.

The lamps flicker at a frequency equal to the *difference* in the frequencies of the alternator and the bus bars. As the frequency of the alternator approaches that of the bus bars, the flicker becomes slower and slower. When the lamps are all dark, the switch may be closed. The fact that the lamps are all dark indicates that the potential difference between each switch blade and its clip is nearly zero and the two alternators are in *opposition* so far as their local series circuits are concerned. Two points across which the potential difference is

zero may be connected without any resulting disturbance, so that the switch now may be safely closed, and the two alternators are thus put in parallel.

The disadvantage of this method is that an incandescent lamp is dark even though a considerable voltage may exist across its terminals and the alternators may be connected in parallel, therefore, when considerable voltage difference exists between them. This may do no harm with slow-speed or small-capacity units; but with high-speed turbine-driven units, which have little armature reactance and are

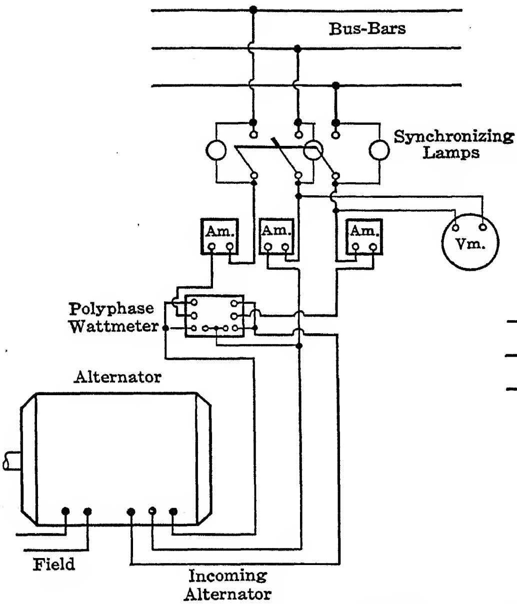


FIG. 210.—Connections for "three-dark" method of synchronizing with lamps.

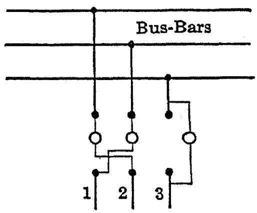


FIG. 211.—Connections for "two-bright-and-one-dark" method of synchronizing with lamps.

quite "sensitive," there may be considerable disturbance if there exists a substantial phase difference at the time of connecting in parallel. Another objection to this "three-dark" method is that the lamps do not show whether the incoming alternator is fast or slow.

The foregoing difficulties may be eliminated in part if the connections of two of the lamps, as 1 and 2, Fig. 211, be crossed. When the alternator and bus bars are in synchronism, 1 and 2 are bright and 3 is dark. As one of the bright lamps is increasing and the other is decreasing in brilliancy near the point of synchronism, it is possible to determine very accurately the instant at which the switch should be closed. This is called the *Siemens-Halske* or *two-bright-and-one-dark*

method. By noting the sequence of brightness of the lamps, it can be determined whether the incoming alternator is fast or slow.

The best method is to use the synchronism indicator, or synchroscope, described in Chap. IV (p. 114). Such an instrument shows accurately the position of synchronism. The synchroscope is connected across one phase only. It is possible that one phase of each alternator may be in synchronism while the other two are out of phase owing to incorrect phase rotation. The correct phase rotation must be determined by lamps or by other means before depending entirely on the synchroscope. Synchronizing lamps are often used in conjunction with a synchroscope so that the operator has a check on the instrument.

In central stations, alternators ordinarily operate at voltages of 600 to 13,800 volts and higher, so that it becomes necessary to use potential and current transformers (p. 300) with the instruments. The lamps and synchroscope then would be connected to the secondaries of the potential transformers, which usually operate at 115 volts or thereabouts. Also, the lamps and synchroscope are usually connected directly to a synchronizing bus, which in turn is connected to the secondaries of the potential transformers whose primaries are connected to the main bus. Connections then are made between the incoming alternator (through the potential transformers) and to the synchronizing bus by means of a plug connector.

147. Hunting.—The driving torque of a reciprocating engine, or of a gas engine, is not uniform during a revolution of the flywheel but varies from zero at the dead centers to a maximum at some intermediate position. Even with a heavy flywheel, this variation of torque may impart impulses to the induced emf, causing it to be ahead of its correct position at some instants and behind it at other instants. This causes large synchronizing currents to flow between alternators in parallel and often causes their rotating members to "oscillate" about their average speed as they are rotating. The angular effect of the crank position can be appreciated when it is realized that in a 60-pole alternator a displacement of 1 mechanical degree, or space degree, in the rotating member makes a difference of 30 electrical degrees in the phase angle of the emf. The impulses often are communicated to the system, causing synchronous motors and converters to oscillate. These oscillations are called *hunting.* Hunting may become serious if the engine governors have a natural frequency of oscillation nearly the same as that of the machine rotors. The oscillations may then become cumulative and may even cause the alternators to go out of synchronism.

Remedies for hunting are to use heavy flywheels, to put dashpots on the engine governors, and to use amortisseur or squirrel-cage windings around the field (see Fig. 159, p. 174). Where several engine-driven units are used, they are often paralleled when their cranks occupy different angular positions. This minimizes the effect of the engine impulses on the system, although this effect is increased so far as the local interchange currents between alternators is concerned.

CHAPTER VIII

THE TRANSFORMER

The static transformer is a device for transferring electrical energy from one alternating-current circuit to another without a change in frequency. This transference is usually, but not always, accompanied by a change of voltage. A transformer may receive energy at one voltage and deliver it at a *higher* voltage, in which case it is called a *step-up* transformer. A transformer may receive energy at one voltage and deliver it at a *lower* voltage, in which case it is called a *step-down* transformer. A transformer may receive energy at one voltage and deliver it at the *same* voltage, in which case it is called a *one-to-one* transformer.

A static transformer has no rotating parts; therefore, it requires little attention, and its maintenance is low. The cost per kilowatt of transformers is low as compared with other apparatus, and the efficiency is much higher. As there are no teeth, slots, or rotating parts, and the windings can be immersed in oil, it is not difficult to insulate transformers for very high voltages.

Because of these many desirable characteristics, the transformer is a very useful piece of apparatus. As it can transform from low to high voltage, and from high to low voltage economically, it is largely responsible for the extensive use of alternating current.

148. Transformer Principle.—The transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both sets of coils are on a common magnetic circuit.

Electromotive forces are induced by a change in flux linkages. In the generator, the flux is substantially constant in magnitude. The flux linking the armature coils is changed by the relative *mechanical* motion of flux and coils. In the transformer, the coils and magnetic circuit are all stationary with respect to one another. The emfs are induced by the change in the *magnitude* of the flux with time. This is illustrated in Fig. 212.

A core such as is shown in Fig. 212 is made up of rectangular stampings of sheet steel, clamped together.

A continuous winding P is placed on one side, or leg, of the iron core. Another continuous winding S, which may or may not have

the same number of turns as P, is shown diagrammatically as being placed on the opposite side, or leg.[1] An alternator A supplies current to the primary winding P. As this winding is linked with an iron core, its mmf produces an alternating flux ϕ in the core. This alternating flux links the turns of the winding S. As this flux is alternating, it induces in the winding S an emf of the same frequency as the flux. Because of this induced emf, the secondary winding S is capable of *delivering* current and energy. The energy, therefore, is transferred from P, the primary, to S, the secondary, by means of the magnetic flux.

At the instant shown in Fig. 212 the upper conductor is positive so that the direction of the flux in the core is clockwise.

The winding P, which *receives* energy, is called the *primary*. The winding S, which *delivers* energy, is called the *secondary*. In a transformer, either winding may be the primary, the other being the secondary, depending upon which winding receives and which delivers energy.

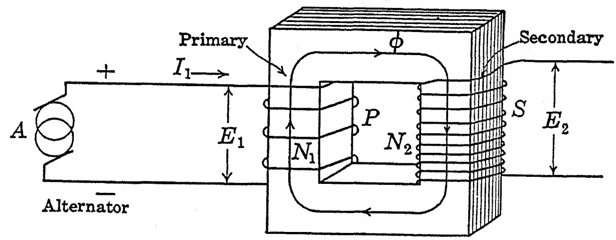


FIG. 212.—Simple transformer, secondary open circuited.

149. Induced Electromotive Force.—The flux ϕ, called the *mutual flux*, in passing through the magnetic circuit formed by the iron core, links not only the turns of the secondary winding S but also the turns of the primary winding P. An emf, therefore, must be induced in both the windings S and P. As this flux ϕ is the same for each of the two windings, it must induce the *same emf per turn* in each winding. The *total induced emf* in each winding then must be proportional to the number of turns in that winding; that is,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}, \qquad (159)$$

where E_1 and E_2 are the primary and secondary *induced* emfs and N_1 and N_2 are the number of turns in primary and secondary, respectively. In the ordinary transformer, the terminal voltage differs

[1] Actually, the primary P and the secondary S will be on the same leg to reduce magnetic leakage (see p. 268 *et seq*).

from the induced emf only by a very small percentage, so that for most practical purposes it may be said that the primary and secondary terminal voltages are proportional to the respective number of turns.

The induced emf in a transformer is proportional to three factors—the frequency f, the number of turns N, and the maximum instantaneous flux ϕ_m. The equation for the induced emf, assuming a sine wave of flux may be derived as follows:

Figure 213 shows the mutual flux ϕ varying sinusoidally with the time. Between points a and b the total change of flux is $2\phi_m$ maxwells. This change of flux occurs in a half-cycle, or in a time $T/2$ sec, where T is the *period*, or the time required for the wave to complete one cycle. The time $T/2$ is equal to $1/2f$ sec.

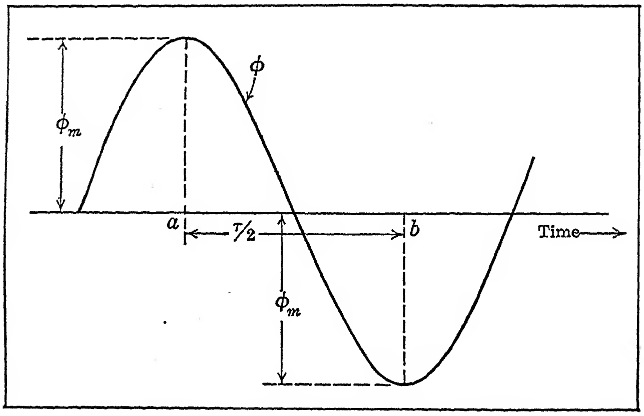


FIG. 213.—Sinusoidal variation of flux with time.

The *average* induced emf is equal to the total change of flux divided by the time (see Vol. I, Chap. IX). That is,

$$e' = -N \frac{2\phi_m}{T/2} 10^{-8} \text{ volts}$$
$$= -N \frac{2\phi_m}{1/2f} 10^{-8} \text{ volts}$$
$$= -4fN\phi_m 10^{-8} \text{ volts}$$

Since with a sine wave the ratio of rms to average volts is 1.11 (see Sec. 7, p. 13), the *rms* induced emf is

$$E = 4.44fN\phi_m 10^{-8} \text{ volts,} \tag{160}$$

the negative sign being dropped. The factor 4.44 is four times the form factor, which is 1.11 for a sine wave (see Chap. I, Sec. 7, p. 13).[1]

[1] Compare (160) with (139) (p. 177), using $2N$ instead of Z in (139).

If the flux varies other than sinusoidally with the time, a factor k_f called the *form factor* must be substituted for 1.11 in Eq. (160).

Equation (160) may be proved more rigorously as follows:

$$\varphi = \phi_m \sin \omega t \tag{I}$$

$$e = -N\frac{d\varphi}{dt}10^{-8} = -N\phi_m\,\omega\cos\omega t(10^{-8}) \text{ volts.} \tag{II}$$

The maximum emf

$$E_m = N\phi_m\,\omega\,10^{-8} = 2\pi f N\phi_m 10^{-8} \text{ volts.}$$

$$E = \frac{2\pi}{\sqrt{2}}fN\phi_m 10^{-8} = 4.44fN\phi_m 10^{-8} \text{ volts.}$$

If the mks system is used, φ and ϕ_m are expressed in webers and (160) becomes

$$E = 4.44fN\phi_m \quad \text{volts.} \tag{160a}$$

The maximum flux $\phi_m = B_mA$, where B_m is the maximum flux *density* and A is the core cross section. (160) may then be written

$$E = 4.44fNB_mA10^{-8} \quad \text{volts.} \tag{161}$$

Frequently, this equation is more convenient to use, for transformer cores are designed on the basis of permissible flux density. The use of (161) is illustrated in the following example:

Example.—The core of a 60-cycle transformer has a net cross section of 20 sq in., and the maximum flux density in the core is 60,000 maxwells per sq in. There are 700 turns in the primary and 70 turns in the secondary.

Determine the induced emf in primary and secondary.

$$E_1 = 4.44 \cdot 60 \cdot 700 \cdot 60{,}000 \cdot 20 \cdot 10^{-8} = 2{,}237 \text{ volts.} \quad \textit{Ans.}$$

$$E_2 = 4.44 \cdot 60 \cdot 70 \cdot 60{,}000 \cdot 20 \cdot 10^{-8} = 223.7 \text{ volts.} \quad \textit{Ans.}$$

Also,

$$E_2 = \frac{2{,}237}{10} = 223.7 \text{ volts.} \quad \textit{Ans.}$$

Equation (I) giving the flux φ is a sine function, and Eq. (II) giving the induced emf is a negative cosine function. Hence, *a sinusoidal emf lags the flux inducing it by* 90°.

150. Ampere-turns.—Figure 214 shows a transformer having a primary and a secondary winding. The directions of the flux, of the voltages, and of the currents, as indicated in the figure, are those at the instant when the upper primary conductor is positive and the current is increasing. Assume, first, that there is no load on the secondary. Under these conditions a very small current I_0 flows in the primary, usually from 1 to 3 per cent of rated current.

This no-load current, I_0, called the *exciting* current, supplies the mmf that produces the mutual flux ϕ and also supplies the core, or no-load, losses (see Sec. 155). It can be resolved therefore into two components, one I_m, in phase with the flux ϕ, which supplies the mmf that produces ϕ, and the other, I_e, in quadrature with I_m, which supplies the losses, Fig. 221 (p. 260). Since the losses are small and the primary circuit is a highly inductive one, I_0 lags the terminal voltage V_1 by nearly 90°. Also, at all ordinary loads the emf E_1, induced in the primary by the mutual flux ϕ, is nearly equal in magnitude to the primary terminal voltage V_1, differing only by the small impedance drop in the primary. Hence, since V_1 is constant, the induced emf E_1 must be nearly so. It follows then from (160) or (160a) that since E_1 is nearly constant the mutual flux ϕ also must be nearly constant at all normal loads and therefore the mmf producing it as well as the

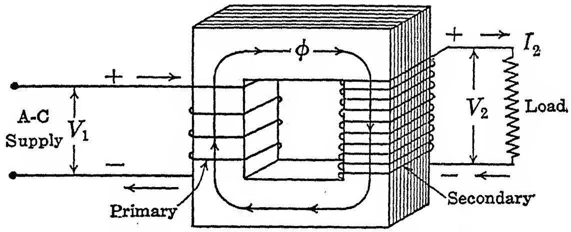


FIG. 214.—Simple transformer, load applied to secondary.

iron losses must be nearly constant. Thus, the exciting current I_0 must be nearly constant at all normal loads on the transformer. Also, I_0 is small in magnitude, ordinarily being only 1 to 3 per cent of the rated current.

The primary induced emf E_1 is a counter emf that opposes current entering the primary and *is analogous* to the counter emf of a motor. At no load it is identical with the emf of self-induction e', Fig. 28 (p. 31), which also opposes the current.

The magnetizing current I_m produces flux ϕ in the core, Fig. 212, the direction of the flux at the instant under consideration being as shown (corkscrew rule). The value of this flux must be such as to make the emf induced in the primary practically equal to the primary terminal voltage.

Apply load to the secondary, Fig. 214. Now there will be a secondary current I_2 whose magnitude and phase relation with respect to the secondary terminal voltage V_2 will be determined by the character of the load. However, at every instant the direction of the secondary current must be such as to oppose any *change* in the flux. This is in accord with Lenz's law that an induced current always has such a

direction as to oppose the cause which produces it. In Fig. 214 it is assumed that the direction of the flux ϕ is clockwise and is increasing. If the secondary current I_2 were producing the flux ϕ, the current by the corkscrew rule would flow *in* at the upper terminal, Fig. 214. Since I_2 opposes the flux ϕ, it must actually flow *out* at the upper terminal. The secondary current I_2 then tends to reduce the value of the mutual flux ϕ in the transformer core. If the flux is reduced, the counter emf of the primary also is reduced. This permits more current to flow into the primary, which supplies the increase in power due to a load being applied to the secondary and also restores the flux to nearly its initial value. This is the sequence of reactions that follow the application of load to the secondary, enabling the primary to take from the line the increased power demanded by the secondary.

The change in the counter emf in the primary from no load to full load is ordinarily about 1 or 2 per cent. As the counter emf is proportional to the mutual flux ϕ, *the value of ϕ, therefore, changes only slightly over the working range of the transformer.* If this flux changes only slightly, the *net* ampere-turns acting on the core remain substantially constant. The increased ampere-turns due to the secondary load must be balanced, therefore, by the additional ampere-turns due to the increase in primary current. Since the flux remains practically constant, it follows that the exciting current must remain substantially constant.

The effect of any *increase* of primary ampere-turns, when not opposed by equal secondary ampere-turns, would be to increase the mutual flux. This would increase the counter emf and tend to cause the primary to deliver power to the power source, which is in violation of the law of the conservation of energy. Any primary ampere-turns in excess of the exciting ampere-turns, therefore, must be balanced by equal and opposing secondary ampere-turns.

The exciting current is of small magnitude and generally differs considerably in phase from the total primary current, as shown by I_0 in Fig. 216 (p. 252). It is usually neglected, therefore, in comparison with the total primary current. If it be neglected, *the primary and secondary ampere-turns are equal and opposite,* and

$$N_1I_1 = N_2I_2.$$

Therefore,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}, \tag{162}$$

that is, *the primary and secondary currents are inversely as the respective turns.*

The above relation also follows from the law of the conservation of energy. If the transformer losses be neglected and unity power factor be assumed,

$$V_1I_1 = V_2I_2,$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

151. Leakage Reactance.—In the preceding discussion, it has been assumed that *all* the flux which links the primary also links the secondary. In practice, it is impossible to realize this condition. All the flux produced by the primary does not link the secondary, but a part completes its magnetic circuit by passing through the air rather than around through the core, as shown by ϕ_1, Fig. 215. That is, between planes a and b, Fig. 215, there is a mmf due to the primary ampere-turns, plane a being at a higher magnetic potential than plane b at

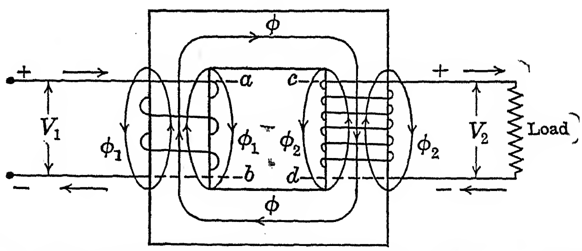


FIG. 215.—Mutual flux, primary-leakage flux, and secondary-leakage flux in transformer.

the instant shown. This mmf is proportional to the primary current and tends to send flux from a to b through the air and around through the core. That part of the flux which passes from a to b through the air follows a magnetic circuit that is acted upon by the primary ampere-turns only. This flux ϕ_1 is called the *primary leakage flux. It is proportional to the total ampere-turns of the primary alone,* as the secondary turns do not link the magnetic circuit of ϕ_1, which, therefore, induces an emf in the primary but not in the secondary. The flux ϕ_1 is in time phase with the total primary current I_1. The emf e_1 induced by ϕ_1 must lag ϕ_1 and I_1 by 90° (see p. 247). The emf necessary to balance this counter emf is opposite and equal to it and, therefore, leads the current I_1 by 90°. As this emf, induced by the primary leakage flux, is proportional to the current and lags it by 90°, it is nothing more than a reactance voltage and is denoted by $-I_1X_1$. The component of line voltage that balances this emf is $+I_1X_1$, Fig. 216(a). A reactance drop exists in a transformer primary, therefore, in precisely the same manner that a reactance drop exists in an alterna-

tor armature. The effect of the primary leakage flux, therefore, is to induce an emf that opposes the current *to* the transformer.

The mmf of the secondary coil, Fig. 215, acting alone, is such that the top of the coil is at a higher magnetic potential than the bottom of the coil. That is, plane c is at a higher magnetic potential than plane d; therefore, a flux ϕ_2 tends to pass from c to d through the air, as shown. Flux ϕ_2 is called the *secondary leakage flux*. As its path is not linked by the primary, *the secondary leakage flux is proportional to the secondary ampere-turns only.* ϕ_2 induces an emf in the secondary, lagging the secondary current I_2 by 90° [see e_2, Fig. 216(a)]. This is also a reactance voltage, and the component that balances it leads the secondary current by 90°. This last voltage is denoted by I_2X_2, Fig. 216(a). The secondary reactance opposes the current flowing *out* of the secondary, just as the armature reactance of an alternator opposes the current flowing out of the armature. Both the primary and secondary leakage reactances of the transformer have the same effect on the regulation of the transformer as the armature leakage reactance of the alternator has on the regulation of the alternator.

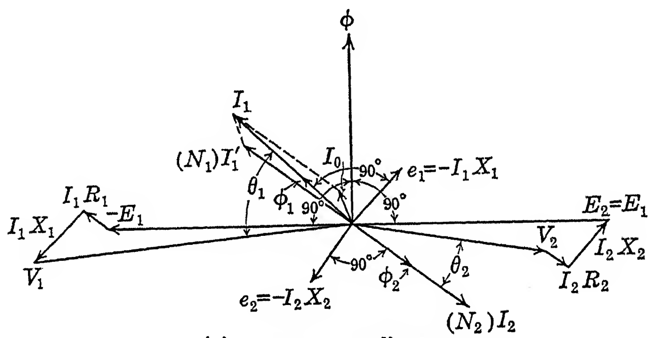
In that part of the core which is surrounded by the secondary winding, the mutual flux ϕ and the secondary leakage flux ϕ_2 are shown in opposition. As ϕ is produced by the joint ampere-turns of primary and secondary and ϕ_2 by the ampere-turns of the secondary alone, ϕ and ϕ_2 are almost never in direct opposition but are usually out of phase by an angle less than 180°, Fig. 216(a). Two separate fluxes in the core actually do not exist at the same instant, but merely the resultant flux, found by combining ϕ and ϕ_2. The primary leakage flux ϕ_1 and the secondary leakage flux ϕ_2 have the same general direction in the space between the primary and secondary coils.

In actual transformers, the primary and secondary windings are not placed on separate legs, as in Figs. 212, 214, 215; for as they are widely separated, large primary and secondary leakage fluxes would result. These large leakage fluxes would cause the transformer regulation to be too poor for commercial use. To reduce the leakage, the primary and secondary should be interleaved. Each is usually split, therefore, into a number of coils, and alternate primary and secondary coils are placed close together, as in Figs. 226 to 228 (pp. 269 and 270).

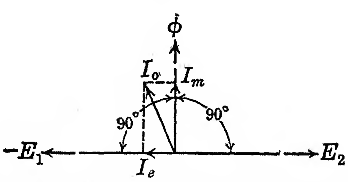
Hence, in actual transformers, the leakage-flux paths are not so simple as those indicated in Fig. 215. Because of the small spaces between primary and secondary windings, the paths of the leakage fluxes are much more restricted. The entire primary and secondary leakage fluxes ϕ_1 and ϕ_2, moreover, link, not all the turns of their respective windings, but only a portion of them. The equivalent

effect of ϕ_1 and ϕ_2, however, is readily determined in the ordinary transformer by simple measurements, as is described later.

152. Transformer Vector Diagram.—Figure 216(a) shows the relations existing among the currents, voltages, and fluxes in a transformer, when the secondary is delivering a current I_2 at terminal voltage V_2 and power factor cos θ_2. A one-to-one ratio of transformation is assumed, in order that the lengths of all the vectors in the diagram shall be of the same order of magnitude. The same diagram may be made applicable to any ratio of transformation, merely by multiplying the proper vectors by the ratio of transformation.



(a) Complete vector diagram



(b) Energy and magnetizing components of no-load or exciting current

FIG. 216.—Vector diagrams for transformer.

The secondary induced-emf vector E_2 along the X-axis is chosen as the reference vector. The secondary current vector I_2 lags V_2 by the angle θ_2, the power-factor angle of the secondary load. The secondary resistance drop I_2R_2, in phase with I_2, is added vectorially to V_2. The secondary leakage flux ϕ_2 is in time phase with I_2 and induces an emf e_2 lagging the flux ϕ_2 by 90°. This emf opposes the current to the external circuit and must be counteracted by a component of the secondary induced emf, or I_2X_2, leading I_2 by 90°. The induced emf E_2 of the secondary is determined by adding vectorially to V_2 the secondary resistance drop I_2R_2, and the secondary reactance drop I_2X_2, due to ϕ_2, in quadrature with I_2 and leading. As both the primary and secondary induced emfs are induced by the same mutual flux ϕ, Figs. 212 and 214, and as both windings have the same

number of turns, since the ratio is one-to-one, the primary and secondary induced emfs will be equal in magnitude and will be in phase with each other. Therefore, $E_1 = E_2$. An emf induced by a flux varying sinusoidally with time is a sine wave and lags the flux by 90° (Sec. 19, p. 30; also, rigorous proof, p. 247). In Fig. 216(a), therefore, the mutual flux ϕ *leads* the induced emfs E_1 and E_2 by 90°.

The line must supply a voltage equal to the primary induced emf and in opposition thereto, before current can flow *into* the primary. This is analogous to the direct-current motor, where the line must supply a voltage equal to the counter emf and in opposition thereto, before current can flow into the armature. A voltage or emf $-E_1$, therefore, opposite and equal to E_1, must first be supplied by the line. The emf $-E_1$ is the *counter* emf of the primary, discussed in Sec. 150.

If the mutual flux ϕ is not to change appreciably, Sec. 150, the primary must supply a sufficient number of ampere-turns to balance the ampere-turns of the secondary. These primary ampere-turns and secondary ampere-turns are equal and opposite. If there are N_2I_2 ampere-turns in the secondary, therefore, there must be at least an equal number of ampere-turns in the primary to balance these. These primary ampere-turns $(N_1)I_1'$, Fig. 216(a), are 180° out of phase with $(N_2)I_2$. It is not customary to show the ampere-turns on the diagram, however, but only the currents, Fig. 216. The ampere-turns may be obtained by multiplying each current by its proper number of turns as indicated by the terms N_1 and N_2, which are shown in parentheses. [In Fig. 216(a), $N_1 = N_2$.]

Since N_1 and N_2 are numerics, the vectors that represent $(N_1)I_1'$ and $(N_2)I_2$ will be changed only in magnitude but not in phase if (N_1) and (N_2) are omitted. Hence the vectors $(N_1)I_1'$ and $(N_2)I_2$ to scale can represent also the primary and secondary currents. In addition to the $(N_1)I_1'$ ampere-turns of the primary, there must be ampere-turns N_1I_m to produce the mutual flux ϕ (Sec. 150), where I_m is the magnetizing component of the exciting current I_0. Also, there must be an energy component of I_0 to supply the core losses. This is illustrated in Fig. 216(b), which shows I_m in phase with the mutual flux ϕ (N_1 being omitted) and I_e, the energy component of I_0, which supplies the core losses and is in phase with $-E_1$. The exciting current I_0, the resultant of I_m and I_e, is in (b) and is also in the vector diagram in (a).[1]

[1] Owing to the fact that the permeability of the iron of the transformer varies widely during each cycle, the magnetizing current I_m is not sinusoidal, Fig. 104 (p. 118), and strictly cannot be represented by a vector. However, I_m is so small compared with the load current that negligible error results if its equivalent sine wave or fundamental component is represented by a vector.

The total primary current I_1 is the vector sum of I_0 and I_1'.

The primary leakage flux ϕ_1 is in phase with I_1 and induces emf $e_1 = -I_1X_1$. The primary terminal voltage V_1 may now be found by adding to $-E_1$, vectorially, I_1R_1 in phase with I_1 and I_1X_1 leading I_1 by 90°. The angle θ_1 between V_1 and I_1 is the power-factor angle of the transformer.

By means of the vector diagrams, Figs. 216(a) and 217, it becomes possible to determine the *regulation* of transformers. This is discussed in detail in Sec. 157 (p. 262).

In most transformer vector diagrams, it is necessary to exaggerate greatly the magnetizing-current and voltage-drop vectors. For example, I_0 is 1 to 3 per cent of I_1; I_2R_2, 1 per cent of V_2; etc. If these quantities be drawn in their actual proportions, they will be too small to be significant. *Hence, changes in* $-E_1$ *and* E_2 *with changes in load are usually very small.*

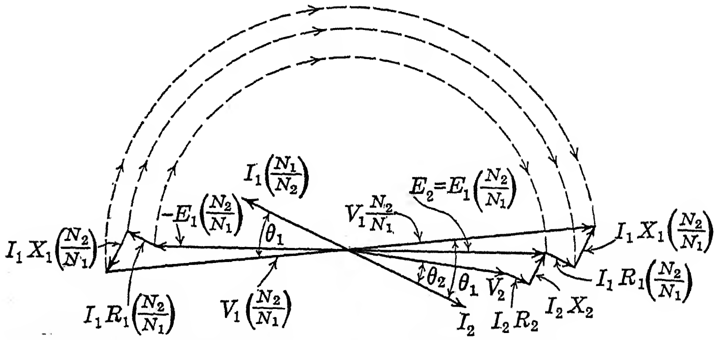


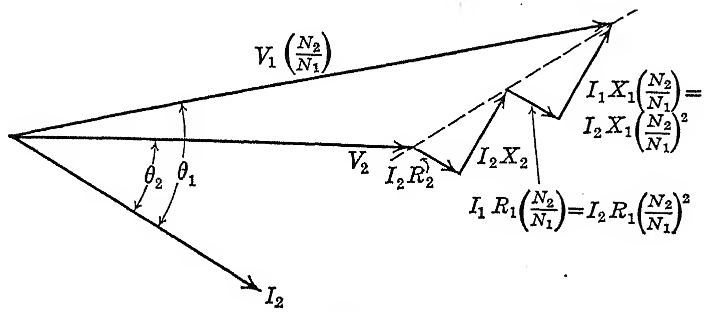
FIG. 217.—Transformer diagram with primary voltages rotated to secondary side of diagram.

153. Simplified Diagram.—The diagram of Fig. 216(a) may be materially simplified if the exciting current I_0 be neglected. As I_0 is usually 1 to 3 per cent of I_1 and the two are considerably out of phase, I_0 may be neglected ordinarily without serious error. Figure 217 shows the diagram of Fig. 216 with I_0 omitted. The ratio of transformation, however, is no longer one to one. The primary has N_1 turns, and the secondary N_2 turns. Hence the ratio of transformation is N_1/N_2. With usual transformers it is not practicable to use the same scale for primary and secondary voltages and currents. For example, with only a 20-to-one ratio of transformation, either one scale would be so small as to be useless or the other so large as to be impracticable.

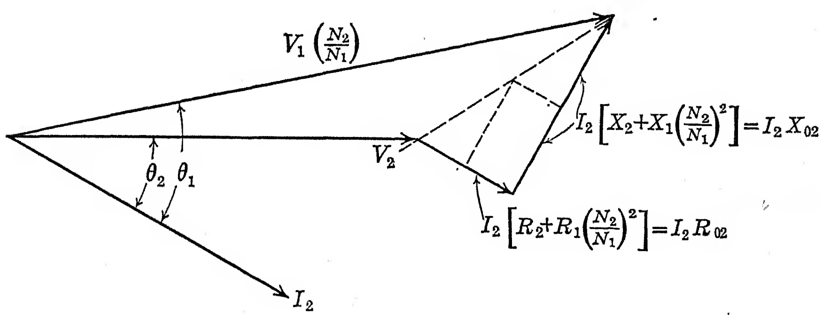
In Fig. 217, V_2, I_2, θ_2 are given, and E_2 is obtained by adding vectorially I_2R_2 and I_2X_2 to V_2. Each of the primary voltages, how-

ever, is multiplied by the inverse ratio of transformation N_2/N_1, and the primary current is multiplied by N_1/N_2, the ratio of transformation. Hence, $E_1(N_2/N_1) = E_2$, etc., and now both primary and secondary vectors are of the same order of magnitude. Thus the diagram for *any* ratio of transformation can be made substantially the same as that for a one-to-one ratio.

Referring to Fig. 217, $-E_1(N_2/N_1)$ is 180° from E_2 and is equal in magnitude to E_2; $I_1(N_1/N_2)$ is 180° from I_2; $I_1R_1(N_2/N_1)$ is 180° from



(a)



(b)

FIG. 218.—Equivalent diagram of transformer.

I_2R_2; and $I_1X_1(N_2/N_1)$ is 180° from I_2X_2. If, therefore, the entire left-hand side of the diagram be rotated through 180° about the origin, Fig. 217, $-E_1(N_2/N_1)$ and E_2 coincide and $I_1R_1(N_2/N_1)$ and $I_1X_1(N_2/N_1)$ become parallel to I_2R_2 and I_2X_2, respectively.

The right-hand side of the diagram, Fig. 217, now gives a simple method for determining the regulation of the transformer, as will be shown in the following sections.

154. Equivalent Resistance and Reactance.—Figure 218(a) gives the right-hand side of the transformer diagram of Fig. 217, E_2 and

$E_1(N_2/N_1)$ being omitted. The resistance drop $I_1R_1(N_2/N_1)$ is parallel to I_2R_2; and since $I_1 = (N_2/N_1)I_2$, this resistance drop is also equal to $I_2R_1(N_2/N_1)^2$. Likewise, the reactance drop $I_1X_1(N_2/N_1)$, parallel to I_2X_2, is equal to $I_2X_1(N_2/N_1)^2$.

The two separate resistance drops may be combined into a single resistance drop; the two separate reactance drops likewise may be combined into a single reactance drop, Fig. 218(*b*), without in any way affecting the relation of V_2 to $V_1(N_2/N_1)$.

The equivalent resistance drop, for the transformer as a whole, becomes $I_2[R_2 + R_1(N_2/N_1)^2]$; the equivalent reactance drop becomes $I_2[X_2 + X_1(N_2/N_1)^2]$.

The quantity

$$R_2 + R_1\left(\frac{N_2}{N_1}\right)^2 = R_{02} \tag{163}$$

is the *equivalent resistance* of the transformer referred to the *secondary*.

The quantity

$$X_2 + X_1\left(\frac{N_2}{N_1}\right)^2 = X_{02} \tag{164}$$

is the *equivalent reactance* of the transformer referred to the *secondary*.

The secondary voltage vectors on the right-hand side might equally well be rotated 180° and then combined with the primary voltage vectors on the left-hand side. Since primary and secondary are interchangeable,

$$R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 = R_{01}, \tag{165}$$

$$X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 = X_{01} \tag{166}$$

are the *equivalent resistance* and *equivalent reactance* referred to the *primary*.

It follows that

$$\frac{R_{02}}{R_{01}} = \frac{X_{02}}{X_{01}} = \left(\frac{N_2}{N_1}\right)^2. \tag{167}$$

The equivalent impedance referred to the primary

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2}. \tag{168}$$

The equivalent impedance referred to the secondary

$$Z_{02} = \sqrt{(R_{02})^2 + (X_{02})^2}. \tag{169}$$

Also,

$$\frac{Z_{01}}{Z_{02}} = \left(\frac{N_1}{N_2}\right)^2. \tag{170}$$

That is, the equivalent resistance, reactance, and impedance, referred to the primary, are to the equivalent resistance, reactance, and impedance, referred to the secondary, as the ratio of primary to secondary turns *squared.*

Consider the copper loss in a transformer

$$P_c = I_1^2R_1 + I_2^2R_2 \text{ watts.}$$

Since $I_2 = I_1(N_1/N_2)$, and $I_1 = I_2(N_2/N_1)$

$$P_c = I_1^2\left[R_1 + R_2\left(\frac{N_1}{N_2}\right)^2\right] = I_1^2R_{01} \tag{171}$$

$$= I_2^2\left[R_2 + R_1\left(\frac{N_2}{N_1}\right)^2\right] = I_2^2R_{02} \text{ watts.} \tag{172}$$

That is, the total copper loss in a transformer is equal to the primary current squared, multiplied by the equivalent resistance of the transformer referred to the primary; likewise, the total copper loss in a transformer is equal to the secondary current squared, multiplied by the equivalent resistance of the transformer referred to the secondary.

It is thus seen that the equivalent resistance of a transformer, when used in conjunction with the current in the side to which this resistance is referred, may be used to determine the equivalent resistance drop in both primary and secondary combined; the equivalent resistance may be used to determine the total copper loss in the transformer. The equivalent reactance may be used in a similar way to determine the equivalent reactance drop in the transformer.

It is interesting to note that, if the copper losses of primary and secondary are equal,

$$I_1^2R_1 = I_2^2R_2,$$

$$\frac{R_1}{R_2} = \frac{I_2^2}{I_1^2} = \left(\frac{N_1}{N_2}\right)^2, \tag{173}$$

or the primary and secondary resistances are proportional to the squares of their numbers of turns. (173) also holds when the mean lengths of primary and secondary turns are equal, the primary and secondary current densities in the copper also being equal.

Example.—A 50-kva 4,400- to 220-volt transformer has a primary resistance and reactance of 3.45 and 5.40 ohms, respectively. The secondary resistance and reactance are 0.0085 and 0.014 ohm, respectively. Determine (*a*) equivalent resistance referred to primary; (*b*) equivalent resistance referred to secondary; (*c*) equivalent reactance reterred to both primary and secondary; (*d*) equivalent impedance referred to both primary and secondary; (*e*) total copper loss, using individual resistances of two windings and using equivalent resistance referred to each side.

Primary current $I_1 = \dfrac{50{,}000}{4{,}400} = 11.36$ amp.

Secondary current $I_2 = \dfrac{50{,}000}{220} = 227$ amp.

Ratio of transformation $\dfrac{N_1}{N_2} = \dfrac{4{,}400}{220} = \dfrac{20}{1}$.

(a) $R_{01} = 3.45 + (2\%_1)^2\, 0.0085 = 3.45 + 3.40 = 6.85$ ohms. *Ans.*

(b) $R_{02} = 0.0085 + (1/20)^2\, 3.45 = 0.00850 + 0.00863 = 0.0171$ ohm. *Ans.*

Also,

$$R_{02} = R_{01}\left(\frac{1}{20}\right)^2 = \frac{6.85}{400} = 0.0171 \text{ ohm } (check).$$

(c) $X_{01} = 5.40 + (2\%_1)^2 0.014 = 5.40 + 5.60 = 11.00$ ohms. *Ans.*

$X_{02} = 0.014 + (1/20)^2 5.40 = 0.014 + 0.0135 = 0.0275$ ohm. *Ans.*

Also,

$$X_{02} = \left(\frac{1}{20}\right)^2 X_{01} = \frac{11.00}{400} = 0.0275 \text{ ohm } (check).$$

(d) $Z_{01} = \sqrt{(6.85)^2 + (11.0)^2} = 12.96$ ohms. *Ans.*

$Z_{02} = \sqrt{(0.0171)^2 + (0.0275)^2} = 0.0324$ ohm. *Ans.*

Also,

$$Z_{02} = Z_{01}\left(\frac{1}{20}\right)^2 = \frac{12.96}{400} = 0.0324 \text{ ohm } (check).$$

(e) $P_c = (11.36)^2 3.45 + (227)^2 0.0085 = 883$ watts. *Ans.*

$P_c = I_1^2 R_{01} = (11.36)^2 6.85 = 883$ watts. *Ans.*

$P_c = I_2^2 R_{02} = (227)^2 0.0171 = 883$ watts. *Ans.*

The equivalent resistance, reactance, and impedance referred to either side may be used in determining the transformer characteristics, such as regulation, efficiency, etc. (see Secs. 156 and 157).

155. Open-circuit Test.—Figure 219 shows a transformer having the low side connected to an alternating source of supply and the high side open-circuited. Either an autotransformer or a voltage divider is a means of varying the voltage supplied to the low side of the transformer, Fig. 219. A voltmeter, an ammeter, and a wattmeter are connected in the primary circuit. The voltmeter gives the voltage across the primary terminals, the ammeter gives the no-load current, and the wattmeter reads the power taken by the transformer under these conditions. As connected, however, the ammeter measures also the current to the wattmeter potential circuit and to the voltmeter. Since the core loss ordinarily is small, this error cannot be neglected as a rule. Either these two potential circuits should be opened when the ammeter is read, or correction should be made. Likewise, the wattmeter as connected measures the power loss in its own current coil and the power to the voltmeter, and correction is usually necessary (see p. 98). The power (corrected) goes to supply the primary I^2R-loss and the core loss of the transformer. As the exciting current is very small, the primary I^2R-loss due to it may be

neglected. The wattmeter, therefore, when corrected for instrument losses, measures the transformer core, or excitation, loss. If the primary voltage be varied and the core loss be determined for different values of voltage, a curve is obtained showing the relation of core loss to voltage. At no-load, the flux is practically proportional to the terminal voltage, as the primary impedance drop due to the no-load

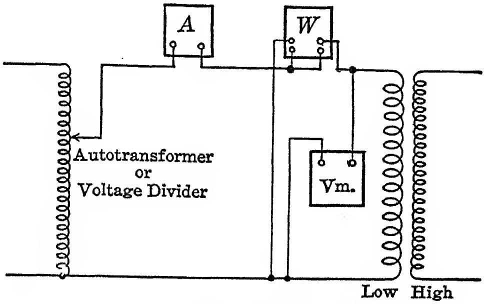


FIG. 219.—Connections for open-circuit test.

current is negligible [see Eq. (160), p. 246]. The eddy-current loss varies as the square of the flux and hence of the voltage and the hysteresis loss as the 1.6 power of the flux and hence of the voltage. The core loss will increase, therefore, nearly as the square of the voltage, as shown in Fig. 220(*a*).

Transformers are designed usually so that the most economical use of materials is obtained. The core is operated, therefore, at a flux

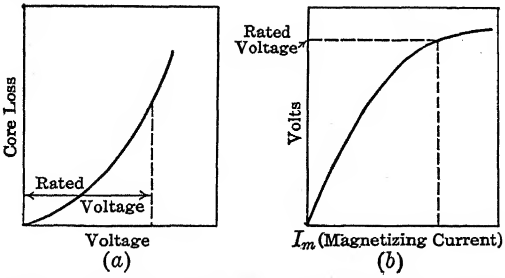


FIG. 220.—(*a*) Relation of core loss to voltage in transformer; (*b*) relation of magnetizing current to voltage in transformer.

density as high as the allowable core loss will permit. A study of Fig. 220(*a*) shows that a slight increase of voltage, above rated voltage, produces a large percentage increase in core loss. As most commercial transformers are rated by their maximum safe operating temperatures, this increased core loss may cause overheating of the transformer. The effect, therefore, of operating such transformers at overvoltage is to produce a marked increase in temperature.

If the magnetizing current be plotted as abscissas and the voltage as ordinates, a saturation curve similar to that of Fig. 220(*b*) is obtained. The point marked "rated voltage" is the point on the saturation curve at which transformers are generally operated and is well beyond the knee of the curve. Outside the question of increased core loss, the usual transformer cannot be operated at a voltage very much in excess of its rated voltage, for the magnetizing current increases very rapidly with small increase in voltage, Fig. 220(*b*).

The flux density in the core is determined primarily by the permissible core loss. Open-hearth annealed sheet steel, such as is used in dynamos, can be used for transformer cores. For a given flux density and frequency, however, silicon steel has much less core loss per unit volume than open-hearth steel, the effect of the silicon being to increase the electrical resistance and, hence, to reduce the eddy-current loss. Because of its small core loss, silicon steel may be operated safely at high flux densities. The greater cost of silicon steel is more than offset by the saving in iron and copper and in the general reduction of the transformer dimensions.

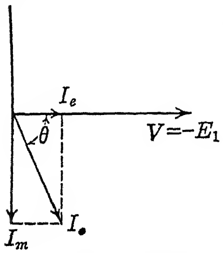


FIG. 221.—Magnetizing and core-loss currents in transformer.

To obtain the value of the magnetizing current, the exciting current I_0, measured by the ammeter, Fig. 219, should be resolved into two components, one of which is in phase with the voltage $-E_1$ or V and is shown as I_e in Fig. 221 ($-E_1$ and V are practically equal at no-load). This current $I_e = I_0 \cos \theta$ is the *energy* component of the exciting current and supplies the core losses. The quadrature component

$$I_m = I_0 \sin \theta$$

is the true magnetizing current, shown plotted in Fig. 220(*b*). In most commercial transformers, $I_0 = I_m$ very nearly [also see Fig. 216(*b*)].

156. Short-circuit, or Impedance, Test.—Figure 222 shows the transformer of Fig. 219 reversed and the low side short-circuited. The reversal is made in order that the line current may not be excessive and also in order that a reasonable voltage drop may be obtained. In a transformer, the impedance drop seldom exceeds 5 per cent of the rated voltage. If the 2,200-volt side of a transformer, Fig. 222, be used as the primary, the voltage necessary to send rated current through the windings on short circuit is about 5 per cent of 2,200, or 110 volts, which is a standard voltage for instrument coils. If the secondary of

the transformer were rated at 220 volts, its voltage at short circuit would be only 11 volts and also the current would be high. At this low voltage, high precision would not be readily obtainable with ordinary instruments.

When the primary current is I_1 amp, Fig. 222, the secondary current I_2 is equal to $I_1(N_1/N_2)$ amp. An ammeter to measure I_2 is therefore not necessary. In fact, such an ammeter, particularly if used in connection with a current transformer, may introduce more error than it is intended to correct. The power to the transformer, Fig. 222, goes to supply three losses: the primary copper loss, $I_1^2R_1$; the secondary copper loss, $I_2^2R_2$; and the core loss at short circuit. As in the open-circuit test, instrument losses usually are not negligible, and correction should be made if necessary. The core loss is negligible, as 5 per cent primary voltage means only about 2.5 per cent of the rated value of flux, since approximately half the impressed voltage on short circuit is consumed in the primary impedance drop. The core loss at 2 or 3 per cent of the rated flux is so small as to be negligible, for the core loss varies nearly as the square of the flux, Fig. 220(a). The power at short circuit, therefore, is

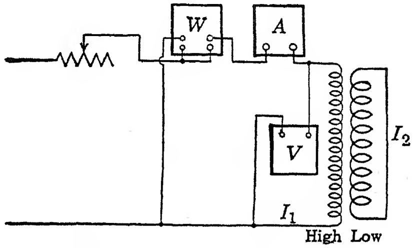


FIG. 222.—Connections for short-circuit test of transformer.

$$P_z = I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02},$$

where R_{01} and R_{02} are the transformer *equivalent effective resistances* referred to the primary and secondary, respectively (Sec. 154).

$$R_{01} = \frac{P_z}{I_1^2}. \tag{174}$$

$$R_{02} = \frac{P_z}{I_2^2}. \tag{175}$$

The values of equivalent effective resistances found in this manner may be checked with the values determined by measuring the resistance of each winding with direct current and applying (163) or (165) (p. 256). The ratio of equivalent effective to equivalent ohmic resistance so determined varies from a few per cent greater than unity in smaller transformers to as high as 20 to 25 per cent in transformers having conductors of large cross section.

Figure 223 shows the equivalent-circuit vector diagram for the short-circuit test. This diagram is that of Fig. 218 except that now V_2 equals zero and all quantities are referred to the primary side. It will be recognized, in Fig. 223, that the entire voltage E_z is consumed in the impedance drops of the two windings. That is,

$$E_z = I_1 Z_{01} = \sqrt{I_1^2\left[R_1 + R_2\left(\frac{N_1}{N_2}\right)^2\right]^2 + I_1^2\left[X_1 + X_2\left(\frac{N_1}{N_2}\right)^2\right]^2}$$
$$= I_1\sqrt{R_{01}^2 + X_{01}^2}.$$

Hence,

$$Z_{01} = \frac{E_z}{I_1}; \tag{176}$$

where Z_{01} is the equivalent impedance of the transformer referred to the primary side. Also, from Eq. (170) (p. 256),

$$Z_{02} = Z_{01}\left(\frac{N_2}{N_1}\right)^2. \tag{177}$$

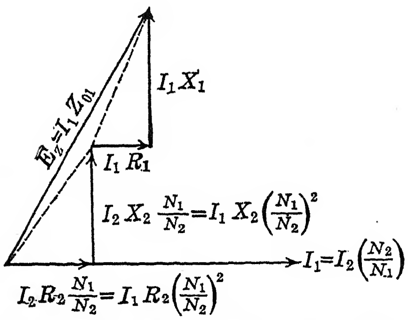


FIG. 223.—Vector diagram for short-circuited transformer.

The equivalent resistance [(174) and (175)] and the equivalent impedance [(176) and (177)] being known, the equivalent reactance is readily found.

$$X_0 = \sqrt{Z_0^2 - R_0^2} \tag{178}$$

for either primary or secondary side.

In making the short-circuit and the open-circuit tests, the question of instrument losses should be investigated and correction made if this be found necessary. As the losses in a transformer are very small, the power taken by the instruments may be a considerable percentage of the power being measured.

157. Regulation.—[1]*The regulation of a constant-potential transformer is the change in secondary voltage, expressed in per cent of rated secondary voltage, which occurs when rated kva output at a specified power factor is reduced to zero, with the primary impressed voltage maintained constant.*

Thus, if the no-load secondary terminal voltage is V_2' volts, the regulation is

$$\frac{V_2' - V_2}{V_2}\,100 \text{ per cent.} \tag{179}$$

[1] American Standards for Transformers, Regulators, and Reactors, ASA C57.1, 57.2, 57.3 of 1942, Standard 1.065. This corresponds to Standard 15.20.235 of the American Standard Definitions of Electrical Terms C42 (1941).

In a one-to-one transformer the regulation becomes,

$$\frac{V_1 - V_2}{V_2} 100 \text{ per cent.} \tag{180}$$

practically.

Knowing the equivalent resistance and the equivalent reactance of the transformer, it is possible to determine the regulation. Referring to Fig. 218(*b*) (p. 255), the no-load secondary voltage is $V_1(N_2/N_1)$, since at no-load the impedance drop in the transformer is negligible. Hence, with lagging current,

$$V_1\left(\frac{N_2}{N_1}\right) = \sqrt{(V_2 \cos \theta_2 + I_2R_{02})^2 + (V_2 \sin \theta_2 + I_2X_{02})^2}, \tag{181}$$

$$\text{Regulation} = \frac{V_1(N_2/N_1) - V_2}{V_2} 100 \text{ per cent.} \tag{182}$$

Also,

$$V_1\left(\frac{N_2}{N_1}\right) = V_2 + I_2(\cos \theta_2 \mp j \sin \theta_2)(R_{02} + jX_{02}), \tag{183}$$

the + sign being used for leading current. Also, in (181), with leading current, $+I_2X_{02}$ becomes $-I_2X_{02}$. (181) and (183) should be compared with (147) and (148) (pp. 201 and 202); also, see Eqs. (149) (150) (p. 203).

(181) to (183) are applicable to the primary side if the subscripts are changed. The regulation is the same in either case.

The vector diagram, Fig. 218(*b*), shows that after the correction for ratio is made the transformer is a low impedance in series with its load. This is illustrated in Fig. 224, in which the secondary quantities are expressed in terms of the primary (see example, Sec. 158, p. 264).

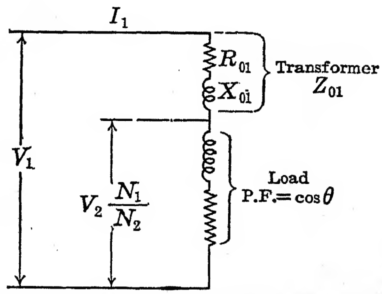


FIG. 224.—Equivalent circuit of transformer.

158. Efficiency.—The ordinary transformer has a very high efficiency (see table, p. 268). Hence, the efficiency cannot be determined with high precision by direct measurement of output and input, since the losses are of the order of only 1 to 3 per cent. The difference between the readings of the output and input instruments is then so small that an instrument error as low as 0.5 per cent would cause an error of the order of 15 per cent in the losses. It is easier, therefore, and more precise to determine the efficiency from the losses.

It has been pointed out that with constant voltage the mutual

flux of the transformer is practically constant from no-load to full load. It usually does not vary more than from 1 to 3 per cent. The core loss, therefore, is practically constant at all loads and may be determined by the open-circuit test, Fig. 219. For most purposes, it is necessary merely to measure the loss at the rated voltage of the transformer.

The only other losses are the primary and secondary copper losses. These can be calculated readily, knowing the resistances of primary and secondary, or they may be computed from the equivalent resistance determined at short circuit. The efficiency of the transformer then may be computed, since the losses are known. That is, the efficiency

$$\eta = \frac{V_2 I_2(\text{P.F.})}{V_2 I_2(\text{P.F.}) + \text{core loss} + I_1^2 R_1 + I_2^2 R_2} \tag{184}$$

$$= \frac{V_2 I_2(\text{P.F.})}{V_2 I_2(\text{P.F.}) + \text{core loss} + I_2^2 R_{02}}. \tag{185}$$

Example.—A 20-kva 2,200- to 220-volt 60-cycle distributing transformer is tested for efficiency and regulation as follows: A wattmeter, an ammeter, and a voltmeter are used to measure the input to the low side, the high side being open-circuited, Fig. 219 (p. 259). The corrected instrument readings are 148 watts; 4.2 amp; 220 volts. The transformer is then reversed, the low side being short-circuited and 86.0 volts applied to the high side. Instruments having the proper ranges are connected in circuit, Fig. 222 (p. 261). The corrected instrument readings are now 360 watts; 10.5 amp; 86.0 volts.

Determine (*a*) transformer core loss; (*b*) equivalent resistance referred to high side; (*c*) equivalent resistance referred to low side; (*d*) equivalent reactance referred to high side; (*e*) equivalent reactance referred to low side; (*f*) regulation of transformer at 0.8 power factor, lagging current; (*g*) efficiency of transformer at full load and half load, at 0.8 power factor, lagging current.

(*a*) Core loss is given directly by the corrected wattmeter reading and is 148 watts. *Ans.*

(*b*) $R_{01} = \dfrac{360}{(10.5)^2} = 3.26$ ohms. *Ans.*

(*c*) $R_{02} = 3.26\left(\dfrac{220}{2{,}200}\right)^2 = 0.0326$ ohm. *Ans.*

(*d*) $Z_{01} = \dfrac{86.0}{10.5} = 8.19$ ohms.

$X_{01} = \sqrt{(8.19)^2 - (3.26)^2} = \sqrt{56.45} = 7.51$ ohms. *Ans.*

(*e*) $X_{02} = 7.51\left(\dfrac{220}{2{,}200}\right)^2 = 0.0751$ ohm. *Ans.*

(*f*) High-side quantities will first be used.

The rated high-side current is $\dfrac{20{,}000}{2{,}200} = 9.10$ amp.

$$V_1 = \sqrt{(2{,}200 \cdot 0.8 + 9.10 \cdot 3.26)^2 + (2{,}200 \cdot 0.6 + 9.1 \cdot 7.51)^2}$$
$$= \sqrt{5{,}131{,}000} = 2{,}265 \text{ volts [see (181)].}$$

$$\text{Regulation} = \frac{2{,}265 - 2{,}200}{2{,}200} = 0.0295, \text{ or } 2.95\% \text{ [Eq. (179)].} \quad \textit{Ans.}$$

The same result is obtained using the low-side constants.

$$V_1\left(\frac{N_2}{N_1}\right) = \sqrt{(220 \cdot 0.8 + 91.0 \cdot 0.0326)^2 + (220 \cdot 0.6 + 91.0 \cdot 0.0751)^2}$$
$$= \sqrt{51{,}131} = 226.5 \text{ volts.}$$

$$\text{Regulation} = \frac{226.5 - 220}{220}100 = 0.0295, \text{ or } 2.95\%. \quad \textit{Ans.}$$

Also, using (183),

$$V_1\left(\frac{N_2}{N_1}\right) = 220 + 91.0(0.8 - j0.6)(0.0326 + j0.0751)$$
$$= 220 + 6.50 + j72.5 = 226.5 + j7.25.$$
$$\left|V_1\left(\frac{N_2}{N_1}\right)\right| = \sqrt{(226.5)^2 + (7.25)^2} = 226.5 \text{ volts } (\textit{check}).$$

(g) Full-load efficiency (using high-side constants)

$$\eta = \frac{20{,}000 \cdot 0.80}{20{,}000 \cdot 0.80 + 148 + (9.10)^2 \cdot 3.26} = \frac{16{,}000}{16{,}420} = 0.974, \text{ or } 97.4\%. \quad \textit{Ans.}$$

$$\eta = \frac{10{,}000 \cdot 0.80}{10{,}000 \cdot 0.80 + 148 + (4.55)^2 \cdot 3.26} = \frac{8{,}000}{8{,}216} = 0.973, \text{ or } 97.3\%. \quad \textit{Ans.}$$

The same values of efficiency are obtained if the low-side current and resistance are used.

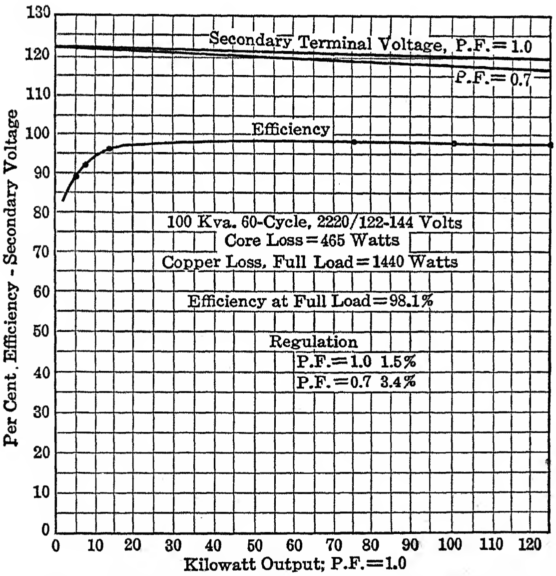


FIG. 225.—Characteristics of 100-kva 60-cycle transformer.

Figure 225 shows the voltage characteristic and the efficiency, plotted against load, of a 100-kva 60-cycle 2,200/122- to 144-volt transformer. It will be noted that the efficiency is high and is practically constant from one-eighth load to 25 per cent overload.

Regulations and efficiencies for typical transformers are given on p. 268.

159. Unit Values.—Regulation computations are frequently simplified if unit values are used. The resistance and reactance drops are then expressed as a proportion of rated voltage and are called *resistance factor* and *reactance factor*.[1] For example, the *resistance factor*

$$r = \frac{I_2R_{02}}{V_2};$$

the *reactance factor* $x = I_2X_{02}/V_2$; the power factor $\cos\theta_2 = p$; $\sin\theta_2 = q = \sqrt{1-p^2}$.

Factoring the right-hand side of Eq. (181) with V_2,

$$V_1\left(\frac{N_2}{N_1}\right) = V_2\sqrt{\left(\cos\theta_2 + \frac{I_2R_{02}}{V_2}\right)^2 + \left(\sin\theta_2 + \frac{I_2X_{02}}{V_2}\right)^2}$$
$$= V_2\sqrt{(p+r)^2 + (q+x)^2}.$$
$$\text{Regulation} = \frac{V_2\sqrt{(p+r)^2+(q+x)^2} - V_2}{V_2}$$
$$= \sqrt{(p+r)^2 + (q \pm x)^2} - 1 \tag{186}$$

the minus sign being used for leading current.

In ASA C57[1] this is given as

$$\text{Regulation} = \sqrt{(r+p)^2 + (x \pm q)^2} - 1. \tag{187}$$

When the power factor changes from lag to lead, θ_2 changes sign. Since the second parenthetical term is squared, it is immaterial which term is negative.

Example.—Determine (*f*) in the example, Sec. 158.

$$r = \frac{91.0 \cdot 0.0326}{220} = 0.0135.$$
$$x = \frac{91.0 \cdot 0.0751}{220} = 0.0310.$$
$$\text{Regulation} = \sqrt{(0.0135 + 0.8)^2 + (0.0310 + 0.6)^2} - 1$$
$$= \sqrt{(0.8135)^2 + (0.6310)^2} - 1$$
$$= \sqrt{1.060} - 1 = 0.0295 \ (\textit{check}).$$

160. All-day Efficiency.—Transformers frequently must be connected to give service 24 hr a day, although the load may be light for a considerable portion of the time. This is particularly true of lighting transformers, which must be ready always to give service but which are lightly loaded except during the house-lighting period. The

[1] See ASA C57, p. 69 (footnote, p. 262).

performance of a transformer under these conditions must be judged by its *all-day* efficiency. This is equal to the ratio of the *energy* output over 24 hr to the *energy* input over the same period.

Example.—Determine the all-day efficiency of the transformer (p. 264) with the following unity-power-factor loads: five-fourths load, 2 hr; full load, 6 hr; half load, 5 hr; one-eighth load, 7 hr; no-load, 4 hr.

Energy output

$$W_1 = 25{,}000 \cdot 2 + 20{,}000 \cdot 6 + 10{,}000 \cdot 5 + 2{,}500 \cdot 7$$
$$= 237{,}500 \text{ watt-hr.}$$

Energy input

$$W_2 = 237{,}500 + [(5/4 \cdot 9.10)^2 \cdot 3.26 \cdot 2] + (\overline{9.10}^2 \cdot 3.26 \cdot 6) + \left[\left(\frac{9.10}{2}\right)^2 \cdot 3.26 \cdot 5\right] + \left[\left(\frac{9.10}{8}\right)^2 \cdot 3.26 \cdot 7\right] + (148 \cdot 24)$$
$$= 237{,}500 + 844 + 1{,}620 + 338 + 30 + 3{,}550$$
$$= 237{,}500 + 6{,}380 = 244{,}000 \text{ watt-hr, nearly.}$$
$$\eta = \frac{237{,}500}{244{,}000} = 0.974.$$

At rated load and unity power factor the efficiency is 0.979. Note that at rated kilowatt load the core loss is 148 watts and the copper loss is 270 watts. Since the core loss exists 24 hr a day irrespective of load, it is desirable for most services that the core loss be substantially less than the copper loss in order to obtain high all-day efficiency. In the foregoing example even with small core loss, the copper loss is only 55 per cent of the 24-hr loss.

161. Commercial Transformers.—Constant-potential transformers are classified, for convenience, as *power transformers* and *distribution transformers*. Power transformers are used on primary transmission lines for the transformation and distribution of relatively large amounts of power. Distribution transformers are used for distributing the power from transmission lines and networks for local consumption. In the table on page 268 are the efficiencies and other operating data for typical power and distribution transformers.

TYPES OF TRANSFORMERS

162. Core- and Shell-type Transformers.—Transformers are divided into two general types, the core type and the shell type. These two types differ in the arrangement of the iron and copper with respect to each other.

In the core type, the winding or the copper nearly surrounds the iron core. Figures 212, 214, 215 (pp. 245, 248, 250) are diagrammatic

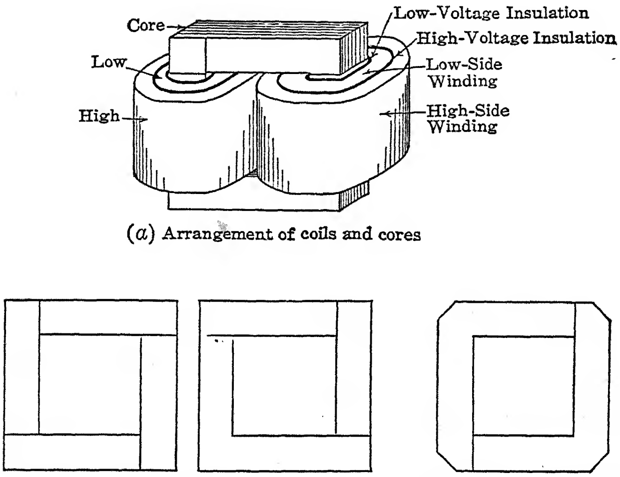
SINGLE-PHASE 55°C SELF-COOLED OIL-IMMERSED TRANSFORMER
Manufactured by Westinghouse Electric Corporation

Kva	Iron loss, watts	Total loss, watts	% efficiency			% regulation		Total weight, lb
			Half load	Three-quarter load	Full load	P.F. 1.0	P.F. 0.8	
2,400-volt primary: 60 cycles, 480/240-volt secondary								
5	40	136	97.3	97.4	97.2	2.0	2.4	175
15	83	320	98.1	98.1	97.9	1.7	2.2	282
50	186	810	98.6	98.5	98.4	1.4	2.3	811
200	760	2,865	98.7	98.7	98.6	1.2	3.3	2,685
500	1,590	6,240	98.9	98.8	98.7	1.0	3.6	4,960
13,200-volt primary: 60 cycles, 2,400-volt secondary								
1,000	2,730	10,350	99.00	99.00	98.00	1.0	4.2	7,600
2,500	5,750	21,850	99.18	99.17	99.07	0.95	4.2	15,000
5,000	10,600	39,700	99.29	99.28	99.21	0.81	4.1	26,000
2,400-volt primary: 25 cycles, 240/120-volt secondary								
5	48	193	96.7	96.7	96.3	3.0	3.0	310
50	253	1,303	97.9	97.8	97.5	2.2	3.2	1,550
200	700	4,320	98.4	98.2	97.9	2.0	4.4	4,800
13,200-volt primary: 25 cycles, 2,400-volt secondary								
1,000	2,210	12,910	98.98	98.84	98.62	1.49	4.7	14,500
5,000	9,700	60,000	99.11	98.99	98.81	1.2	4.5	43,000

merely, representing core-type transformers. Figure 226(*a*) shows the general arrangement of the core-type transformer. The core is in the form of a hollow square made up of sheet-steel laminations about 14 mils thick. The core may be built up with rectangular laminations the joints of which butt in the individual layers. The joints lap in alternate layers, however, Fig. 226(*b*), which shows the arrangement of joints in two adjacent layers. When a large number of transformers of a single type are being manufactured, the laminations are often made of L-shaped punchings, Fig. 226(*c*), stacked so that the joints alternate, (also see Fig. 229).

If a transformer were made with the primary and secondary coils on separate legs, as indicated in Figs. 212, 214, 215, an unsatisfactory transformer would result, as the large leakage flux for both primary and secondary would give very poor regulation. By having both a pri-

mary and a secondary on each leg, as shown in Figs. 226(*a*) and 228(*a*), the leakage flux is reduced to a small value. If the high-voltage winding were placed next the core, it would be necessary to insulate it from both the core and the low-voltage winding and two layers of



(*a*) Arrangement of coils and cores

(*b*) Arrangement of joints in adjacent lamination layers

(*c*) Core made of L-shaped punchings

FIG. 226.—Core-type transformer.

high-voltage insulation would be necessary. By placing the high-voltage winding outside and around the low-voltage winding, only one layer of high-voltage insulation, that between the high- and low-voltage windings, is necessary.

In the shell type of transformer, the iron nearly surrounds the copper, Fig. 227. The core has the form of a figure 8. The entire flux passes through the central part of the core, but outside of this central core it divides, half going in each direction, Fig. 227. The coils are made in the shape of pancakes and are usually wound with strip copper. These coils are taped, and the primary and secondary usually are stacked so that each primary is adjacent to a secondary. In this manner the leakage flux of both primary and secondary is reduced to a very small value. The secondaries, or low-side coils, are placed adjacent to the iron in order to minimize the amount of high-voltage insulation required.

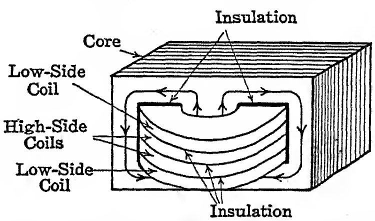
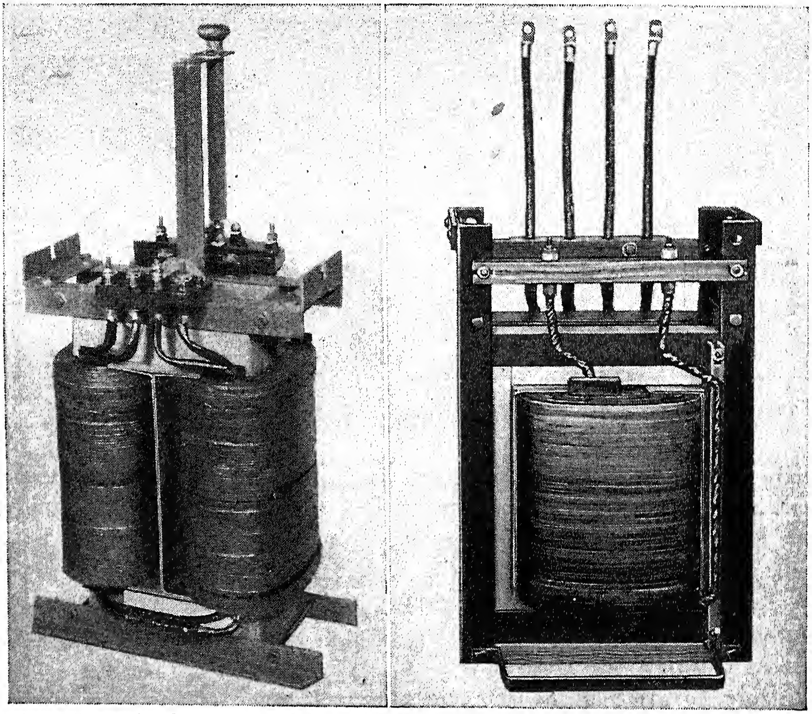


FIG. 227.—Coils and core, shell-type transformer.

To compare the two types of transformer in general, the core type has a longer mean length of core and a shorter mean length of turn. The core type has a lesser cross section of iron and therefore a greater number of turns. The core type is better adapted for high voltage since there is more space for insulation. In the shell type the coils are better braced mechanically so that they are less easily displaced by the high electromechanical forces that frequently develop during short circuits.



(*a*) Core type (*b*) Shell type

FIG. 228.—Core and windings of distribution transformers. (*Wagner Electric Corp.*)

In Fig. 228(*a*) are shown the assembled core and winding of a core-type distribution transformer and in Fig. 228(*b*) are shown the assembled core and windings of a shell-type distribution transformer. The method of clamping the laminations and bringing out the leads should be noted. In (*a*) the vertical insulated handle is a rotary tap changer by means of which the ratio may be changed without draining the oil. A position finder permits the desired ratio to be selected.

163. Wound-core Transformer.—In the construction of a transformer it is necessary that the coils and the core link each other. In

the past the almost universal method has been to assemble the flat laminations by hand through and around preformed coils (Sec. 162). This method has the disadvantage of the considerable cost of punching the laminations, including the necessary wastage and the time and labor involved in the assembly. Other disadvantages are the magnetic reluctance at the joints and the difficulty of making a mechanically rigid core construction.

The improved methods of producing silicon steel have so decreased the losses and increased the permeability in the direction of grain orientation that there have developed methods of assembling the cores with the coils so as to avoid flux transverse to the grain. For example, in the pack-rolling process of manufacture, silicon-steel sheets, about 120 by 30 in., are stacked and rolled hot. This process develops an orientation of the grain in the direction of rolling that makes the losses 10 to 15 per cent greater when the flux is transverse to the direction of

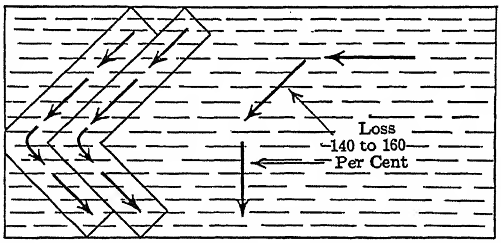


FIG. 229.—Flux and grain direction in high-reduction cold-rolled silicon steel.

the grain than when it is in that direction. However, when the flux makes an angle of 45° with the direction of grain orientation, the losses are increased but little. Hence the L-stampings, Figs. 226(*c*) and 229, are cut at 45°. In order to avoid waste at the end of the sheet, a process was developed whereby the sheets were welded end to end. However, in L-stampings cut at 45° the flux at the corners crosses the grain at right angles, introducing the extra loss.

The foregoing rolling process was followed by the cold-rolled high-reduction process whereby the steel could be produced in continuous sheets, 100 ft or more in length. However, although this process improved markedly the magnetic characteristics in the direction of rolling or of the orientation of the grain, the losses transverse to this direction, or even at 45° were now increased by 40 to 60 per cent, Fig. 229. This made it uneconomical to use such steel for either straight or L-stampings.

Hence, in order to utilize the excellent magnetic characteristics of this new type of steel, it became necessary to develop cores in which the direction of flux was always in the direction of the grain. The

only types of core that meet this condition are either the wound-strip core or the bent-iron core. Many attempts had been made to utilize the wound-strip core, but it had been found impracticable to assemble the coils with the cores on a production basis. To be sure, such wound cores had been used in transformers where the turns were few and winding could be done by hand, such, for example, as the bar-type current transformers (p. 301) and also the bushing type.

Recently the General Electric Company, the Line Material Company, and the Westinghouse Electric Corporation have each developed a practicable method of manufacturing transformers using wound-strip cores.

164. Spirakore Transformer.—In the General Electric *Spirakore* transformer the coils and core are assembled by coiling the steel strip or ribbon rapidly about preformed transformer coils. In this method,

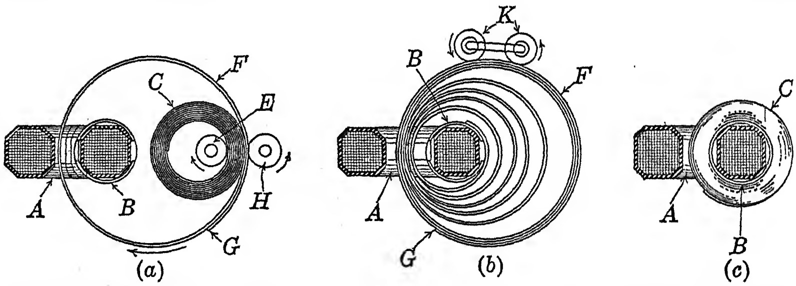


FIG. 230.—Winding core about coil in spirakore transformer. (*General Electric Co.*)

the high-reduction cold-rolled steel, which is received in rolled sheets, is unrolled and at the same time is slitted to the required width and is then wound rapidly in a tight spiral on a metal mandrel the cross section of which is identical in size and shape with the coil section about which it is to be ultimately assembled. The winding process distorts or so strains the steel that its magnetic properties are seriously impaired. However, the strains are relieved and the good magnetic properties are restored by annealing at high temperature, and in the annealing process at the same time the steel becomes "set" to the coiled form or shape to which it is wound.

The annealed coiled steel strip must now be wound about the preformed coil assembly without subjecting the steel to sufficient mechanical strain to impair its magnetic properties again. In its final disposition, the turns must have the same relation to one another as when the coil was removed from the annealing furnace, that is, the inside turn must still be inside and the outside turn outside. Also, the dimensions of the coil must not change. The method of doing this is shown in Fig. 230. In (a) the wound core C, which has been taken from the

annealing furnace and the mandrel removed, is placed over a roller E, and the end of the strip is carried in a clockwise direction through the coil window A and brought around to form a rather large loop F, the end of the strip being tack-welded to the next underlying turn at G. The core and large loop are then rotated rapidly by means of the rollers E and H. When this process is completed, the core is left in the form of a large, loose coil linking the preformed winding coils, as shown in (*b*). Two rollers K then press in against the outer turn, then rotate rapidly and tighten the loose spiral into a compact core C,

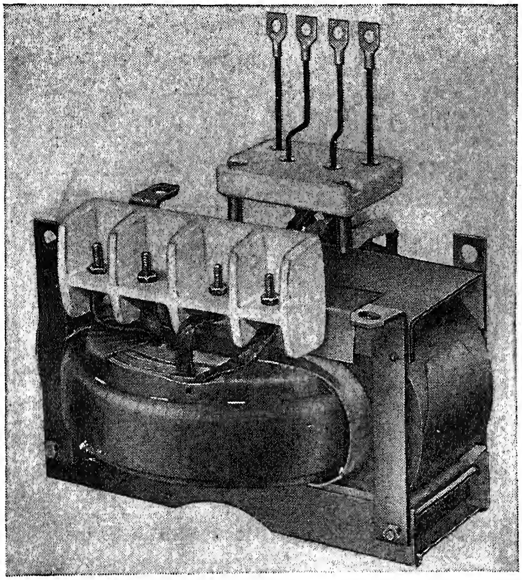


Fig. 231.—Assembled core and coils for single-phase Spirakore distribution transformer, 3 kva, 6,900/115/230 volts, 60 cycles. (*General Electric Co.*)

shown in (*c*), fitting tightly around the coil section at B. The spiral is prevented from unrolling by spot-welding the end to the adjacent turn. By reason of the set imparted by the heat-treatment, the core tends to assume the same shape that it had when annealed. The foregoing process is repeated in winding the second spiral-wound core about the other side of the coil assembly through the other half of the window A. In Fig. 231 are shown a completed coil and core assembly. The advantages of this construction are that the direction of the flux is always in the direction of the orientation of the grain so that the iron losses are a minimum, there is no wastage, practically there are no joints in the path of the flux, the core is rigid, there are no strains produced in the iron as in the clamping of punchings, and the assembly

requires only a fraction of the time required to stack punchings by hand. This type of core is used for distribution transformers of 5 kva and under.

Spirakore for Rectangular Coils.—In the larger transformers, better characteristics and performances are obtained when the coils are rectangular in cross section and the ratio of length to width is large. This necessitates that the opening, or "window," in the core also must be rectangular. This makes the foregoing method of core and coil assembly impracticable. However, a second method involving, in part, the foregoing method has been developed, whereby large rectangular wound-iron cores may be assembled with preformed coils without straining the iron so as to impair its magnetic properties.

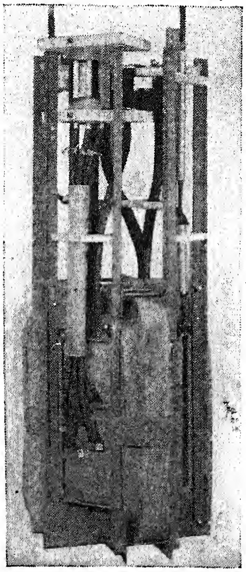


FIG. 232.—Assembled core and coils for single-phase, distributed-core Spirakore transformer, 500 kva, 2,400/4160 Y to 240/480 volts, 60 cycles. High-voltage-side view. (*General Electric Co.*)

As with the round spiral core, the steel sheet as received is slitted to the desired width and is then wound on an iron mandrel having a section that is the same as the section of the coil assembly about which the core is to be assembled. As before, the core is then annealed to remove the strains and to "set" the core. After annealing, however, the core is unwound and simultaneously cut in two-turn lengths, no care being taken as to exactly where the cuts are made. When being cut the two-turn lengths are "nested" in the order in which they are cut. They are then assembled by hand about the preformed coil assembly, the innermost two-turn length first being "snapped" about the coil assembly and the others in order. These two-turn lengths spring readily back into their original form without being subjected to undue bending strains. A butt joint occurs in alternate layers, but, owing to the manner of cutting, the butt joints are more or less staggered throughout the core and have negligible effect on the permeance. This type of core has most of the advantages of the round core, that is, the direction of the flux is the same as the direction of the orientation of the grain, the permeance is high, the core is rigid, and clamping strains are negligible.

In Fig. 232 are shown the assembled core and coils of a 500-kva Spirakore transformer with coils of rectangular cross section. The

magnetic circuit of the transformer consists of a central core and four outer legs 90° apart. This is a *distributed-core* type of transformer. Several different core sections are used so that when the four cores are combined at the center the central core will nearly fill the circular window of the winding. The cores of the transformer of Fig. 232 are wound with two different widths of strip, Q and P, Fig. 233, giving cross sections that are readily combined in the central core.

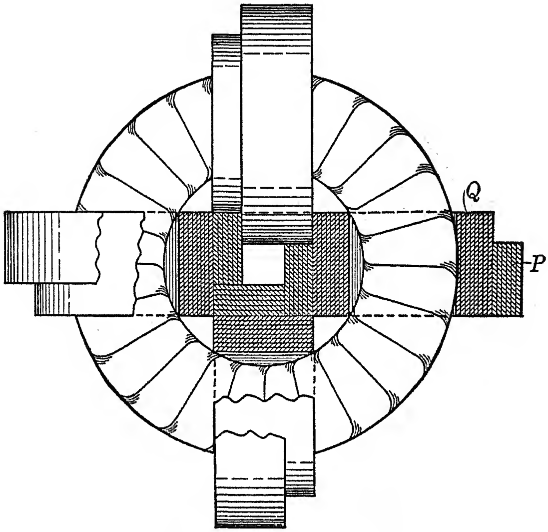


FIG. 233.—Core sections, Spirakore transformer with distributed core. (*General Electric Co.*)

165. Hipersil Cores.—Hipersil (*high-per*meability *sil*icon steel) is a grain-oriented low-loss steel developed by the Westinghouse Electric Corporation. Unlike the Spirakore and Line Material core the wound core is cut transversely and opened in order to assemble it with the windings, which have been made into preformed coils. The material is prepared by (1) winding the steel strip continuously on a mandrel having the dimensions of the coil leg; (2) annealing the core at high temperature to remove the winding strains and make the form permanent; (3) while at high temperature, vacuum impregnating with a molten glass, which when cooled leaves a microscopically thin glass layer between laminations, which insulates them and because it is so hard and adhesive makes the core a solid unit that can be cut and machined; (4) cutting the core into two parts and machining the ends to make closely coinciding surfaces when reassembled. The two parts of the core are then assembled with the preformed coils, after which they are clamped and held tightly together with metal bands.

Figure 234 illustrates the Hipersil assembly.